THE ASTROPHYSICAL JOURNAL

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS

Agents

THE CAMBRIDGE UNIVERSITY PRESS
LONDON AND EDINBURGH

THE MARUZEN-KABUSHIKI-KAISHA
TOKYO, OSAKA, KYOTO, FUKUOKA, SENDAI

THE MISSION BOOK COMPANY
SHANGHAI

KARL W. HIERSEMANN
LEIPZIG

THE

ASTROPHYSICAL JOURNAL

An International Review of Spectroscopy and Astronomical Physics

VOLUME XLIV

JULY–DECEMBER, 1916

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS

Published
July, September, October, November, December, 1916

Composed and Printed By
The University of Chicago Press
Chicago, Illinois, U.S.A.

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THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME XLIV | JULY 1916 | NUMBER 1

ANOMALOUS DISPERSION IN THE SUN. II

By SEBASTIAN ALBRECHT

In an earlier paper[1] with the foregoing title the writer has shown that lines in the solar spectrum which have close companions of sufficient intensity are displaced relative to wave-lengths obtained in the laboratory, and that these displacements are strikingly in accord, both in direction and in relative amounts, with the requirements of the anomalous-dispersion theory. The validity of these results and the methods by which they were derived have been brought into question in valued contributions by Dr. T. Royds[2] and by Director Evershed.[3] That the slight regard in which Evershed and Royds hold Rowland's Table of Solar Spectrum Wave-Lengths is not justified seems amply shown by the confirmation, by means of the Rowland tables, of the Mount Wilson pressure groups and pressure displacements. Further, when Dr. Royds questions the validity of the purely relative comparison between Rowland's solar wave-lengths, which are still the best that have been published, and the best laboratory wave-lengths of the iron arc, he assumes the responsibility of *proving* in what essential way his so-called direct-comparison methods are "more direct" for such a purely relative comparison of what may be termed "internal"

[1] *Astrophysical Journal*, **41**, 333, 1915.

[2] *Kodaikanal Observatory Bulletin*, No. 48, 1915.

[3] *The Observatory*, **39**, 59, 1916.

differences in the two systems. Systematic differences between the two systems as a whole or for any continuous section of spectrum are in no way involved. In either case the comparison is between two entirely distinct things, a solar spectrum and a laboratory spectrum. In the so-called direct method the advantage—which is of minor importance in this comparison—of having the two distinct spectra obtained nearly simultaneously and with the same instrument is in a measure counterbalanced by the disadvantage that the conditions under which the solar spectrum is taken are not necessarily the most favorable for the arc spectrum.

The principal object of this article is to offer a material addition to the evidence contained in the previous paper rather than indulge in a lengthy discussion of opinions. Before introducing the new material it may be well to recall the principal facts brought out in the former paper, p. 354:

The evidence adduced above is quite definite as to the observed facts. These are: Fraunhofer lines, as given in Rowland's tables, are displaced when they have close companions; the displacement is (*a*) toward the violet when the companion is toward the red, and (*b*) toward the red when the companion is toward the violet. The displacement in (*a*) is greater than in (*b*), and in both it increases as the separation between the lines diminishes.

I wish next to quote from a recent contribution by Sir Joseph Larmor:

Thus *very close spectrum lines ought to repel each other* to a degree that experiment alone can reveal.

But this conclusion implies that the adjacent lines represent independent vibrations. If they are two components of a single compound vibration of the molecule, the argument is not applicable.

If an iron line in the solar spectrum has a very close adjacent line *due to another substance*, while in the arc spectrum that substance is not present, then a displacement of the solar line relative to the arc line may be looked for.

The displacement is—at any rate, on this theory—proportional to the density of the substance that is present and produces the adjacent disturbing line.[1]

In the original statement of the anomalous-dispersion theory Julius made no distinction, I believe, between pairs of lines in which the components are due to one element and those in which they are due to two different elements. In a more recent article[2]

[1] *The Observatory*, **39**, 103, 1916. [2] *Astrophysical Journal*, **43**, 49, 1916.

he says that the dispersion theory requires "the existence of a definite mutual influence of Fraunhofer lines separated from each other by very short distances, irrespective of the elements to which the lines are due"; and in a footnote, "provided those elements coexist in the mixtures at the same levels."

A provisional test for the distinction made by Larmor can be made from the data contained in my Tables V and VI (*op. cit.*, pp. 348 and 349). These are substantially reproduced in Tables I and II of the present article, with the modifications that the elements are indicated to which the companion lines are due, and the displacements[1] are inclosed in parentheses in the cases in which the companion lines are also due to iron. The lines thus differentiated are distributed among all the pressure groups.

As was pointed out in the former article (pp. 352 and 353), the results are not dependent upon the adopted system of weighting. However, since the displacement is distinctly a function of the separation of the line from its companion, and since the *certainty* of such displacement should be greater when the companion line is relatively strong, the weighted results deserve the preference. No proportionality between displacement and intensity was proposed, nor was there any intention to convey the impression that such proportionality exists. Nevertheless, the great multiplicity of very faint lines in the solar spectrum necessitates the tacit adoption of the principle that lower limits of intensity, and probably also of relative intensity, of the companion line exist, below which the line is incapable of producing the anomalous-dispersion effect here considered. With the moderate amount of material which is at present available it is not feasible to define, with great precision, such a lower limit. However, as can readily be verified by means of Tables I, V, and VI of the former article, moderately definite and, I believe, safe lower limits of intensity and of relative intensity of the companion line were adopted for the exclusion of lines for which the "certainty" of power to produce such an effect seemed too small. In view of the fact that it has been possible to determine a fairly satisfactory relation between separation and displacement,

[1] For this column I retain the heading "To Reduce Point to Curve" and the signs of the original tables. The term "displacements" as used in this article shall be irrespective of sign.

we may also hope for an actual determination, in the future, of approximate lower limits for the intensities. The essential requisite I believe to be a several-fold accumulation of data.

TABLE I

Fe Lines with Companions toward the Red in the Solar Spectra

λ Rowland	Elements and Intensities (Companion: Line)	Separation	Weight	To Reduce Point to Curve	Group	Remarks
	Fe	A		A		
3705.708	2: 9	0.14	1	+.007	*a*1	
3735.014	Fe 4:40	0.47	½	(+.004)	*b*1	
3743.508	Ti 2: 6	0.12	1	+.006	*b*1	Also Fe—Burns
3834.364	Mn 4:10	0.14	1	+.009	*b*1	And 3:10, 0.36 to vi.
3997.547	2: 4	0.09	1	+.001	*b*4	And 1:4, 0.29 to vi.
4191.595	Fe 3: 6	0.25	1	(+.011)	*a* or *b*	
4204.101	La 4: 3	0.06	2	+.048	*b*3	And 2:3, 0.37 to vi.
4210.494	3: 4	0.07	3	+.027	*c*5	
4337.216	Cr 3: 5	0.51	½	+.002	*b*3	
4454.552	Ca, Zr 5: 3	0.40	1	+.003	*b*3	
4461.818	Fe–Mn 3: 4	0.36	1	+.008	*a*3	Provisionally retained on account of Mn
4476.185	Ag 3: 4	0.07	3	+.008	*b*4	
4489.911	Mn–Fe 3: 4	0.34	1	+.004	*a*3	Provisionally retained on account of Mn
4637.685	Fe 4: 5	0.51	½	(+.004)	*d* ?	
4679.027	2: 6	0.38	1	+.005	*c*4	Ni? Also Fe (1)—Burns
4707.457	2: 5	0.22	1	(+.006)	*c*5	? Burns gives Fe 2 in arc
4727.582	Mn 2: 3	0.09	2	+.003	*d* ?	
4938.997	Fe 2: 4	0.42	½	(+.005)	*c*	
4957.480	Fe 8: 5	0.30	2	(+.003)	*c*5	
4982.682	2: 4	0.31	1	+.013	*d*	
4985.432	Fe 3: 3	0.30	1	(+.009)	*c*	
5005.896	Fe 5: 4	0.41	1	(+.002)	*c*	
5012.252	1: 4	0.08	½	−.002	*a*	
5107.619	Fe 4: 4	0.20	2	(+.008)	*a*	
5125.300	Ni 1: 3	0.12	1	−.002	*d*	
5139.427	Fe 4: 4	0.22	2	(+.006)	*c*	
5195.113	Fe 2: 4	0.53	½	(+.006)	*a*	
5208.596	Fe 2: 5	0.18	0	+.003	......	Cr line; weight 0—"group" is unknown
5273.339	Fe–Cr 2: 3	0.22	1	+.004	sub-*d*	Provisionally retained on account of Cr
5328.236	Cr 2: 8	0.28	½	−.007	*a*1	Double in sun?
5365.069	Fe 3: 5	0.53	½	(.000)	*e*	
5463.174	Fe 3: 3	0.32	1	(+.009)	*e*	
5476.500	Fe 3: 1	0.28	2	(+.011)	*a*	
5476.778	Ni 5: 3	0.34	2	+.015	*d*	And Fe 1:3, 0.28 to vi.
5598.524	Ca 4: 1	0.19	3	+.008	*e*	
6008.186	Fe 6: 4	0.60	½	(+.012)	*e*	
6078.710	Fe 2: 5	0.52	½	(−.002)	*e*	
6136.829	Fe 3: 8	0.38	1	(+.004)	*b*	
6400.217	Fe 2: 8	0.32	½	(+.001)	*d*	

TABLE I—*Continued*

	Summary*			Percentages	
	(1)	(2)	(3)	(3)÷(1)	(3)÷(2)
Weighted mean.........	+.0094	+.0115	+.0062	66	54
Sum of weights.........	45.5	27.5	18	....	....
Straight mean..........	+.0068†	+.0080	+.0055	81	69
Number of lines........	38	20	18	....	....

* (1) = Results for all lines—(previously published).
(2) = Results when lines with Fe companions are omitted.
(3) = Results for the lines with Fe companions.

† In the original Table V (*op. cit.*) this was erroneously given as +.005.

The evidence summarized at the bottom of Tables I and II seems to show quite definitely that when the companion line is also due to iron the displacement is only half as great (average is 53 per cent) as when the companion is due to some other element. *This material reduction of the displacement for pairs in which both components are due to iron is distinctly in line with Larmor's theory.*

In Table III have been tabulated the ratios, companion toward violet:companion toward red, for the various subdivisions in the summaries of Tables I and II. It is seen that when the companion is toward the violet the displacement is quite closely two-thirds (or possibly five-eighths) as great as when the companion is toward the red. The marked approximation to uniformity in these ratios, whether we consider the weighted means or the straight means, was somewhat surprising. However, this is in part accidental.

Tables I and II of Royds (*op. cit.*) which were intended to prove my results to be "largely fictitious" were next examined from the point of view employed above. Of the 17 Fe lines with companion to the red, 10 (=60 per cent) of the companions are due to Fe and only 7 to other elements; of the 30 lines with companions to the violet, only 12 (=40 per cent) of the companions are due to Fe and 18 to other elements. While this disparity between the two sets of lines does not account for the smaller relative shift for the sun-arc displacements as compared with my residuals,[1] it does in part

[1] A satisfactory direct comparison between the sun-arc displacements of Evershed and my displacements does not seem feasible.

TABLE II

FE LINES WITH COMPANIONS TOWARD THE VIOLET IN THE SOLAR SPECTRUM

λ Rowland	Elements and Intensities (Companion: Line)	Separation	Weight	To Reduce Point to Curve	Group	Remarks
	Fe	A		A		
3647.988	Fe 4:12	0.43	½	(+.002)	*b*1	
3680.069	2: 9	0.25	½	−.007	*a*1	And 2:9, 0.46 to red
3722.729	Ni 3: 6	0.09	2	−.014	*a*1	Line also Ti?
3737.281	Ca–Mn 5:30	0.22	½	+.003	*a*1	
3746.058	Fe 8: 6	0.34	2	(−.008)	*a*1	
3748.650	Fe 10: 1	0.24	3	(−.008)	*b*1	
3887.196	Cr 3: 7	0.25	1	.000	*b*1	
3888.671	2: 5	0.11	1	−.008	*b*1	And Fe–Mn 2:5, 0.30 to red
3895.803	Mn 3: 7	0.22	1	.000	*a*1	
3969.413	6:10	0.53	½	−.004	*b*1	
4132.235	V 2:10	0.14	1	−.012	*b*1	Line is Fe–Co
4134.840	Fe? 3: 5	0.25	½	+.006	*b*4	Also V 1:5, 0.16 to vi. Weight ½ for V
4144.038	Fe, Mo 4:15	0.47	½	−.005	*b*1	Mo, 2:15. 0.37 to vi. Retained on account of Mo
4147.836	2: 4	0.33	1	+.002	*a* or *b*	
4191.843	Fe 6: 3	0.25	2	(+.003)	*e*	
4227.606	Ca 20: 4	0.70	½	−.004	*d*5	And 1:4. 0.13 on same side
4233.772	Mn 4: 6	0.44	½	+.002	*d*5	
4291.630	Ti 2: 2	0.35	1	+.007	*a*	And 1:2. 0.26 on same side
4294.301	Ti 2: 5	0.10	2	−.005	*b*2	
4308.081	Ca 3: 6	0.17	2	−.006	*b*1	And fainter line on each side
4315.262	Ti 3: 4	0.12	2	−.007	*b*3	
4407.871	V 2: 4	0.06	2	+.011	*c*4	Also V, 2:4, 0.49 to red
4427.482	Ti 2: 5	0.22	1	.000	*a*3	
4531.327	Fe?, Co 2: 5	0.20	½	−.002	*b*3	And 2:5, 0.47 to red
4556.306	Fe 3: 4	0.24	1	(+.002)	*e*	
4592.840	Ni 2: 4	0.13	2	−.010	*c*4?	
4668.331	2: 4	0.09	2	−.010	*d*?	
4727.676	Fe 3: 2	0.09	0	−.029	......	Mn, weight 0—"group" unknown
4787.003	Ni 3: 2	0.28	1	−.012	*c*4	
4789.849	Cr 2: 3	0.32	1	−.012	*c*4	
4872.332	1: 4	0.22	½	−.006	*c*5	
4878.407	Ca 3: 4	0.09	2	−.008	*c*5	
4939.868	Fe 2: 3	0.45	½	(−.002)	*a*	
4957.785	Fe 5: 8	0.30	1	(−.004)	*c*5	
4985.730	Fe 3: 3	0.30	1	(+.003)	*c*	
5006.306	Fe 4: 5	0.41	1	(−.002)	*c*	
5028.308	Fe 1: 2	0.37	½	(.000)	*a*	Companion not in Burns's Fe table
5041.255	Fe 3: 4	0.19	1	(−.002)	*a*	
5041.936	Ca 2: 4	0.14	1	+.001	*a*	
5079.409	Fe 3: 4	0.25	½	(−.006)	*a*	
5098.885	Fe 1: 3	0.13	½	(−.005)	*a*	
5107.823	Fe 4: 4	0.20	2	(−.004)	*a*	

TABLE II—*Continued*

λ Rowland	Elements and Intensities (Companion: Line)	Separation	Weight	To Reduce Point to Curve	Group	Remarks
	Fe			A		
5139.644	Fe 4: 4	0.22	2	(+.002)	*c*	
5167.678	Mg 15: 5	0.18	3	−.016	*a*	
5195.647	Fe 4: 2	0.53	½	(−.001)	*e*	
5202.516	Fe? 2: 4	0.08	1	−.006	*a*	Companion not in Burns's Fe table. Weight reduced
5208.776	Cr 5: 2	0.18	3	−.006	sub-*d*	
5227.362	Fe–Cr 3: 5	0.32	1	−.003	*a*4	Provisionally retained on account of Cr
5270.558	Ca 3: 4	0.12	2	−.026	*a*4	
5273.558	Fe 3: 2	0.22	2	(−.014)	sub-*d*	
5333.089	Co, Fe 1: 4	0.24	½	+.005	*a*4	Provisionally retained on account of Co
5365.596	Fe 5: 3	0.53	½	(+.003)	*a*	
5371.734	Cr? 4: 3	0.08	3	−.042	*a*1	
5404.357	Fe 2: 5	0.33	1	(−.012)	*e*	
5447.130	Ti 2: 6	0.33	½	+.003	*a*	Double in sum?
5455.834	Fe? 2: 4	0.16	2	(−.007)	*a*	Fe—Burns
5463.494	Fe 3: 3	0.32	1	(−.002)	*e*	
5594.884	Ca 4: 1	0.19	2	+.009	*e*	Measured only by Burns with large probable error.
5603.186	Ca 3: 4	0.10	2	−.011	sub-*d*	
5615.877	Fe 2: 6	0.36	1	(−.004)	sub-*d*	
5659.052	Fe 1: 4	0.30	½	(−.003)	sub-*d*	? Also Cr 0:4, 0.17 to vi
6020.401	H, Fe? 2: 4	0.17	1	−.008	*b*?	
6148.040	2: 3	0.09	2	−.005	*d*	
6191.779	Ni 6: 9	0.39	1	−.004	*b*	
6254.456	1: 5	0.07	½	+.017	*b*	Doubtful

	Summary*			Percentages	
	(1′)	(2′)	(3′)	(3′)÷(1′)	(3′)÷(2′)
Weighted mean.........	−.0066	−.0077	−.0040	61	52
Sum of weights.........	81.5	53.0	27		
Straight mean..........	−.0041	−.0047	−.0030	73	64
Number of lines........	64	41	23		

* (1′) = Results for all lines previously published.
(2′) = Results when lines with Fe companions are omitted.
(3′) = Results for the line with Fe companions.

explain why the displacement for companion to the violet is not smaller than for companion to the red, as would be required by the anomalous-dispersion theory.

TABLE III

RATIOS: COMPANION TOWARD VIOLET ÷ COMPANION TOWARD RED

	(1′)/(1)	(2′)/(2)	(3′)/(3)	Mean of Three Ratios
For weighted means	0.70	0.67	0.65	**0.67**
For straight means	0.60	0.59	0.55	0.58
Average	(0.65)	(0.63)	(0.60)	(0.62)

As the displacement has not entirely disappeared for pairs in which both lines are due to iron, we must conclude that the components of these pairs represent only in part actual physical connection in the molecule, and in part entirely independent vibrations. As was pointed out by Larmor (*op. cit.*) concerning two adjacent lines of the *same* substance:

If these lines are in physical connection in the molecule, as components of one compound mode of vibration, no sensible difference would be expected, as regards their separation, between the solar spectrum and the arc spectrum; if they are independent lines, then they should separate from one another as the density of the substance that produced them increases.

In order that lines of the *same* substance may be independent, the most natural inference is that they originate at different levels in the solar atmosphere. The actual way in which lines may depend upon level cannot be determined here. We can, however, make a provisional—though far from satisfactory—test of our problem on the hypothesis, elaborated by St. John,[1] "that the lines of any one element originate at depths increasing with decrease of solar intensity, that the enhanced lines are higher than unenhanced lines of equal solar intensity, and that we see into the sun to greater depth at the red end of the spectrum than at the violet." With this as a working hypothesis, the components of a close pair of lines of equal intensity would have their origin at the same effective solar level and thus have a common origin, representing, at least in a large measure, two components of a single compound vibration of the molecule; lines differing in intensity are supposed to originate at different levels in the solar atmosphere, i.e., in

[1] *Astrophysical Journal*, **37**, 377, 1913; **38**, 157, 1913; **40**, 45, 1914; *Mt. Wilson Contr.*, Nos. 69, 74, and 88.

separate and distinct masses of the same gas. In the latter case the components of a pair would have no connection within the molecule, and, for the purposes of our problem, should be as independent of each other as though they were due to two different elements. This independence would necessarily be complete only in so far as there is a complete separation of the respective masses of gas in which the two components originate.

TABLE IV

DISPLACEMENTS, FOR COMPANION TOWARD THE RED, ARRANGED ACCORDING TO RATIO OF INTENSITIES, FE COMPANION : FE LINE

λ	Ratio of Intensities	Separation	Weight	To Reduce Point to Curve	Straight Means	No. of Lines	Weighted Means	Sum of Weights	Mean Separation	Mean Ratio	Weighted Means for Separation of 0.26A
		A		A	A		A		A		A
4985.4	3: 3	0.30	1	+.009	+.0080	4	+.0077	6	0.26	1:1	+.0077
5463.1	3: 3	0.32	1	+.009							
5107.6	4: 4	0.20	2	+.008							
5139.4	4: 4	0.22	2	+.006							
4637.6	4: 5	0.51	½	+.004	+.0040	6	+.0050	3½	0.46	0.6:1	+.0086
5365.0	3: 5	0.53	½	±.000							
4938.9	2. 4	0.42	½	+.005							
5195.1	2: 4	0.53	½	+.006							
4191.5	3: 6	0.25	1	+.011							
6078.7*	2: 5	0.52	½	−.002							
4707.4	2: 5	0.22	1	+.006	+.0038	4	+.0042	3	0.35	0.3:1	+.0058
6136.8	3: 8	0.38	1	+.004							
6400.2	2: 8	0.32	½	+.001							
3735.0	4:40	0.47	½	+.004							
5005.8	5: 4	0.41	1	+.002	+.0070	4	+.0064	5½	0.40	1.8:1	+.0089
6008.1	6: 4	0.60	½	+.012							
4957.4	8: 5	0.30	2	+.003							
5476.5	3: 1	0.28	2	+.011							

* Grouped with lines of higher ratio because companion is enhanced.

Accordingly, the pairs of lines in which both components are due to iron were selected from Tables I and II and arranged in the order of their ratios of intensities as shown in Tables IV and V. The order is that of progressively increasing difference from equality in the intensities of the two components. It was deemed desirable to keep separate the small number of lines for which the ratio of intensities is greater than unity, and these were all placed into one group at the bottom in each table.

Since the displacement is a function of the separation of the line from its companion the means in Tables IV and V require a reduction to a uniform separation before they can be directly

compared. The curves a and b in the former article, Fig. 2, p. 353, are represented sufficiently well by the linear equations

$$y_a = -0.0182x_a + 0.0124$$
$$y_b = -0.0173x_b + 0.0087$$

These two lines are nearly parallel, and we may adopt a mean value of -0.018 for the slope. This holds for the complete data

TABLE V

DISPLACEMENTS, FOR COMPANION TOWARD THE VIOLET, ARRANGED ACCORDING TO RATIO OF INTENSITIES, FE COMPANION: FE LINE

λ	Ratio of Intensities	Separation	Weight	To Reduce Point to Curve	Straight Means	No. of Lines	Weighted Means	Sum of Weights	Mean Separation	Mean Ratio	Weighted Means for Separation of 0.26A
		A		A	A		A		A		A
4985.7	3: 3	0.30	1	+.003							
5463.4	3: 3	0.32	1	−.002							
5107.8	4: 4	0.20	2	−.004	+.0002	5	−.0001	7	0.26	1:1	−.0001
5139.6	4: 4	0.22	2	+.002							
4556.3*	3: 4	0.24	1	+.002							
5006.3	4: 5	0.41	1	−.002							
5041.2	3: 4	0.19	1	−.002	−.0033	3	−.0028	2½	0.28	0.8:1	−.0032
5079.4	3: 4	0.25	½	−.006							
4939.8	2: 3	0.45	½	−.002							
4957.7	5: 8	0.30	1	−.004	−.0032	4	−.0048	4	0.32	0.6:1	−.0059
5028.3	1: 2	0.37	½	±.000							
5455.8	2: 4	0.16	2	−.007							
5404.3	2: 5	0.33	1	−.012							
5098.8	1: 3	0.13	½	−.005							
5615.8	2: 6	0.36	1	−.004	−.0044	5	−.0054	3½	0.31	0.3:1	−.0063
3647.9	4:12	0.43	½	+.002							
5659.0	1: 4	0.30	½	−.003							
3746.0	8: 6	0.34	2	−.008							
5273.5	3: 2	0.22	2	−.014							
5365.5	5: 3	0.53	½	+.003	−.0042	6	−.0061	10	0.35	3.1:1	−.0077
5195.6	4: 2	0.53	½	−.001							
4191.8	6: 3	0.25	2	+.003							
3748.6	10: 1	0.24	3	−.008							

* Grouped with lines of higher ratio because companion in enhanced.

of Tables I and II of the present article. What the correct slope may be for the corresponding curves for Tables IV and V cannot be determined satisfactorily from the data there given. Qualitatively the results will not be seriously affected by adopting the slope found above. The last column in Tables IV and V gives the weighted means of column 7[1] reduced to the uniform separation of 0.26A with the use of slope 0.018.

[1] The reasons for giving preference to the weighted results were stated above. The straight means give substantially the same results.

The evidence thus furnished is not as definite as could be desired. Table IV indicates practically no relation between displacement and difference in intensity of the components of close pairs, while Table V seems to show zero displacement for equality in the intensities of the two component lines and a progressive increase of displacement with increasing difference in the intensities. Whether this lack of accord between the two cases be due to insufficient data alone, or whether St. John's theory represents only imperfectly the actual relation between solar levels and intensities, must be left in abeyance. The principal difficulty which the writer experiences for a complete acceptance of St. John's theory is the requirement that the lines separate in the different solar levels strictly according to intensity, whereas we are accustomed to expect marked differences in intensity in spectra from very limited and presumably homogeneous portions of light-sources in the laboratory. The unqualified acceptance of St. John's theory would seem to require the conclusion that it is impossible (or, at least, it has been impossible up to the present) to obtain substantially homogeneous conditions in a very limited portion of the arc, or spark, or electric furnace. Or, are we to conclude that the complete significance of St. John's observed facts is still obscure? For the present, therefore, the moderate preponderance of evidence in favor of a relation between displacement and ratio of solar intensities is to be considered as possibly a crude indication of such a relation, though even this is to be taken with reserve.

In conclusion it may be well to recall that results similar to those published in my former article had been obtained[1] from a much larger number of lines discussed entirely without reference to pressure-effect. This justifies the belief that the introduction into the problem of density-effect and pole-effect—and these should be taken into account as soon as they have entered upon a sufficiently advanced stage of development—will leave the evidence adduced above qualitatively intact, while it may alter it quantitatively. In fact, these effects have possibly been taken into account indirectly. According to Royds, pressure-shifts and shifts due to differences in vapor-density are somewhat interlaced; with

[1] *Op. cit.*, p. 335, n. 2.

increased atmospheric pressure, he believes there will also be increased vapor-density in the arc. If this reasoning be applied to the Mount Wilson pressure displacements—and it should probably be extended to apply also to conditions in the sun—then not only true pressure-shifts but in part shifts due to vapor-density have been indirectly taken care of in my reductions.

Naturally, no position taken in regard to the interpretation of observed facts need be irrevocable. While the evidence originally adduced is still substantially in favor of anomalous dispersion in the sun, and while it has been strengthened by the separation of the pairs according as the components are due to one element or to two different elements, there is nevertheless no immediate necessity for making a final decision one way or the other. The reopening of the problem for dicussion by several investigators and from new points of view is certainly giving promise to ample rewards, both for the main problem and for other problems which are indirectly involved.

SUMMARY

In a former article it was shown that iron lines with close companions in the solar spectrum (Rowland's Preliminary Table of Solar Spectrum Wave-Lengths) are displaced relative to their positions in the arc spectrum; when the companion is to the violet the displacement is toward the red, and when the companion is to the red the displacement is toward the violet; in the former case the displacement is only two-thirds as great as in the latter, and in both cases it diminishes progressively with increasing separation of the two lines. These observed facts are strikingly in accord with the requirements of the anomalous-dispersion theory of Julius. Personality in the measurement of close pairs of lines was also suggested as a possible cause for the observed facts. However, it would be difficult to explain, on this ground, the observed inequality of the displacements for the two components.

In the present investigation a marked distinction was found to exist between pairs of lines in which both components are due to iron, and those in which one of the components is due to some other element. In the latter case, both for companion to the red and for companion to the violet, the displacement is only one-half as

great as in the former case. As above, so also in each of these subdivisions, the displacement for companion to the violet is only two-thirds as great as for companion to the red. On the anomalous-dispersion theory the observed smaller displacement for pairs of lines in which both components are due to iron is explained on the basis that the components of these pairs represent only in part physical connection in the molecule and in part entirely independent vibrations. These observed facts are also in accord with the anomalous-dispersion theory as modified by a recent suggestion of Sir Joseph Larmor.

For the lines with Fe companions an attempt was made to determine a relation between displacement and difference in solar level between the line and its companion, by utilizing the hypothesis, elaborated by St. John, "that the lines of any one element originate at depths increasing with decrease of solar intensity." The moderate preponderance of evidence in favor of a relation between displacement and ratio of intensities of the two components is not sufficient to be regarded as more than possibly a crude indication of such an effect.

The observed facts outlined in paragraphs one and two of the summary seem sufficiently definite to be considered established. In regard to their interpretation it may be said that the theory of anomalous dispersion in the sun as developed by Julius and modified by Larmor does account for them. However, as this subject is quite clearly still in the early stages of development, final judgment may well be suspended for the present.

Dudley Observatory, Albany, N.Y.
May 5, 1916

NOTE

While the manuscript for the above article was in the hands of the printer an interesting communication appeared in which St. John shows,[1] by two lines of evidence, that "no factor of transformation nor any curve can yield true differences between the Rowland and international wave-lengths for all lines even for a limited spectral region." A careful perusal of my first paper on anomalous dispersion in the sun will show that I am entirely in accord with this

[1] *Proceedings of the National Academy of Sciences*, 2, 226, 1916.

conclusion. However, as my two articles rest upon the basis of a comparison between the Rowland and international wave-lengths, and in order to avoid an erroneous inference—which might easily be possible—it was deemed advisable to call special attention to the fact that no such direct comparison of the two systems of wave-lengths is involved in my work. A more accurate reference to my method of comparing the two systems would be: "Albrecht uses as ordinates the quantities, Rowland minus *corrected international*, and from the mean curve derives directly the systematic difference for *a very much restricted set* of lines." In fact, my method of comparison was devised because I was fully aware of both the pressure-effect and the effect according to solar intensity, and because I wished the method to be sufficiently flexible to allow of future correction for other possible effects which at present are entirely unknown.

July 10, 1916

THE ACCURACY OBTAINABLE IN THE MEASURED SEPARATION OF CLOSE SOLAR LINES; SYSTEMATIC ERRORS IN THE ROWLAND TABLE FOR SUCH LINES[1]

By CHARLES E. ST. JOHN and L. W. WARE

I. INTRODUCTION

Close pairs of solar lines play such a rôle in the search for evidence of mutual influence between neighboring lines in the solar spectrum—a deduction from the anomalous-dispersion hypothesis—that it is important to have an idea of the accuracy obtainable in their measurement, and hence of the relation that the errors bear to the probable magnitude of the suggested effect. The investigation thus necessitated is also preliminary to a determination of the wave-lengths of the solar lines in international units, and in particular is an effort to develop a method of measuring the wave-lengths of such as have lines closely adjacent.

In investigations on the resolving power of spectroscopes, a complex line is considered to be resolved when the intensity at the midpoint of the overlap between two lines of equal intensity is 0.81 of the maximum for the single lines. For precision in the measurement of close doubles in the solar spectrum the important consideration is the lower limit for the difference in wave-length between lines of a specified intensity and character, that can be measured to a given accuracy with the spectrographs and photographic plates now available, rather than the resolving power which shows the line as double. Though the above standard of resolution is sufficient for determining the complex character of a solar doublet, the accuracy of the wave-length determination for the separation implied by it has not previously been investigated. Moreover, since the conditions presented by the unequal intensity of the background on the two sides of each component are such as may introduce systematic personal errors, it has seemed desirable to examine this question

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 120.

in detail; and, in fact, the results of the examination form an important portion of this contribution.

2. APPARATUS

For the present investigation an excellent grating by Anderson has been at our disposal through the kindness of the Physical Laboratory of the Johns Hopkins University. The ruled surface is 114×160 mm and the number of lines is 95,000. The definition is all that can be asked, even when the whole surface is used. As determined from the iodine absorption lines, the excellent performance is retained up to and including the fifth order. It has been used as a Littrow spectrograph, focus 30 feet (9.14 m), in connection with the 60-foot tower telescope. For comparing the results given by large-scale spectrograms obtained by long focus and high resolving power, respectively, a series of plates was taken with a large grating by Michelson, number of lines 77,500, used in the 75-foot spectrograph of the 150-foot tower equipment. The slit-width was usually four-normal, but for the very difficult separations a two-normal slit was employed, giving 94.3 per cent of the resolution for an indefinitely narrow slit[1] with monochromatic light. Commercial photographic plates have been used as follows: for the blue-violet, Seed's Process plates; for the green, Cramer's X-ray for the most part, but in special cases where fineness of grain was particularly important, Process plates sensitized with pinaverdol; for the yellow-red, Wratten "M" plates and Seed's Process sensitized according to the Wallace formula.[2] Backed plates have been used with advantage for some of the most difficult separations.

Upon a plate of an intensity suitable for the general run of lines, the measurement of the separation and, hence, of the wave-lengths of the components of a close pair, in which one or both components are strong lines, is difficult; but with exposure and development adapted to the case, somewhat too strong for weak lines, the difficulties and uncertainties are decreased with a corresponding in-

[1] Schuster, *Astrophysical Journal*, **21**, 207, 1905; Zeeman, *K. Akad. Amsterdam Proceedings*, **18**, 412–415, 1915.

[2] *Astrophysical Journal*, **26**, 317, 1907.

crease of confidence in the results. The exposure times and the intensity of development have been chosen to obtain the best conditions for the particular separation under consideration.

For measuring the spectrograms the usual filar-micrometer instruments have been employed; in addition a registering microphotometer of the Koch form has been used for purposes of checking, and also for some absolute determinations where it appeared especially difficult to eliminate the personal equation.

3. METHODS OF MEASUREMENT

In the measurement of the separation of close pairs of solar lines some means of determining whether the results, barring accidental errors, represent the real separation or are affected by systematic errors is an important desideratum. That concordant values are obtained by repeated measurements of the same or of other similar plates by a single observer has little bearing upon the question, as possible systematic errors introduced by the personal equation of the observer are not thereby eliminated. This comes into play in estimating the position of the maximum intensity of a line, one of whose edges is seen against a strong continuous spectrum background, and the other against a background—the region of overlap—whose intensity may differ but little from that of the line itself. In order to obtain a check upon the measurements for ordinarily close pairs, all determinations have been made by at least two observers, and in order to vary the conditions widely, they have been made upon a series of six spectrograms representing the first five orders of the 30-foot and the first order of the 75-foot spectrograph. The appearance of a close pair of lines differs greatly under these conditions, as the scale of the spectrograms ranges from 1.8 to 0.25 A per mm and the resolving power changes by fivefold. Under such widely different circumstances any personal error depending upon the proximity and character of the adjacent lines will not remain constant, and concordance among the results can be taken to indicate the practical elimination of such error. For selecting the data to be used in forming the final mean separation for a given pair of lines the criterion is, therefore, the degree of concordance between the orders. Beginning with

the highest, those orders are included for which the agreement between the orders is comparable to that between the measurements within the orders.

For pairs which with the highest resolving power and the largest scale employed are not completely separated, but yet appear accurately measurable with the filar micrometer, curves have been drawn by the registering microphotometer. Separations so obtained appear to be least affected by errors, either systematic or accidental, and to represent most nearly the true separation of the components.

For doublets at or very near the limit of resolution, whose components cannot be sufficiently separated for micrometric settings upon their maxima to be made without influence by the neighboring line, and are not reproduced as discrete curves by the registering microphotometer, the following method has been used to obtain the separation and furnish a check upon the attempted micrometer measurements. In such cases it is generally possible to make intensity classifications for the components as precisely as for the free-standing lines. An inspection of the plate enables one to select the free-standing lines that correspond most nearly to the components of the complex line. Microphotometer curves are obtained for the complex line and the selected list of isolated lines, the sensitiveness of the instrument being held constant during the run over the plate. From the curves of the isolated lines one selects those that conform to the violet and red branches of the compound curve, superposes the negatives so that the corresponding branches of the three curves are coincident, and measures the distance between the axes of the constituent curves. The scale of the spectrograms used for this purpose, 1 mm = 0.30 A, is increased fifty-fold by the microphotometer, so that the scale of the curves is 1 mm = 0.006 A. The measurements are made with sufficient accuracy with a fifth-millimeter scale. The operation is an analysis of the doublet, since one determines not only the separation but also the form and intensity of the components.

A less satisfactory procedure is to measure upon the original spectrogram the widths of a large number of isolated lines of the same nominal intensities as the components of the doublet by

setting upon their edges as in the measurement of star-images. Similar settings are made upon the free edges of the complex line. Its width decreased by the half-widths of the two sets of free-standing lines furnishes a measure of the separation of the components. The weak point of the method lies in the fact that lines of the same nominal intensity vary greatly, even in the same spectral region, as shown by the curves of widely different heights and characters which they yield; but by including several reference lines of each intensity the results are in fair agreement with those given by the microphotometer.

4. ILLUSTRATIONS OF THE METHODS

a) *Filar micrometer.*—A number of doublets in the violet region were measured for separations upon a fifth-order plate of excellent definition, scale 1 mm=0.29 A. One exposure was measured by an observer five times at intervals extending over two months; three exposures were measured by another observer, once each. Both observers had had experience in the measurement of solar lines. The results are shown in Table I. In the first column between the Rowland wave-lengths, an indication is given of the appearance of the doublets. The symbol $s<<<<c$ means that the space between the maxima is very much less dark than the continuous background bounding the free edges of the components, i.e., the doublet is barely resolved; while for the pair at λ 4457, $s=c$, there is practically complete resolution. In the eighth and ninth columns are the means of the series of 5 and 3 measures, and in the tenth column the differences between the Rowland and the mean Mount Wilson separation. These differences are all positive and indicate an average systematic error of 0.006 A, either in the Rowland tables or in the Mount Wilson measures, carrying with it a corresponding error in the wave-lengths of the constituent lines. In the last column are given the separations obtained by the same observers upon a third-order spectrogram of excellent quality. The agreement with the fifth-order plate is mutually corroborative. The systematic character of these discrepancies between the Rowland and Mount Wilson results lends them a greater significance than their magnitude alone would imply. Further evidence of

their reality and the reasons for considering them actual errors in the Rowland values will be given in later sections of the paper.

b) *Registering microphotometer.*—The Mg-Fe pair at λ 5167 and the Fe pair at λ 5169 furnish examples of the unrecognized difficulty and the unsuspected systematic errors that occur in filar-micrometer determinations of separations apparently within the

TABLE I

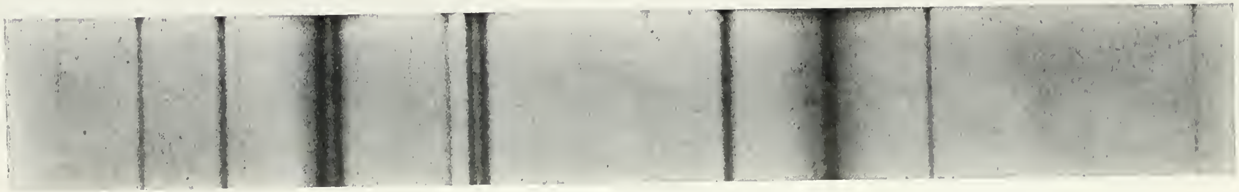
MICROMETRIC MEASURES ON CLOSE PAIRS OF SOLAR LINES

Pairs	Int.	Order V							Rowland *minus* Mt. Wilson	Order III
		1	2	3	4	5	S (5)	W (3)		
4455.980 $s<<<<c$ .064	2 3	0.067	0.078	0.072	0.072	0.075	0.073	0.078	+0.009	0.074
4457.600 $s=c$ .712	2 2	.108	.107	.105	.106	.104	.106	.106	+ .006	.108
4459.199 $s<<<c$ .801	2 3	.093	.098	.093	.094	.096	.095	.093	+ .008	.091
4464.844 $s<c$ .938	2 1	.088	.086	.088	.088	.088	.088	.090	+ .005	.091
4472.884 $s<<<c$ .967	1 0	.079	.080	.068	.078	.082	.077	.082	+ .004	.076
4476.185 $s<<<<c$ .253	4 3	.071	.065	.067	.064	.064	.066	.068	+ .001	.070
4482.338 $s<<<<c$ .438	5 3	0.091	0.091	0.089	0.096	0.092	0.092	0.097	+0.006	0.094
Mean	.....	.....	.....	.....	.....	.....	.....	.....	+0.006	......

range of ordinary methods. In these cases the resolution under high power is so nearly complete that it has not been easy to convince one's self that the micrometer settings upon the components are subject to the systematic errors that appear when the results are compared with those obtained from the registered curves. In Plate I (*a*) a spectrum of the fourth order is reproduced, magni-

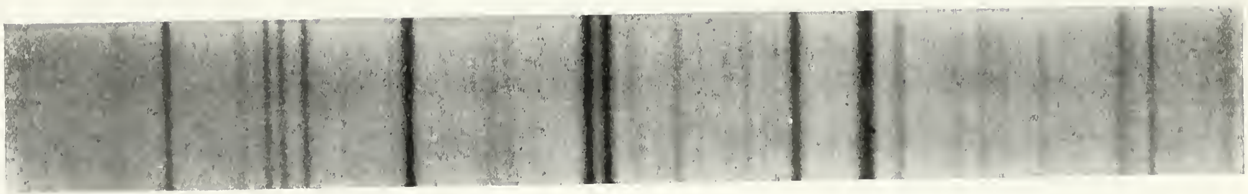
PLATE I

a



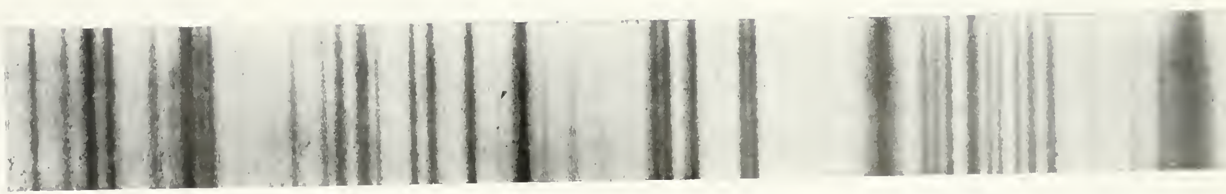
λ5167 λ5169 Fourth Order. Enlargement 4.7

b



λ5107 Fourth Order. Enlargement 4.7

c



λ3918 λ3919 Fourth Order. Enlargement 4.7

		λ3918	λ3919
Separation:	Rowland	0.099	0.101
	Mount Wilson	0.099	0.093

fication 4.7, and in Figs. 1 and 2 are the corresponding curves drawn by the registering photometer.

Under the title "An Adaptation of the Koch Registering Microphotometer to the Measurement of the Sharpness of Photographic Images"[1] Tugman shows that the upper slit of the microphotometer

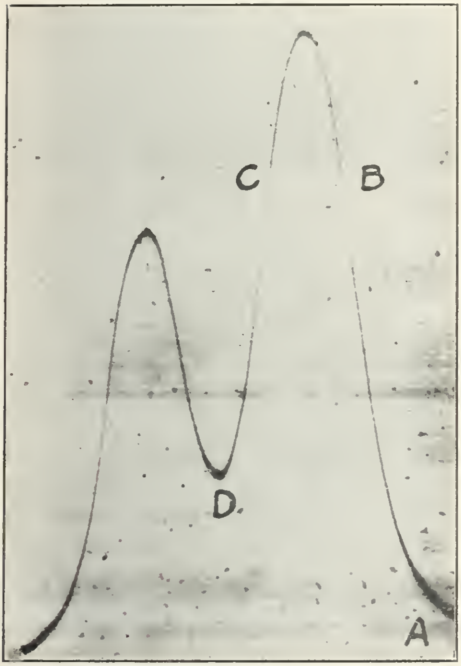


FIG. 1.—Microphotometer curve of Mg–Fe pair λ 5167.

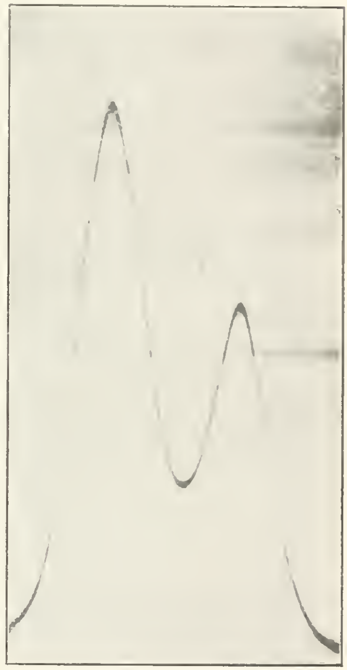


FIG. 2.—Microphotometer curve of Fe pair λ 5169.

should not cover more than one-fifth of the total width between maximum and minimum density, i.e., the half-width of the spectrum line; but that with slit-widths less than 0.75 mm the pinhole effect begins to enter, which falsifies the density-gradient by widening the base of the curve. Such an effect, while distorting the curve and lowering the resolution by the basal widening, does not shift the axis of the curve for an isolated line; and the free portions

[1] *Astrophysical Journal*, 42, 321, 1915.

of the components of a nearly separated doublet, such as the pair at λ 5167 or at λ 5169, appear to be unaffected.

The shortest distance between maximum and minimum intensity occurs between the lowest point in the region of overlap and the peak of the smaller curve. Upon the high-dispersion spectrogram used the horizontal distance is 200 μ for λ 5167. The objective magnifies fivefold; with a slit-width of 100 μ one-tenth of the image of the half-line is covered. The symmetrical character of the free portions of the two curves strikes one at a glance. The distance between their axes is 430 μ=0.158 A. Three observers with long experience in the measurement of solar lines by the filar micrometer found separations of 0.175, 0.169, and 0.172 A, respectively. That of the Rowland table is 0.181 A.

Third- and fifth-order spectrograms yield separations consistent with those of the fourth order, showing similar discrepancies between micrometer measures and the values derived from the registered curves. If the Rowland value represents the true separation, there must be a relative instrumental displacement of the axes of the curves of 3 to 4 mm, which seems inadmissible. The two methods show similar results for the pair at λ 5169, namely, 0.122, 0.136, and 0.151 A for the registering microphotometer, filar micrometer, and Rowland, respectively. To ascribe such discrepancies to systematic errors in the filar-micrometer settings, as seems the probable explanation, is a decided shock to all the observers, since they thought in advance that the settings could be made without systematic errors. The resolution upon the original spectrogram is more nearly complete than the curves indicate, as the pinhole effect has accentuated the overlap.

c) *Analysis.*—For pairs with components of intensities 3 and 4 on the Rowland scale the limit of resolution measurable by micrometer appears to be in the neighborhood of 0.10 to 0.15 A, increasing with the wave-length. In Table II are given the separations for very close pairs determined by micrometric settings upon (1) the maxima of the components, (2) the edges of the doublets and the reference lines, and (3) by analysis of the curves registered by the microphotometer. The spectrograms were taken in the fifth order, the scale varying from 0.23 to 0.30 A per

millimeter in passing from the green to the violet. The seventh column in Table II contains the unweighted means of the three methods of determination, and the last the Rowland *minus* Mount Wilson differences, which again are all positive with a

TABLE II

Comparison of Methods for the Separation of Pairs at the Limit of Resolution

Rowland			Filar Micrometer		Analysis	Mean Δλ	R.−Mt. W.
Wave-Lengths	Int.	Δλ	On Max.	On Edges			
4204.101 $s<<<<<c$ .163	3 4	0.062	0.046	0.040	0.041	0.042	+0.020
4219.516 $s<<<<<c$ .580	4 3	.064	.053	.045	.049	.049	+ .015
4476.185 $s<<<<c$ .253	4 3	.068	.068	.063	.068	.066	+ .002
4482.338 $s<<<<c$ .438	5 3	.100	.094	.081	.087	.087	+ .013
4878.313 $s<<c$ .407	3 4	.094	.093	.086	.086	.088	+ .006
5264.329 $s<<<<c$ .415	4 3	.086	.079	.068	0.75	.074	+ .012
5270.438 $s<<<c$ .558	3 4	0.120	0.104	0.108	0.098	0.103	+0.017
Mean R.−Mt. W.							+0.012 A

mean magnitude of 0.012 A. The registered curve for the pair at λ 4219, Rowland intensities 3 and 4, is reproduced in Fig. 3, with curves of the isolated lines λ 4201, intensity 3, coincident with the red branch, and λ 4196, intensity 4, coincident with the violet branch. The separation derived from these component curves is 0.051 A, the Rowland value being 0.064 A. Although the gradation-curve corresponding to the free edge of the doublet does not

represent the actual distribution of energy in the line, it does represent the slope and height of a curve due to a free-standing line which in intensity and character is identical with the corresponding component of the doublet. Any errors, due to the pinhole effect, in reproducing the density-gradient of the free edges of the components occur also in the curves of the isolated lines of the same

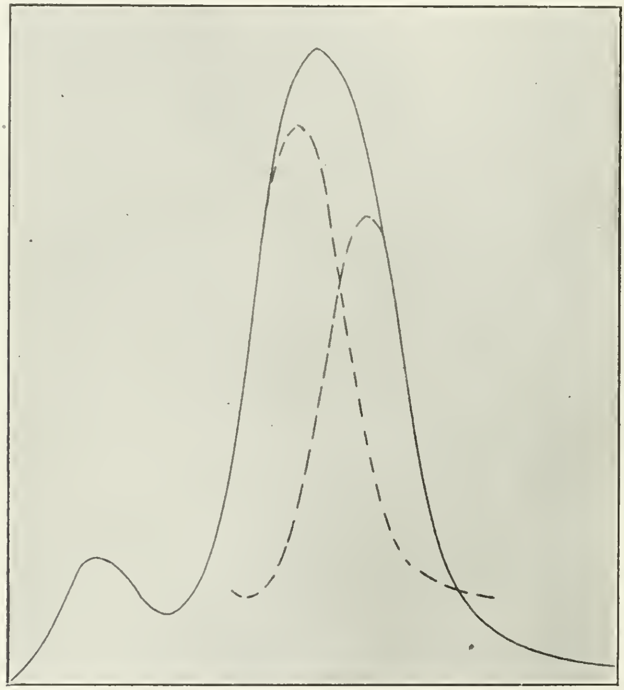


FIG. 3.—Microphotometer curve of solar pair λ 4219, intensity 3 and 4, with curves of isolated lines λ 4196 and λ 4207, intensities 3 and 4, superimposed.

form and intensity. A simple compounding of the curves would yield too high a resultant for the center. This apparent discrepancy is referable to the fact that the areas of the registered curve, as Tugman shows, do not measure the energy of the spectral lines.

5. DEFINITIVE RESULTS

Pairs consisting of lines of intensities 3 and 4 occur more frequently than any others and offer the most favorable basis for a

comparative study. To obtain an extended series of observations, thirty pairs with components of these Rowland intensities were taken for the measurement of the separations. These are the pairs between λ 3900 and λ 5270, the separation of whose components is under 0.35 A. As the conditions upon which the discrepancies between the Rowland and the Mount Wilson values depend vary with the spectral region—the width of the lines of the same nominal intensity increasing with the wave-length—separate consideration is given to the pairs to the red of λ 4000.

In Table III are assembled the data relative to these thirty pairs, which, as far as possible, were measured on each of the six series of plates. Vacant spaces in the table mean that the resolution was not sufficient for the present purposes of measurement. The parentheses indicate that, though the measurement seemed possible at the time, the results were not considered in obtaining the means because of the lack of agreement with the more concordant results for the higher orders. Whether or not they should be included is a matter of judgment; their inclusion, however, would increase the discrepancy between the Rowland and the Mount Wilson values. The pairs are arranged in the order of decreasing separation; therefore, as one passes down the table, the first-order measures soon become uncertain, then impossible, a course followed by the other orders in succession. This furnishes the means for fixing a minimum separation measurable to the indicated degree of precision with the given resolving powers and dispersions. This is influenced, however, by the wave-length and the character and intensities of the lines. The results for the 75-foot spectrograph recorded in the twelfth column were not used in obtaining the means, as the purpose was to compare the results for the two instruments. As far as the measures are dependable they are in excellent agreement.

The magnitude of the errors in the Rowland separation depends upon the proximity of the components. For the first 6 pairs, mean separation 0.274, it is +0.003; for the following 8 pairs, mean separation 0.145, it is +0.008; and for the last 8 pairs, mean separation 0.075, it is +0.013 A, based upon the mean of the three methods used.

TABLE III

FILAR-MICROMETER SEPARATION OF CLOSELY ADJACENT SOLAR LINES

PART I. PAIRS TO THE RED OF λ 4000

Rowland		Order, N.M. and 1 mm in A for 30-Foot					Mean $\Delta\lambda$	Wt.	Mean Dev.	Rowland *minus* Mt. W.	75-Foot	Remarks
Wave-Length	Int.	I 95,000 1.80	II 190,000 0.88	III 285,000 0.56	IV 380,000 0.36	V 475,000 0.27					I 77,500 0.72	
4461.818 2.165	4 3	0.342	0.344	0.342	0.341	0.340	0.342	31	0.001	+0.005	0.340	
4489.911 90.253	4 3	.339	.338	.341	.340	.340	.340	31	.002	+ .002	.339	
4339.617 .882	4 3	.267	.265	.264	.264	.262	.264	12	.002	+ .001	.263	
5079.158 .409	3 4	.254	.250	.248	.249	.246	.249	17	.003	+ .002	.248	
4556.063 .306	3 4	.238	.240	.240	.241	.238	.239	21	.002	+ .004	.241	
5139.427 .644	4 4	.209	.212	.212	.212	.217	.212	15	.002	+ .005	.210	Poor quality
5107.619 .823	4 4	.190	.193	.192	.194	.192	.192	27	.002	+ .012	.191	Excellent quality
5041.069 .255	3 4	.180	.182	.185	.189	.188	.183	18	.005	+ .003	.175	Violet component double
4024.726 .881	3 4	154	.157	.156	.154	.156	.156	21	.001	− .001	.153	
5169.069 .220	3 4	....	.133	.137	.137	.139	.136	19	.003	+ .015	(.124)	Difficult
4171.068 .213	4 4	(.129)	.134	.136	.136	.136	.135	14	.002	+ .010	.132	

TABLE III

PART I—*Continued*

4667.626 .768	4 3	(.125)	.131	.131	.133	.133	.132	16	.001	+ .010	(.125)	
4315.138 .262	3 4	(0.099)	.111	.114	.119	.114	.115	19	.003	+ .009	(.097)	
4078.515 .631	4 3	....	.107	.106	.110	.112	.109	32	.003	+ .007	(.103)	
4581.575 .693	4 4	....	(0.102)	.109	.107	.106	.107	10	.003	+ .011	(.090)	
5270.438 .558	3 4	....	....	.098	.104	.113	.104	15	.006	+ .016	....	Very difficult
4878.313 .407	3 4	....	....	.092	.094	.093	.093	17	.005	+ .001	....	Very difficult
5264.329 .415	4 3	....	....	.076	.080	.083	.079	11	.004	+ .007	....	Very difficult
4476.185 .253	4 3	....	....	0.069	0.069	.067	.068	2	.003	.000	....	Very difficult
4219.516 .580	4 3	....	....	....	....	.053	.053	1	....	+ .011	....	Very difficult
4204.101 .163	3 4	....	....	....	....	.046	.046	3	.002	+ .016	....	Very difficult
4210.494 .561	4 3	....	....	....	....	0.046	0.046	2	0.002	+0.021	....	Very difficult

TABLE III

PART II. PAIRS TO THE VIOLET OF λ 4000

Rowland		Order, N.M. and 1 mm in A for 30-Foot					Mean Δλ	Wt.	Mean Dev.	Rowland *minus* Mt. W.	75-Foot	Remarks
Wave-Length	Int.	I 95,000 1.80	II 190,000 0.88	III 285,000 0.56	IV 380,000 0.36	V 475,000 0.27					I 77,500 0.72	
3910.984 1.135	4 3	(0.158)	0.153	0.153	0.152	0.150	0.152	14	0.002	−0.001	0.154	Poor quality
3956.476 .603	4 4	(.116)	.121	.119	.120	.120	.120	17	.001	+ .007	.118	
3979.664 .783	4 4	(.114)	.117	.119	.116	.118	.117	14	.002	+ .002	.119	
3948.818 .925	4 4	....	.099	.100	.103	.104	.102	17	.002	+ .005	(.095)	
3918.464 .563	4 4	....	(.091)	.098	.098	.100	.099	11	.001	.000	.091	
3989.912 90.011	4 3	....	(.085)	.094	.094	.094	.094	14	.001	+ .005	.088	
3919.208 .309	3 3	....	(0.082)	.091	.094	.094	.093	11	.002	+ .008	(0.081)	
3926.086 .165	4 3	....	....	(0.069)	0.075	0.074	0.075	8	0.001	+0.004	....	

As is well known, the solar lines decrease in width and the spectrograms improve in definition in passing from the red to the violet. This change is striking when one attempts measurements near the limit of resolution in the two regions in immediate sequence. In Part II of Table III are the data for 8 pairs to the violet of λ 4000, mean separation 0.107 A, for which the mean error in Rowland is +0.004 A, approximately half that for the pairs to the red of λ 4000 with the same mean separation. A difference of this order is consistent with the appearance of pairs in the two regions, for, in the violet, owing to the better resolution, the contrast between the intensities of the background on the two sides of a component is much less marked for pairs of like intensity and separation.

As the Mount Wilson measures, made under the most diverse conditions, differ systematically from the Rowland values, the assumption of corresponding errors in the Rowland tables seems a well-grounded conclusion. The establishment beyond doubt of the errors in a few cases may, however, strengthen the evidence that the discrepancies in general represent actual errors in the Rowland tables. The interpretation of the registered curves for the pairs at λ 5167 and at λ 5169 given in an earlier paragraph (p. 20) does not appear open to serious criticism, as no recognized peculiarity in the action of the registering microphotometer has the effect of displacing the axes of curves for lines so nearly resolved. An example still more free from possible criticism is supplied by the solar pair at λ 5107. These are lines of the best quality, and are completely resolved upon the Mount Wilson plates. A spectrum of the fourth order is reproduced in Plate I (*b*) and the curve in Fig. 4. The Mount Wilson micrometer measures give a separation of 0.192 A for the mean of 33 measures on the six series of spectrograms with an average deviation of 0.002 A. The separation given by the registered curve is 0.190 A, while that from the Rowland wave-lengths of the constituent lines is 0.204 A, a discrepancy of +0.013 A.

Another example that appeals to the eye, even without measurement, is found in the pairs at λ 3918 and λ 3919. A spectrum of the fifth order is shown in Plate I (*c*), and the corresponding registered curves in Fig. 5. The eye decides at once which is the wider

separation, and both the micrometer measurements and the registered curves confirm it. The separations deduced from the Rowland wave-lengths are 0.099 A and 0.101 A, but the separations upon the Mount Wilson plates are 0.099 A and 0.093 A, respectively.

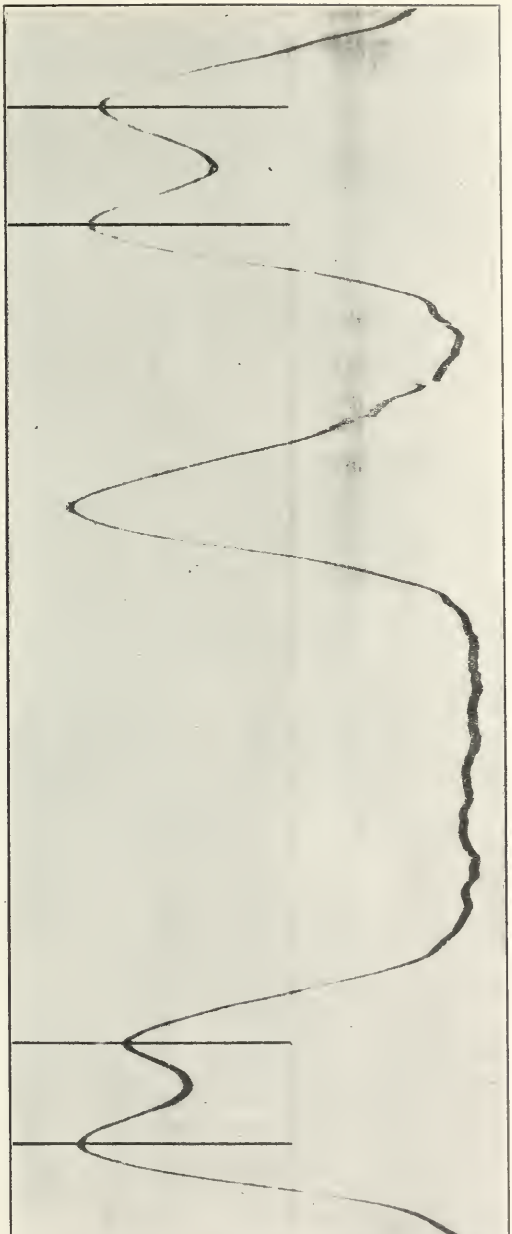
Fig. 4.—Microphotometer curve of Fe pair λ 5107.

6. THE ROWLAND PLATES

Through the kindness of Professor Ames some of the original plates taken by Rowland for the solar spectrum map were loaned to the Observatory. Two plates of the third order are especially good, showing very little grain. In the introduction to the *Preliminary Table of Solar Spectrum Wave-Lengths* Rowland says:

> In some cases when a double line is particularly difficult to separate, measurements are given on the two components and also on the line unresolved. This last measurement is placed in parenthesis between the other two measurements. Thus:
>
> W.-L.
> 3738.454 .... 3 }
> (3738.466) .... 6 } d
> 3738.505 .... 2 }
>
> means that there is a line at W.-L. 3738.466 with the intensity 6; and that with good definition this line may be resolved into two components having intensities 3 and 2 and the given wave-lengths.

Such a complex appearing in the table at λ 3926 is well resolved on the Rowland plates, more plainly resolved, in fact, than other doubles on the plates, which, from the absence of any indication to the con-



λ 3918

λ 3919

Fig. 5.—Microphotometer curve of Fe pair λ 3918 and Fe-Cr pair λ 3919

trary, were measured without difficulty. It is thought, therefore, that the quality must have been considered very satisfactory by Rowland and Jewell. Though the Rowland plates excel the Mount Wilson plates (Seed's Process) in fineness of grain, the slightly

TABLE IV

COMPARISON BETWEEN MOUNT WILSON AND ROWLAND THIRD-ORDER PLATES

Wave-Length	Int.	Resolution		Mt. Wilson Measures		Rowland Table Δλ	Rowland Table *minus* Mt. W. Plate
		Rowland 1 mm = 0.64 A	Mt. Wilson 1 mm = 0.57 A	Rowland Plate Δλ	Mt. Wilson Plate Δλ		
3910.984 .135	4 3	$s=c$	$s=c$	0.151	0.153	0.151	−0.002
3914.426 .477	3 2	$s<<<<<<c$	$s<<<<<<c$	.046	.044	.051	+ .007
3918.464 .563	4 4	$s<<c$	$s<c$	.101	.098	.099	+ .001
3919.208 .309	3 3	$s<<<c$	$s<<c$	.094	.091	.101	+ .010
3926.086 (.123) .165	4 3	$s<<<<<c$	$s<<<c$	.075	.069	.079	+ .010
3948.818 .925	4 4	$s<<c$	$s<c$	.106	.100	.107	+ .007
3956.476 .603	4 4	$s=c$	$s=c$	.124	.119	.127	+ .008
3979.664 .783	4 3	$s=c$	$s=c$	.120	.119	.119	.000
3989.912 .011	4 3	$s<<<c$	$s<<c$	.094	.094	.099	+ .005
4024.726 .881	3 4	$s=c$	$s=c$	0.154	0.156	0.155	−0.001
Mean ...							+0.0045

larger scale of the latter compensates for this, so that, in fact, they are somewhat easier to measure. For comparison ten pairs were measured upon both the Rowland and Mount Wilson plates. The data are shown in Table IV. In the third and fourth columns is given an indication of the resolution. For four pairs $s=c$; the

intensity in the space between the components equals that of the continuous spectrum background at their outer edges, i.e., the resolution is complete. The pair at λ 3914 shows only the merest trace of resolution. For the other five pairs the resolution upon the Mount Wilson plates, though far from complete, is a degree better than upon the Rowland plates.

(*a*) The Rowland values exceed the Mount Wilson measures on the Rowland plates by +0.0023 A; this suggests that the personal equation of the observer plays an important rôle. (*b*) The Mount Wilson measures on the Rowland plates exceed the measures on the Mount Wilson plates by +0.0022 A, i.e., owing to the better resolution the influence of an adjacent line is less on the Mount Wilson plates with a correspondingly lessened tendency toward overspacing. The two effects (*a*) and (*b*) account for the Rowland *minus* Mount Wilson mean of +0.0045 A. The discrepancies between the Mount Wilson and the Rowland separations measured upon the same plates are most manifest for pairs near the limit of resolution, thus:

Fairly Resolved λ	R.−Mt. W.	Near Limit of Resolution λ	R.−Mt. W.
λ 3910	0.000	λ 3914	+0.005
3918	−0.002	3919	+0.007
3948	+0.001	3926	+0.004
3956	+0.003	3989	+0.005
3979	−0.001		
4024	+0.001		

7. DISCUSSION

In view of the diversity in the methods of measurement employed at Mount Wilson and the wide range in the scale and resolution of the spectrographic material, it seems probable that the effects due to personal equation have been greatly reduced, if not eliminated, and that the differences between the Rowland and Mount Wilson measures represent systematic errors in the former. A tendency toward over-separation of the components of close pairs appears to accompany departure from the conditions for finest definition. As the Rowland plates were taken for the average run of lines, it seems probable that the errors for closely

adjacent lines would be systematically positive. That the evidence from the Mount Wilson observations is cumulative is seen from the data in Table V, where the results from various lines of investigation are assembled. The magnitude of the error depends upon several factors, the proximity, the intensity, the character, and the wave-length of the components. For lines of like intensity and character in a definite spectral region the error increases with decrease of separation; for lines of the same separation, character, and region it increases with the intensity; for lines of the same separation, character, and intensity it increases with the wave-length.

TABLE V

Résumé of Data

Source	Table I	Table II			Table III		Table IV		Pairs 5167–5169
Wave-length.	4465	4685	4680	4570	4640	3940	3960	3940	5168
Intensity....	2.5	3.5	3.5	3.6	3.5	3.8	3.8	3.1	6.5
Separation...	0.087	0.073	0.274	0.145	0.075	0.107	0.125	0.080	0.149
R.−Mt. W...	+0.006	0.012	+0.003	+0.008	+0.013	+0.004	0.000	+0.005	+0.017

The difficulties of measurement and the systematic character of the errors encountered in this investigation occur in the determination of the absolute wave-lengths of lines with closely adjacent companions. Differential measures upon similar spectra such as those of the two limbs of the sun would not be subject to such systematic errors, provided the spectra were of equal intensity, a condition that gains in importance when the measurements are upon lines not completely isolated.

A résumé of the circumstances that accompany the systematic differences between the Rowland and the Mount Wilson measurements suggests as a cause a personal equation whose influence appears in estimating or neglecting the effect of contrast. Early in this investigation a tendency toward an increase in the measured separations accompanying a lessened intensity of the photographic plate became evident. For example, six Mount Wilson observers, including the four with the widest experience in solar work, measured the separation of the pair at λ 5455 upon two spectra identical

except for a slight difference in intensity. The measurements agree in showing a separation upon the weaker exposure greater by 0.013 A; an appeal to the microphotometer curves supports the smaller value.

An indication of wider import is that a falling off in definition due to any cause is a condition productive of the errors in question. It is immaterial whether the decrease in the sharpness of the lines with the consequent loss in definition is caused by a slight change in focus or a widening of the slit. Two third-order spectrograms, No. 56 and No. 199, throw light on the question. Upon Plate No. 56, considered at the time to be of excellent quality, seven close pairs were measured by three experienced observers with fairly concordant results. Later, spectrogram No. 199 was taken in the same region with a two-normal slit and a backed plate. The plates were taken with the same focal setting and are practically of the same intensity, but No. 199 is very superior in definition. The five exposures were measured by two of the previous observers with very concordant results, which appear in Table VI, together with those from a fifth-order spectrogram. The separations given by No. 56 are consistently larger than those from No. 199, the average difference being +0.008 A. The differences, moreover, are greater for the pairs nearest to the limit of resolution. For four such pairs it is +0.012, while for the three of better quality it is +0.004 A. The appearance of pairs very near the limit of resolution is extremely sensitive to instrumental conditions, 1 mm change of focus in the 30-foot spectrograph—one part in nine thousand—being recognizable in the spectrum of close pairs in which both lines are of good quality. The close agreement between Plate No. 199 and the fifth-order plate furnishes a criterion for deciding between the two third-order plates and for the rejection of the measurements made upon No. 56.

An explanation of these discrepancies based upon the psychological effects of contrast appears to be in harmony with the observations. On line 5167, Plate I, represented by the curve in Fig. 1, the intensity at the symmetrical points *B* and *C* is the same, but *B* is seen against the relatively strong continuous background at *A*, while *C* is seen against the weaker background at *D*.

Under the influence of the difference in contrast, *B* appears of greater intensity than *C* and is of greater influence in estimating the position of the maximum, which is consequently moved outward. A similar effect operates in fixing the maximum of the adjacent line; the combined result is therefore a separation more or less in error in the positive direction, the amount depending upon the personal equation of the observer. The observations show that the increase in this contrast difference, which occurs whenever the intensity

TABLE VI

INCREASE IN SEPARATION ACCOMPANYING DECREASE IN DEFINITION

λ	Int.	Plate 56	Plate 199	Plate 56 − Plate 199	Order V
4455.980 .064	2 3	0.086	0.074	+0.012	0.073
4457.600 .712	2 2	.112	1.08	+ .004	106
4459.199 .301	2 3	.104	.091	+ .013	.095
4464.844 .938	2 1	.093	.091	+ .002	.088
4472.884 .967	1 0	.082	.076	+ .006	.077
4476.185 .253	4 3	.082	.070	+ .012	.066
4482.338 .438	5 3	0.104	0.094	+0.010	0.092
Mean......	....	0.095	0.086	+0.008	0.085

in the region of overlap is decreased relatively to that of the continuous spectrum, is accompanied by increase in the measured separation. The influence in question appears in the case of the pairs at λ 5167 and at λ 5169, where the micrometer measures always exceed those made by the registering microphotometer (p. 20); also in Table II, where the micrometer results average larger by 0.005 A than those found by analysis, and in Table VI, where the spectrogram taken with a wider slit gives separations greater by 0.008 A than Plate No. 199.

When the spectrograms differ only in intensity, the matter is complicated by photographic effect. Whether or not increase of exposure lessens the difference in contrast on the two sides of a line in a close pair depends upon where on the characteristic curve of the plate the density of the continuous spectrum and that of the region of overlap, respectively, fall—a question not yet investigated. The effect studied by Eberhard[1] may also be of influence, though from an investigation by Koch[2] it appears to be negligible.

The wave-lengths of 54 solar lines separated 0.25 to 0.50 A from adjacent lines have been measured upon plates of high dispersion, the neighboring free-standing lines being used as standards. The mean variation from Rowland is ±0.003 A. As over 250 lines were used for reference, it appears that the accidental errors in the relative wave-lengths of the *Preliminary Table of Solar Wave-Lengths* for lines practically isolated are much less than ±0.01 A, a value frequently assumed. It is probable, as Frost and Adams[3] remark, that errors of this magnitude occur but rarely, and mainly then for lines whose measurement is intensely difficult, such as the very weak or strong lines. The proximity and character of neighboring lines introduced disturbing elements that have apparently affected the results for closely adjacent lines and given the errors a systematic character. Information upon which to base a discriminating judgment has not been available, and the user of the table, who has not solar spectrograms of high dispersion at command, could hardly do otherwise than ascribe equal precision to the data for lines that appear in the table without distinguishing signs, though the errors for certain classes of lines are relatively large and systematic.

SUMMARY

1. The measurement of solar lines near the limit of spectrographic resolution is a matter of extreme difficulty and liable to be systematically in error.

2. The Mount Wilson separations have been determined, as far as possible, upon each of five series of spectrograms with dispersions varying from 1 mm = 1.8 A to 1 mm = 0.23 A.

[1] *Phys. Zeit.*, **13**, 288, 1912. [2] *Annalen der Physik*, **42**, 1, 1913.

[3] *Publications of the Yerkes Observatory*, **2**, 155.

3. Separations obtained from curves produced by the registering microphotometer are, for incompletely separated components, smaller than those found by filar-micrometer settings.

4. For pairs near the limit of resolution analysis based upon microphotometer curves, settings upon the edges of the doublet and of lines similar to its components, and ordinary filar-micrometer measurements have been employed. The first appears most reliable and yields smaller values than the third.

5. Separations equal to the theoretical spectrographic resolution, though serving to detect duplicity, are not sufficient for filar-micrometer measurements of wave-lengths to the third decimal place in angstrom units.

6. Spectrograms of the finest definition yield the lowest values for the separation of the components of doublets near the limit of resolution.

7. Filar-micrometer measurements of the separation between the components of close doublets varies with the width of the slit, the precision of the focal settings, and the density of the spectrograms.

8. Whatever decreases the intensity of the common region relatively to that of the continuous spectrum produces a tendency on the part of the measurer toward increased separation. This seems to be an effect of contrast, the observer locating the maximum nearer that edge of the line for which the contrast is greatest, i.e., nearer the free edge.

9. The differences between the Rowland and Mount Wilson determinations of the separation of the components of close pairs of solar lines are systematic. For pairs to the red of λ 4000, component intensities 3 and 4, with mean separations of 0.276 A, 0.145 A, and 0.075 A, the Rowland values exceed the Mount Wilson values by +0.003 A, +0.008 A, and +0.013 A, respectively.

10. That these differences are errors in the Rowland values is made probable by the agreement between the diverse methods used, by the concordance between the spectrograms of different orders, and by the tendency toward over-separation with any departure from best conditions.

Mount Wilson Solar Observatory
June 1916

ON THE TEMPERATURE AND RADIATION OF THE SUN

By C. G. ABBOT, F. E. FOWLE, and L. B. ALDRICH[1]

The paper of Felix Biscoe[2] on this subject seems to require comment from the point of view of the Smithsonian Astrophysical Observatory.

We are not greatly concerned with the part relating to the temperature of the sun, though that is based on our observations. We of course dissent altogether from the treatment of them. Even if one should admit that the properties of the photosphere and solar atmosphere are as assumed by Biscoe,[3] the conclusion stated by him[4] does not seem to us to follow. Like many others, he treats the transmissions of the solar atmosphere as if all that is necessary is to deal with a beam of parallel rays coming directly through the solar atmosphere. He neglects altogether the rays *scattered into the beam.* It is well known that so slight an envelope as our terrestrial atmosphere scatters something like 10 per cent out of the direct solar beam. If we compare the gigantic sun with the pigmy earth it is apparent that even a "thin" atmosphere on the sun must scatter much more. Under solar conditions, contrasted with terrestrial ones, scattering takes place on rays arising from every direction instead of from one direction alone. Accordingly the beam which seems to come from the center of the sun's disk, as (to use a homely illustration) the handle of an umbrella comes from its center, really comes largely by scattering from all sides, just as the strength of the umbrella handle arises from its ribs. On this account we regard Biscoe's "coefficients of transmission of the solar atmosphere" as quite without foundation.

As for his basic assumption of a sharply defined "black-body" photosphere radiating through a thin gaseous envelope which takes no sensible part in the radiation, we have long regarded this as

[1] Published by permission of the Secretary of the Smithsonian Institution.

[2] *Astrophysical Journal*, 43, 197–217, 1916.

[3] *Ibid.*, p. 205, lines 5–11; p. 199, lines 21–22.

[4] *Ibid.*, p. 199, lines 23–24.

improbable, and it grows more improbable with every new discovery. Our views on this subject are so fully published that we need not occupy space here for them.

Turning now to Biscoe's observations, nothing in our experience has prepared us for such a wide range of results as his values of "S_0," the apparent solar constant from pyrheliometry alone, published on p. 211 of his article. If we may express our opinion, either the observations are erroneous or they are made on many days that could be seen by the eye to be unsuitable. He very justly concludes that they do not warrant the belief that their variations are exclusively of solar origin.

Biscoe then refers to our solar-constant measurements made in 1911 and 1912 at Mount Wilson and Bassour, and says that he attributes their variations, which we believe to indicate a variability of the sun, to terrestrial influences and erroneous methods of reduction. He believes that he has shown that when we get high values of the solar constant it is because the atmospheric transmission is low, and vice versa. This conclusion he regards as strengthened by a reference to several Washington observations.

Mr. H. Knox Shaw in a paper of high merit[1] discusses a similar suggestion, and points out in particular the apparently untrustworthy character of our results for the autumn of 1911, both at Mount Wilson and at Bassour.

We admit at the start:

1. For purposes of determining the variability of the sun, Washington observations are too inaccurate to be worth consideration.

2. In the autumn of 1911 the sky, both at Mount Wilson and at Bassour, was so often overcast by cirrus clouds that a large portion of the days had to be discarded, and others of them are really unfavorable, though they were not obviously so to the eye.

3. In the year 1912 the volcano of Katmai made the atmosphere so turbid that the purpose of our expeditions was near to being defeated. We observed the sun's variability under a most unusual and unfortunate handicap.

4. Our own investigations have convinced us that some of the days we have included in Mount Wilson results of each and every

[1] *Helwan Observatory Bulletin*, No. 17.

year are affected by unfavorable atmospheric conditions. Such days are likely closely to precede and follow storms, periods of cloudiness, or periods of great humidity. We have not thought best hitherto utterly to exclude such days, partly because it is difficult to frame a suitable criterion for exclusion, and partly because we alone are making solar-constant observations, and an unsatisfactory value may be better than none.

These admissions made, we stand by the variability of the sun; by the proof of it in the work of 1912; by the close accuracy of our mean value of the solar constant of radiation; and by the soundness of our methods of solar-constant determination.

The variability of the sun is now confirmed[1] by (*a*) Mount Wilson observations of the solar constant, (*b*) comparison of Mount Wilson and Bassour observations, (*c*) comparison of Mount Wilson and Arequipa observations, (*d*) comparison of Mount Wilson and magnetic observations, (*e*) comparison of Mount Wilson solar-constant work with Mount Wilson solar-contrast work. The cumulative effect of this evidence is overwhelming.

We do not give much weight to the 1911 comparison of Bassour and Mount Wilson work. As for 1912, as shown by Shaw, the comparison yields a correlation coefficient of 58 per cent with a probable error of 7.9 per cent. Biscoe will have us admit that this result depends on the combination of three correlations: (*a*) correlation of solar-constant and atmospheric transmission at Mount Wilson; (*b*) correlation of solar-constant and atmospheric transmission at Bassour; (*c*) correlation of atmospheric transmission at Mount Wilson with atmospheric transmission at Bassour. We call attention to Figs. 1 and 2, which show the first two of these supposed correlations graphically. The factors of correlation are respectively:

(*a*) $r = +0.05 \pm 0.12$
(*b*) $r = -0.26 \pm 0.11$
(*c*) $r = +0.59 \pm 0.08$

Correlation (*a*) would result $r = -0.27 \pm 0.12$ if one day should be omitted. The large correlation coefficient (*c*) is of course due to

[1] See *Annals of the Smithsonian Astrophysical Observatory*, **3**; *Smithsonian Miscellaneous Collections*, **65**, Nos. 4 and 9; and **66**, No. 5; *Terrestrial Magnetism and Atmospheric Electricity*, **20**, 143, 1915.

the volcano, and shows how seriously the undertaking was handicapped. But when all is said against the work, we think no one can fairly draw the conclusion that the combination of the two low and inaccurate correlations (*a*) and (*b*) with the very high correlation (*c*) can reasonably be expected to account for the very high correlation of 0.58 ± 0.08 between the solar-constant values themselves as computed by Shaw. It is further to be noted that not only are higher values found at Bassour when higher ones are

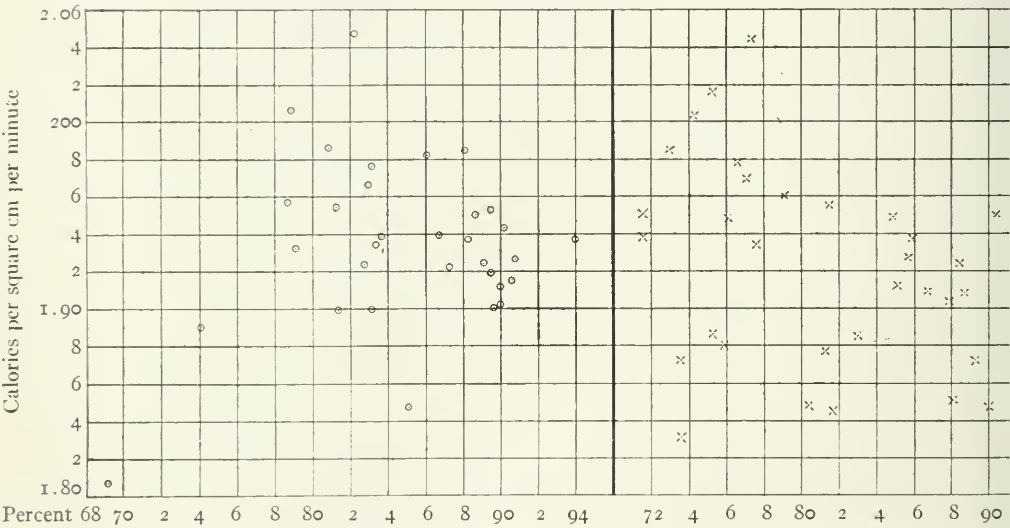


FIG. 1.—Mount Wilson values of solar constants for 1912 and coefficients of apparent atmospheric transmission.

FIG. 2.—Bassour values for 1912

found at Mount Wilson, but that the best representative straight line of Fig. 15 (*Annals of the Smithsonian Astrophysical Observatory*, 3) is at 45° inclination, as predicted according to the hypothesis of solar variation.

Critics should recall besides that if the sun really varies, higher real values of solar radiation may have occurred during part of the time when the atmospheric turbidity due to the eruption of Katmai was great. In that case the small negative correlations (*b*) and (*a*), as altered by removing one day, would be largely apparent, not real. Hence the results of 1912 cannot be regarded as proving

that higher solar-constant results functionally attend lower atmospheric transparency.

In order to see if there is a real negative correlation of our Mount Wilson solar-constant values with the atmospheric transmission, we have selected from all the years 1910–1914 a considerable number of days to which no exception can be taken, either as to atmospheric or as to instrumental conditions. For this purpose we have excluded days which were on the verge of approaching or receding cloudiness or great humidity, days in which the observed solar constant was greatly different from values immediately before and after, days in which cirrus clouds appeared, and days when there were any observations of medium or low grade. The days selected are as shown in Table I. From these data we find the following

TABLE I

Mount Wilson Results, Unexceptionable Conditions

Date	Solar Constant	Apparent Transmission	Date	Solar Constant	Apparent Transmission
1910 Oct. 6...	1.920	0.904	1913 Sept. 28..	1.855	0.889
7...	1.894	.915	29..	1.882	.885
8...	1.892	.906	Oct. 19..	1.873	.889
9...	1.921	.893	20..	1.868	.906
10...	1.940	.906	21..	1.912	.887
Nov. 7...	1.905	.929	22..	1.893	.885
8...	1.907	.929	23..	1.871	.897
			24..	1.882	.900
1911 July 2...	1.911	.885	25..	1.850	.900
3...	1.911	.887	26..	1.871	.887
Aug. 10...	1.927	.933	Nov. 8..	1.902	.906
12...	1.949	.914			
14...	1.925	.912	1914 June 13..	1.943	.895
			14..	1.944	.895
1912 June 7*..	1.939	.908	23..	1.943	.904
18...	1.928	.906	24..	1.936	.895
			Sept. 2..	1.948	.881
1913 Aug. 3...	1.928	.861	3..	1.942	.900
4...	1.916	.867	14..	1.954	.885
6...	1.913	.875	15..	1.965	.887
12...	1.940	.885	20..	1.936	.916
13...	1.927	.890	21..	1.960	.906
14...	1.955	0.883	Oct. 12..	1.951	0.904

* See *Annals of the Smithsonian Astrophysical Observatory*, 3, p. 121, note.

negligibly small correlation coefficient between solar-constant values and apparent atmospheric transmission coefficients:

$$r = -0.06 \pm 0.10.$$

Hence we may conclude that while some of the very best Mount Wilson solar-constant results indicate large solar variation, they show no dependence on the atmospheric transparency.

In conclusion we take this opportunity of expressing our sense of the high value which would attach to the establishment of several additional spectro-bolometric stations for solar-constant observing. Fortunately the Smithsonian Institution will establish such a station in South America soon. But the variability of the sun is so considerable, and its consequences may well be so interesting, that we should welcome steps leading to accurate measurements of the solar variability in at least two other widely separated and highly favorable regions. As we have admitted above, not all cloudless days give good values, even on Mount Wilson. Four independent accurate determinations of the solar constant would not be superfluously many from which to fix the daily values of the intensity of solar radiation outside our atmosphere.

Smithsonian Astrophysical Observatory
Washington, D.C.
May 1916

INTENSITY OF THE CONTINUOUS SPECTRUM OF STARS AND ITS RELATION TO ABSOLUTE MAGNITUDE[1]

By GEORGE S. MONK

Some time ago a comparison[2] was made by Mr. Adams of the intensity of the continuous spectrum of pairs of stars of large and small proper motion photographed upon the same plate, which showed that stars of small proper motion are relatively weaker in the more refrangible portion of the spectrum. Later, a large number of spectrograms obtained for radial velocity were compared and the results tabulated.[3] This comparison showed (1) that in types F0 to K4 stars of small proper motion have spectra which are weaker in the violet than stars of large proper motion; (2) that this difference increases with advancing type from F to K.

Accepting proper motion as an indication of distance, the first result might be interpreted as a consequence of the scattering of light in space; but the second result points to the fact that the absorption in the violet portion of the spectrum is due, at least in part, to physical conditions in the stars rather than to scattering.

In continuation of this method of comparison, the density of the continuous spectrum of most of the stars on the radial-velocity program has been estimated. The method used has been fully described in the second of the articles previously referred to, the only difference being that a negative with five successive exposures of Arcturus has been used as a standard for comparison instead of the plate of α Tauri previously used. Briefly, the method consists of comparing the photographic densities of stellar spectra with the standard plate of Arcturus. The standard plate is then measured under a microphotometer and the densities obtained at the points of comparison. From the estimated relation between

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 119.

[2] *Mt. Wilson Contr.*, No. 78; *Astrophysical Journal*, **39**, 89, 1914.

[3] *Mt. Wilson Contr.*, No. 89; *Astrophysical Journal*, **40**, 85, 1914.

the densities of the standard plate and that of any other star, the actual densities of the spectrum of the star may be deduced. In order to check the accuracy of this method, twenty spectra selected at random were measured under a microphotometer; the results were found to be in good agreement with those obtained by comparison with the standard plate of Arcturus. The densities on about 1200 plates have been estimated and reduced by this method.

It was necessary to reject many plates for the following reasons: (*a*) presence of clouds or haze during exposure; (*b*) great zenith distances; the average zenith distances for groups of stars compared have been made to agree closely; (*c*) photographic blemishes on negatives; (*d*) extremely strong or weak negatives; the average intensities of any two groups compared were made to agree.

By the use of these restrictions, it is believed that the disturbing effects which must be present in a mass of material obtained under such widely differing conditions have been eliminated to a considerable extent. The plate numbers which were used in tabulating the measures show that no one group of stars was photographed within a short range of time, so that variations in color-sensitiveness of the emulsions should not affect the results materially. In most cases two or three plates of the same star have been measured and the mean values of the densities used.

These measures were tabulated for each type, the stars being grouped by large and small proper motion. The results appear in Table I. Slight corrections have been applied to the densities to make the values at λ 4930 equal for each spectrum type, thus affording a direct comparison at λ 4250.

These results are in the general direction of those of Adams and Kohlschütter, but show the connection of proper motion with absorption in the violet and the increase of this effect with spectrum type to a less extent. Soon after Table I had been prepared, Mr. Adams completed his determination of the absolute magnitudes of certain F5–K9 stars on the basis of differences in spectrum lines.

A comparison of the results of Table I with these new data gave Table II, in which "absorption in violet" is the amount of density by which the part of the spectrum about λ 4250 is weaker than that about λ 4930.

This is suggestive of a relationship between absorption in the violet and absolute magnitude—a result which substantiates that obtained by Adams and Kohlschütter, who compared stars of average apparent magnitude 6.0 having small proper motion with

TABLE I

Average Type	Average μ	Density at λ 4250	Density at λ 4930
F1	0.″020	0.41	0.35
F3	0.330	0.39	0.35
F6	0.015	0.40	0.36
F7	0.370	0.42	0.36
G3	0.018	0.27	0.38
G2	0.600	0.33	0.38
G7	0.012	0.29	0.43
G7	0.660	0.33	0.43
K1	0.017	0.23	0.39
K1	0.600	0.28	0.39

stars of fainter average apparent magnitude and large proper motion.[1] Consequently their differences in absolute magnitude were greater than those for the stars of Tables I and II, which were based on proper motion alone, apparent magnitude being disregarded.

TABLE II

Average Type	Absorption in Violet Large μ − Small μ	Differences of Absolute Magnitude Large μ − Small μ
F6–7	+0.02	+1.8
G3–2	0.06	4.0
G7	0.04	2.7
K1	+0.05	+4.4

All of the density measures which could be so used were then tabulated according to absolute magnitudes, giving, as a final result, Table III. The average deviation of a single star from the mean "absorption in violet" in the last column of the table is ±0.09.

[1] *Mt. Wilson Contr.*, No. 89, p. 1; *Astrophysical Journal*, 40, 385, 1914.

Few stars were available having A-type spectra and large proper motion. A comparison of this scanty material shows very little

TABLE III

Type	No. of Stars	Average Type	Average μ	Average M	Absorption in Violet
F3–F9.......	18	F6.1	0".010	+1.7	+0.01
	33	F6.0	0.500	+4.8	−0.05
G0–G4......	17	G2.7	0.015	+0.6	+0.12
	28	G2.0	0.660	+5.2	+0.02
G5–G9......	51	G7.0	0.011	+0.9	+0.20
	22	G7.2	0.690	+6.0	+0.07
K0–K4......	26	K1.4	0.017	+1.5	+0.32
	18	K2.5	0.680	+6.6	+0.16
K5–K9......	....	....	....	....	....
	10	K7.0	0.700	+7.7	+0.22

difference in absorption, the spectra of stars which were used having large proper motion being slightly weaker in the violet.

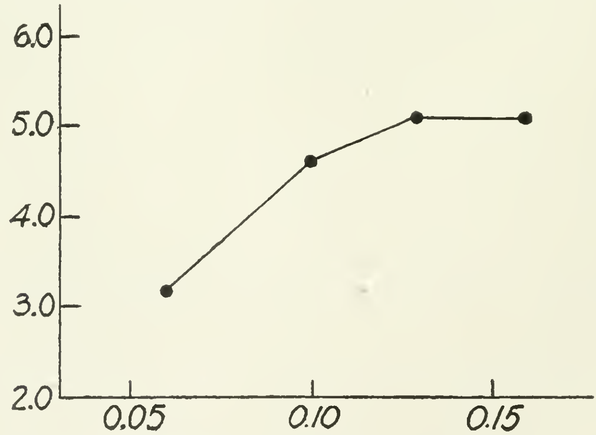


FIG. 1.—Abscissae: Differences of photographic density in the violet for stars of large and small absolute magnitude.
Ordinates: Differences in absolute magnitude.

This result, together with those expressed by Table III, is similar to that obtained by Van Rhijn.[1]

[1] Derivation of the Change of Color with Distance and Apparent Magnitude (dissertation, Groningen, 1915), p. 73.

Plotting the differences in M for each type against the differences in absorption in the violet, the curve in Fig. 1 is obtained. The relationship which seems to exist between absolute magnitude and the relative weakness in the violet of the stars having small

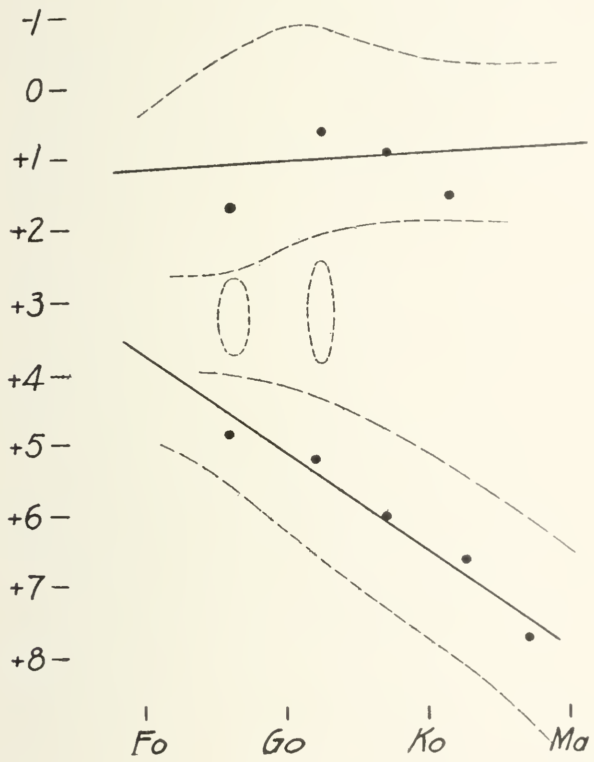


FIG. 2.—Variation of absolute magnitude with spectrum type

proper motion leads to the conclusion that at least the greater part of this effect is due to differences in absolute magnitude rather than to absorption of light in space.

No stars of large absolute magnitude between K5 and K9 were originally available. Seventy-six additional stars of small average proper motion and about 1.5 magnitudes fainter in apparent magnitude than the stars having small proper motion used above have been measured. Mr. Adams has found their average abso-

lute magnitude to be $+2.1$, about one magnitude fainter than those used in Table III. Although the values for the violet absorption fall between the two groups compared, these stars have been omitted, in part because of uncertainty in their proper motions, and in part because many of the photographs are of inferior quality.

An interesting relationship is shown in Fig. 2, in which the average absolute magnitudes of the stars used are plotted against spectrum type. The faint dotted lines show the range of magnitude within each group. The dotted ovals represent seven F and four G stars measured, but not used in this discussion. It will be seen that there is an absence of intermediate magnitudes for the latest types, a result in agreement with the hypothesis of giant and dwarf stars as discussed by Hertzsprung and Russell. It is probable that the greater irregularity of the curve in the case of the stars of small proper motion is due mainly to the relative uncertainty in the determination of their absolute magnitudes.

The photographs of spectra used in this discussion were obtained under a wide variety of conditions, and the results, accordingly, are to be considered mainly from a qualitative point of view. It seems reasonable to conclude from them, however, that with the aid of photographs taken with this purpose directly in view, the relative intensity of the violet portion of the spectrum, together with spectrum type, might be employed to provide values of absolute magnitude of a fair degree of accuracy.

I am indebted to Mr. Adams for much valuable criticism during the progress of the work and in the preparation of this paper.

Mount Wilson Solar Observatory
May 1916

THE PERIOD OF U CEPHEI

By MARTHA BETZ SHAPLEY

Throughout an interval of seventeen years Wendell at Harvard, using a polarizing photometer, made systematic measures of the eclipsing variable U Cephei for the special purpose of investigating variations in its light-period.[1] His observations constitute one of the most accurate means now available for the study of perturbations in double-star systems, for the series is not only of long duration but also peculiarly homogeneous—instrument, method of observing, and comparison star remaining unchanged. Moreover, because of the high accuracy of the measures, others made during the interval covered by them need not be considered, and earlier and later observations need be used only to check and extend the results obtained from Wendell's work. The present discussion of this extensive series was undertaken upon the suggestion of Professor E. C. Pickering.

Nearly all the available observations of U Cephei from the date of its discovery in 1880 were discussed thoroughly by Chandler[2] in 1902. He found no definite evidence of variability in the length of the period. The later Harvard observations, however, failed to conform with Chandler's period and in 1909 Wendell published a new linear formula for the light-variations.[3] His observations, begun in 1895, were continued until 1912, but no further discussion of the period was based upon them and no attempt has been made to harmonize the earlier and the later work.

As the light of U Cephei is constant at minimum for about two hours, it is not expedient to attempt a direct determination of the actual time of zero phase. Wendell adopted the plan of observing mainly the steepest part of the ascending or descending branch.

[1] *Harvard Annals*, **69**, Part I, p. 58, 1909, and Part II, p. 135, 1914.

[2] *Astronomical Journal*, **23**, 227, 1903. Yendell, Wilsing, and others had previously investigated the light-elements.

[3] *Harvard Annals*, **69**, Part I, p. 96, 1909. This formula is the one now used in the ephemeris in the *Vierteljahrsschrift*.

Usually four or five sets of measures were made while the variable was between the eighth and ninth magnitudes; and at nearly every epoch at which the star was observed the time of a given magnitude, say $8^{M}_{.}40$, can be determined from his observations with an

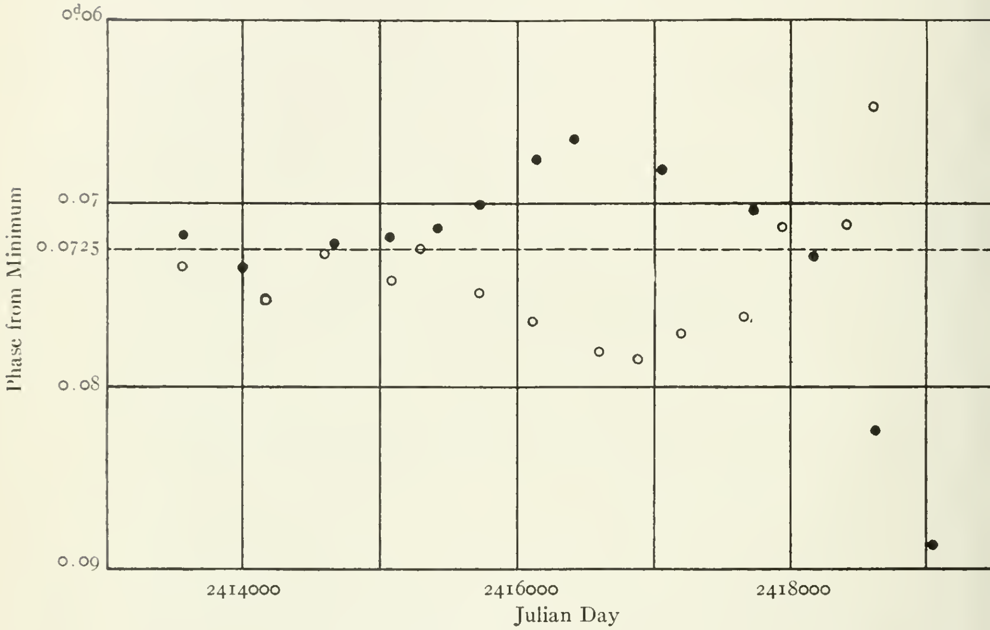


FIG. 1.—Deviations of Wendell's observations of U Cephei from his linear elements
● = Ascending branch
○ = Descending branch

uncertainty of less than a minute. The published phases of all of Wendell's measures were computed from the formula

$$\text{Min.} = \text{J.D. } 2407890.3007 + 2^{d}_{.}4928840 \cdot E,$$

with zero phase at the midpoint of minimum light.

The determination of the mean time of crossing $8^{M}_{.}40$ involved the plotting of each night's work and the superposition and appropriate adjustment of a mean curve upon the individual observations in the chosen interval ($7^{M}_{.}9$ to $8^{M}_{.}9$). All the comparisons made by Wendell, 17,296 in number, are collected into 1081 sets. Of these, 450 fall within the adopted magnitude limits and define the mean

times at 103 different epochs. Table I contains, in order of Julian Day, the data relative to these determinations. The letter in the third column indicates whether the ascending or descending branch was observed at the corresponding epoch.

The 103 determinations in Table I, weighted according to the number of observations involved, were grouped into the 27 normal points of Table II. Ascending and descending branches, which were not combined in forming means, are shown in Fig. 1 as dots and circles, respectively. Variations in the mean phases of both steep branches are immediately apparent from the indicated curves. A study of their form and similarity shows that the variation is not in the width of the minimum, nor due to variability of the comparison star, but is rather a definite change in the length of the light-period—a change, moreover, which is obviously not merely an apparent one due to an erroneous linear formula.

From a discussion of the residuals the average probable errors of an observed phase are found to be as follows:

From one observation,	ascending branch	$\pm 0^{d}.00102$
	descending branch	± 0.00097
From one epoch of minimum,	ascending branch	± 0.00048
	descending branch	± 0.00047
From one normal,	ascending branch	± 0.00023
	descending branch	± 0.00025

Thus the probable error of an average point plotted in Fig. 1 is about 21 seconds of time. But the diagram shows that the greatest deviation from Wendell's elements is about 24 minutes (taking the true time of crossing $8^{M}.40$ as $0^{d}.0725$, indicated by the dotted line). Throughout the seventeen-year interval the two branches of the curve are everywhere in almost perfect agreement as to the amount and sign of the deviation from a uniform period; and the amplitude of the variation from any possible uniform period is as much as 30 times the average probable error of a single normal point.

An investigation of magnitudes at maximum and minimum light yielded some evidence of change in the range of U Cephei at principal minimum, but the data in Table III show that it has no

TABLE I

No.	Julian Day	Branch	No. Obs.	Mean Phase of Crossing 8^{M}40	No.	Julian Day	Branch	No. Obs.	Mean Phase of Crossing 8^{M}40
1...	2413531	D	4	0^d.0720	53..	2416059	A	4	0^d.0680
2...	“	A	3	.0705	54..	6136	D	4	.0794
3...	3581	D	3	.0752	55..	6151	“	4	.0774
4...	“	A	4	.0728	56..	6156	“	3	.0774
5...	3601	“	5	.0717	57..	6226	A	4	.0692
6...	3860	D	1	.0708	58..	6236	“	5	.0678
7...	3950	A	5	.0730	59..	6323	D	4	.0773
8...	3955	“	4	.0751	60..	6383	A	5	.0683
9...	3960	“	5	.0737	61..	6388	“	5	.0678
10...	3965	“	4	.0724	62..	6403	“	3	.0647
11...	3975	“	6	.0754	63..	6408	“	5	.0648
12...	3980	“	5	.0734	64..	6520	D	4	.0795
13...	4067	D	5	.0701	65..	6682	“	4	.0768
14...	4092	A	5	.0735	66..	6697	“	3	.0796
15...	4097	“	7	.0727	67..	6844	“	4	.0827
16...	4189	D	7	.0790	68..	6869	“	4	.0761
17...	4224	“	7	.0758	69..	6874	“	4	.0766
18...	4583	“	3	.0727	70..	6879	“	3	.0793
19...	4588	“	4	.0728	71..	6904	A	3	.0690
20...	4653	A	5	.0719	72..	6909	“	4	.0679
21...	4658	“	2	.0744	73..	7046	D	4	.0774
22...	4663	“	4	.0716	74..	7198	“	4	.0781
23...	4673	“	4	.0720	75..	7208	“	4	.0750
24...	4688	“	5	.0717	76..	7223	“	5	.0772
25...	4947	D	5	.0769	77..	7238	“	3	.0790
26...	5012	A	5	.0717	78..	7258	A	4	.0678
27...	5017	“	5	.0714	79..	7552	D	4	.0753
28...	5114	D	5	.0739	80..	7587	“	4	.0780
29...	5124	“	4	.0708	81..	7632	A	5	.0702
30...	5169	A	5	.0723	82..	7642	“	4	.0702
31...	5271	D	1	.0764	83..	7734	D	4	.0751
32...	5276	“	5	.0715	84..	7739	“	4	.0762
33...	5281	“	6	.0709	85..	7769	A	4	.0700
34...	5316	“	5	.0744	86..	7784	“	4	.0711
35...	5326	“	5	.0734	87..	7794	“	4	.0708
36...	5336	A	5	.0712	88..	7921	D	6	.0677
37...	5361	“	5	.0716	89..	7931	“	5	.0743
38...	5366	“	3	.0705	90..	7936	“	4	.0733
39...	5376	“	3	.0718	91..	7961	A	5	.0708
40...	5513	“	5	.0713	92..	7966	“	5	.0711
41...	5528	“	3	.0716	93..	8290	D	4	.0749
42...	5655	D	5	.0754	94..	8325	A	6	.0771
43...	5665	“	4	.0749	95..	8335	“	6	.0721
44...	5680	A	4	.0688	96..	8442	D	5	.0712
45...	5695	“	5	.0695	97..	8452	“	4	.0676
46...	5710	“	4	.0701	98..	8497	A	4	.0792
47...	5725	“	4	.0724	99..	8609	D	4	.0652
48...	5740	“	5	.0699	100..	8614	“	4	.0643
49...	5812	D	6	.0744	101..	8679	A	5	.0838
50...	5872	A	4	.0704	102..	8684	“	5	.0835
51...	6004	D	5	.0733	103..	9033	“	3	0.0890
52...	6039	A	5	0.0665					

obvious relation to the variation in the length of the period. The table contains nightly groups in order of Julian Day of all observations made by Wendell during maximum light and during the interval of totality at primary eclipse.

TABLE II

No.	Mean Julian Day	Branch	Number		Mean Phase of Crossing 8M40	O−C (Wendell)
			Obs.	Epochs		
1.......	2413552	D	7	2	0ᵈ0734	−0ᵈ0009
2.......	3577	A	12	3	.0718	− 7
3.......	4003	"	41	8	.0736	+ 11
4.......	4154	D	20	4	.0752	− 27
5.......	4586	"	7	2	.0728	− 3
6.......	4668	A	20	5	.0721	− 4
7.......	5066	"	15	3	.0718	− 7
8.......	5071	D	15	4	.0742	− 17
9.......	5299	"	21	4	.0725	0
10.......	5411	A	24	6	.0713	− 12
11.......	5720	D	15	3	.0749	− 24
12.......	5736	A	26	6	.0701	− 24
13.......	6102	D	16	4	.0766	− 41
14.......	6130	A	18	4	.0678	− 47
15.......	6395	"	18	4	.0666	− 59
16.......	6546	D	15	4	.0782	− 57
17.......	6866	"	15	4	.0786	− 61
18.......	7035	A	11	3	.0682	− 43
19.......	7182	D	20	5	.0772	− 47
20.......	7653	"	16	4	.0762	− 37
21.......	7720	A	21	5	.0704	− 21
22.......	7928	D	15	3	.0714	+ 9
23.......	8163	A	22	4	.0729	+ 4
24.......	8398	D	13	3	.0712	+ 13
25.......	8612	"	8	2	.0648	+ 77
26.......	8629	A	14	3	.0824	+ 99
27.......	9033	"	3	1	0.0890	+0.0165

The earlier work on the light-elements by Chandler was based upon more than three thousand observations by Knott, Wilsing, Yendell, Sperra, and Chandler, obtained during an interval of twenty-two years. The material has been adapted in Table IV so as to show the deviations from Wendell's light-elements. The observed times of minimum in the third column are derived by applying the residuals of the fourth column, taken from Chandler,[1] to the times predicted by means of his formula:

$$\text{Min.} = \text{J.D. } 2407890.3229 + 2^{d}.4928761 \cdot E.$$

[1] *Op. cit.*

TABLE III

MAXIMUM AND MINIMUM LIGHT

Julian Day	Mean Phase	Mean Max. Light	No. Obs.	Julian Day	Mean Min. Light	No. Obs.
2413491	$2^{d}.24$	$6^{M}.76$	2	2413531	$9^{M}.18$	7
3589	0.53	6.81	5	3581	9.27	5
3626	0.24	6.78	2	3586	9.20	4
3705	1.92	6.74	5	3601	9.21	6
3734	1.00	6.76	2	3880	9.24	7
3913	0.44	6.78	5	3950	9.18	5
4741	0.83	6.80	12	3955	9.22	5
6430	2.09	6.82	2	4067	9.14	4
6431	0.60	6.78	3	4087	9.13	10
9405	0.60	6.87	5	4092	9.11	10
9415	0.62	6.86	2	4583	9.18	1
9429	2.15	6.94	4	4653	9.15	1
9436	1.70	6.83	4	5827	9.09	1
9441	2.11	6.88	2	6388	9.12	7
				6418	9.14	1
				6423	9.07	4
				6428	9.12	2
				6904	9.12	1
				7258	9.02	1
				8335	9.03	1
				8679	9.04	3
				8684	9.13	1
				9018	9.25	1
				9028	9.13	2

TABLE IV

NORMALS FROM CHANDLER

Number of Minima	Wt.	Mean Observed Time of Minimum	O−C (Chandler)	O−C (Wendell)
3	2	$2408027^{d}.4315$	$+0^{d}.0004$	$+0^{d}.0222$
5	6	08194.4547	+ 9	+ 222
4	6	08386.4071	+ 19	+ 225
6	9	08533.4841	− 8	+ 193
4	4.5	08730.4204	− 17	+ 178
5	8	08887.4709	− 24	+ 166
5	9.5	09064.4640	− 35	+ 149
4	5	09251.4291	− 42	+ 137
2	3	09615.3926	− 6	+ 162
2	4	10313.3934	− 51	+ 95
7	10.5	10597.5931	+ 68	+ 204
3	5	10729.7082	− 6	+ 126
1	2	10921.6664	+ 62	+ 188
2	1	11664.5422	+ 49	+ 151
7	7	12676.6481	+ 31	+ 101
5	9	13167.7398	− 18	+ 37
3	4	13259.9756	− 24	+ 28
1	1	13693.7307	− 78	− 40
5	5	14922.7285	+ 21	+ 20
1	1	15326.5774	+ 51	+ 37
4	3.5	15982.1954	− 33	− 68

The few published observations of minima (complete enough to be of definite value) since the close of Wendell's series are summarized in Table V.[1] Though some are not of high weight, they all tend to confirm Wendell's last observations in showing a large deviation from his linear formula. The fourteen determinations are grouped into normals at the bottom of the table.

TABLE V

LATER OBSERVATIONS

No.	Relative Weight	Observed Time of Minimum	O−C (Wendell)	Observer
1	4	2418886.428	$+0^{d}016$	Bemporad*
2	1	9063.422	.016	"
3	4	9235.440	.024	"
4	3	9240.424	.022	"
5	3	9250.394	.021	"
6	....	9417.417	.020	Lehnert†
7	....	9422.399	.017	"
8	....	9432.374	.020	"
9	....	9442.346	.021	"
10	low	9554.518	.013	"
11	....	9579.446	.013	"
12	5	9711.575	.019	Ginori‡
13	5	9933.443	.020	Bemporad§
14	4	2420115.423	.019	"
		Mean Julian Day		
1, 2	5	2418921	$+0^{d}016$	Bemporad
3, 4, 5	10	9241	.022	"
6, 7, 8, 9	....	9428	.020	Lehnert
10, 11	....	9571	.013	"
12	5	9711	.019	Ginori
13, 14	9	2420014	.020	Bemporad

* *Atti della Accademia Gioenia di scienze naturali in Catania*, Serie V, **5**, pp. 64–68, 1912; *ibid.*, **8**, pp. 5, 16, 17, 1915.

† *Astronomische Nachrichten*, **192**, 199, 1912; **194**, 165, 1913.

‡ *Rivista di Astronomia*, **7**, 245, 1913.

§ *Astronomische Nachrichten*, **199**, 217, 1914.

The normal residuals in Tables II, IV, and V are plotted in Fig. 2, with abcissae in Julian Days and ordinates giving the deviation from Wendell's formula;[2] the dotted line represents the

[1] All times in this paper are heliocentric and refer to the Greenwich meridian.

[2] The plotted square at Julian Day 2408129 is derived from a minimum very carefully observed at Harvard in 1881 by Pickering, Wendell, and Searle (*Harvard Annals*, **46**, 207, 1904).

deviation of Chandler's light-elements. This diagram, which completely illustrates the results of the present study, gives a history of the variations in the period of U Cephei for thirty-three years. Obviously neither Chandler's nor Wendell's elements suffice at present, but the former, with the addition of a sine term of a few

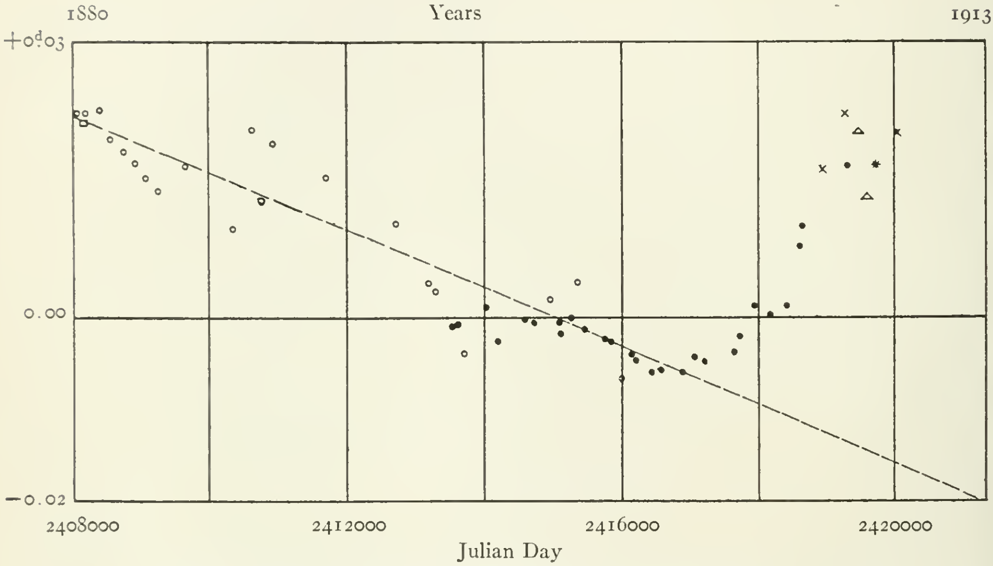


FIG. 2.—Deviations of the period of U Cephei from Wendell's linear elements

○ = Chandler
△ = Lehnert
● = Wendell
✻ = Ginori
× = Bemporad
□ = Pickering-Wendell-Searle

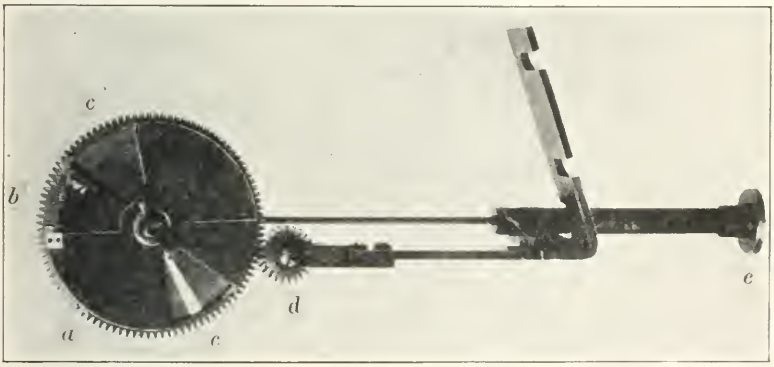
minutes' amplitude and twelve years' period, fulfils requirements up to 1900, though failing since that time. Wendell's elements serve best as a working formula at present, but would probably predict minima too early. The variations are apparently very complex and no attempt is made to obtain an analytical expression for them.

PASADENA, CAL.
April 3, 1916

Minor Contributions and Notes

A NEW DOUBLE OCCULTING SECTOR FOR STELLAR PHOTOGRAPHS

Various devices have been used to reduce the photographic image of a bright star, the parallax of which is sought, to near equality with the images of the comparison stars. The adjustable rotating sector of the type used by Schlesinger in his work at the Yerkes Observatory is commonly employed. Since it is almost always necessary to use comparison stars at least as faint as the



ninth or tenth magnitude, even the maximum reduction, about five magnitudes, makes it impossible adequately to reduce the image of a parallax star brighter than magnitude 3.5 or 4.0.

A simple form of double sector was recently suggested by Mr. Frank R. Sullivan, engineer in charge of the 40-inch refractor, and it has now been constructed in our shops. The photograph reproduced herewith shows the essential parts of the apparatus. It consists of two toothed disks of the same diameter (74 mm), mounted on the same shaft and separated by about 2.5 mm. The second disk, visible through the openings in the main sector at *cc*, has 99 teeth and runs freely on the axis. The main sector *b* has 100 teeth

and clamps down tight on a shoulder by means of a winged nut. A beveled gear behind the disks transmits the motion to the main sector. Both disks mesh with the free pinion *d*, with the result that the second "creeps" one revolution in every hundred. The opening in the second sector is 6°, while each of the two opposite openings *cc* in the main sector can be varied from 0° to nearly 90°. The beveled gear *e* engages a small pinion on the motor shaft.

Assuming 2.75 as the exposure factor for a difference of one stellar magnitude, the reduction of light with this double sector in magnitudes is $2.28 \log \frac{36000}{2n^\circ}$ if both openings in the main sector are left open. For the greatest reduction one of the openings in the main sector may be covered and the denominator in the foregoing expression becomes simply n. Table I gives directly the reduction in magnitudes for scale settings (read at *b*) on the main sector.

TABLE I

Scale Divisions	Degrees Open	Reductions For	
		Single Opening	Double Opening
1	2	9.7	9.0
2	4	9.0	8.3
3	6	8.6	7.9
4	8	....	7.6
5	10	....	7.4
10	20	....	6.7
20	40	....	6.0
40	80	....	5.4

Our usual speed for this sector is five revolutions per second. If the main sector is opened three divisions or 6°, its other sector being covered, we shall get in a 10-minute exposure a total exposure for the occulted star of $\frac{1}{10}$ second made up of at least 30 uniformly distributed exposures. The double sector is interchanged with the usual single sector in a few seconds by removing the one and clamping in the other. All the stars north of declination -15° as bright as magnitude 2.0 have been placed on the program for a determination of parallax with the 40-inch refractor. A plate of the field of

Sirius recently obtained with this sector shows Sirius as a star of magnitude 8.5 on the *B.D.* scale and having a round image suitable for measurement with five minutes' exposure. Another plate of the field of Procyon shows this bright star like one of *B.D.* magnitude 9.1. The five-minute exposure gave an almost ideal image for measurement.

Oliver J. Lee

Yerkes Observatory
April 18, 1916

REVIEWS

The Solar Rotation in June 1911. By J. B. HUBRECHT. "ANNALS OF THE SOLAR PHYSICS OBSERVATORY, CAMBRIDGE," VOL. III, PART I. Cambridge: University Press, 1915. 4to, pp. 77. 9 *s*.

The investigation of the rotation period of the sun by spectroscopic methods has received an exceptionally large amount of attention from astrophysicists during the past ten years. The remarkable character of the law of the sun's rotation and the possibilities of variation in rotation period and in the behavior of different elements have combined to add to the attraction which a radial velocity investigation of high precision naturally possesses for astronomers.

In this volume of the "Annals of the Solar Physics Observatory of Cambridge" are contained the results of an investigation of the solar rotation by Dr. Hubrecht in June, 1911. The instruments used were the McClean horizontal telescope, consisting of a coelostat and 12-inch objective in conjunction with a Littrow grating spectrograph of 14 feet focal length. The spectrograph may be rotated about a horizontal axis in such a way as to secure any desired position angle, and the sun's image is moved with reference to the slit by slow-motion controls on the auxiliary plane mirror. The exposures upon each photograph were made successively, the spectrograph remaining unchanged in position and the image of the sun being moved to bring upon the slit the points between which it was desired to institute a comparison. The most interesting feature of Dr. Hubrecht's observations is the fact that these comparisons were made between points distant from one another by 90° of latitude instead of 180°, as in investigations by previous observers. This has made it possible to discuss the question of the symmetry of the rotation law in the two solar hemispheres.

A description of the method of measurement and reduction of the photographs and the detailed results for the individual plates follow the account of the method of observation. In this connection the criticism should perhaps be made that no details are given regarding the color correction of the image-forming objective. If the focus of the solar image upon the spectrograph slit is made for visual light the color

curve of the objective must be of quite an exceptional character in order that the diameter of the image in light of wave-length 4300 be changed by but one part in one thousand.

From a discussion of his results Dr. Hubrecht arrives at the important conclusion that there is a variation of velocity with wave-length amounting to about 3 per cent at a maximum in the interval between λ 4300 and λ 4400. This difference, after an extended discussion of other possible causes, the author concludes to be due to ray curving in the solar atmosphere, a result which will probably be accepted with some question by astronomers. Naturally the chief objection to this explanation is that if carried to its logical conclusion the difference between the values obtained in the violet and the yellow and red portions of the spectrum should be very large, much larger than there is as yet any reason to suppose.

The result of a comparison of individual lines and elements indicates no certain differences of velocity for different elements, a conclusion in agreement with that of other observers except those at Mount Wilson and Marseilles. Attention may, however, be called to the fact that the enhanced lines should be distinguished from the arc lines in such a comparison. Thus for titanium we have as a percentage residual in the table given:

3 arc lines $+0.3$ 3 enhanced lines -0.3

These quantities are small and may possess no significance. They are, however, in the direction found by some other observers.

The lack of symmetry found by Dr. Hubrecht between the Northern and Southern hemispheres as regards rotational values is a result of great interest, and provides ample justification for the method of observation adopted by him. The existence of local drifts during the comparatively short interval of time covered by the observations is a most probable explanation of the differences obtained. This may perhaps account as well for at least a part of the discrepancy between his result of $13^{\circ}.2$ for the angular velocity at the equator and that of Plaskett of $14^{\circ}.4$ for a slightly later epoch. In view of the probability of a variation in the sun's period of rotation, as shown by recent observations, such a comparison should be limited to results of closely the same date.

Although some of the results obtained from this investigation of Dr. Hubrecht must await confirmation from future observations, the questions raised will add materially to the interest already attaching to the study of this important problem.

W. S. Adams

KARL SCHWARZSCHILD

It is with deepest regret that we record the death, on May 11 at Potsdam, of Karl Schwarzschild, director of the Royal Astrophysical Observatory of Potsdam. His name has been associated with the *Journal* as a collaborator since 1911. In the death of Schwarzschild, science has suffered an irreparable loss. Hardly more than forty-two years of age, he had already distinguished himself by his contributions to mathematics, physics, and to both practical and theoretical astronomy. We hope to publish in a future issue an appropriate sketch of his life and work.

THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XLIV — SEPTEMBER 1916 — NUMBER 2

THE POLE-EFFECT IN A CALCIUM ARC

By WALTER T. WHITNEY

In the task of establishing accurate standards of wave-length in the spectrum of the iron arc, Goos[1] observed that the wave-lengths of certain lines changed in passing from center to pole of the arc, and that similar changes were caused by increase of current and shortening of the arc. These changes were similar to those caused by increase of pressure, and Goos accounted for the displacements observed on the hypothesis that there existed differences of pressure along an arc and between arcs burning under different conditions.

St. John and Babcock[2] in their work on the pole-effect in the iron arc found displacements of the *d* lines which, on the pressure hypothesis, would demand differences of 1.5 to 2 atmospheres, while certain other lines sensitive to pressure remained unaffected. They state: "It appears, therefore, from the data, that the displacements shown by the lines of groups *c*5, *d*, and *e* are not due to a general increase of pressure in the vapors near the pole of the arc."

Royds[3] suggested that the inconsistencies between Swaim's[4] observations of the pressure-shift of the series lines of zinc and the

[1] *Astrophysical Journal*, **38**, 141, 1913.

[2] *Ibid.*, **42**, 131, 1915.

[3] *Ibid.*, **41**, 154, 1915.

[4] *Ibid.*, **40**, 137, 1914.

general conclusions put forth by Humphreys regarding pressure-shift and series are due in part "to the existence of a density-effect superposed upon the true pressure-effect," since the series lines dealt with are very unsymmetrically widened. The same objection is applicable to the pole-effect measures, but the observations reported in a previous paper,[1] as well as those of St. John and Babcock, point to no change of wave-length with large differences of vapor-density. This point bears more weight in the case of the Ca series lines measured, both first and second subordinate, where much dissymmetry is present.

The purpose of this investigation was to determine the pole-effect for all the principal lines of the Ca spectrum. The photographs were taken in the second order of a 21-foot Rowland concave grating on plates 2×19 inches, which embrace 600 angstroms. The plate scale is 1.32 A per mm. Cramer "Crown" plates were employed in the violet and blue, Cramer "Inst. Iso" in the yellow and green, and Cramer "Spectrum" in the red.

The calcium arc, burning in a horizontal position, was formed between rods of metallic calcium 9 mm in diameter and 2–5 cm long which were firmly clamped in brass holders. In this work the arc was kept about 4 mm long and carried 4 amperes at 110 volts. An image of the arc, enlarged four times, was projected upon the slit of the spectrograph by means of a quartz lens of sufficient aperture to more than completely fill the grating. The negative or cooler electrode was rather sharply pointed and the arc here remained as steady as could be desired. The positive electrode, however, was made with a flat face, since it was found that the arc at the positive pole would not remain at the tip of a pointed terminal but would shift back and forth and finally become extinguished by increasing its length.

In order to avoid overheating of the positive terminal and the consequent rapid oxidation, accompanied by unsteadiness and bad flaring, this electrode was made longer than the negative and surrounded by a small water-jacket (Fig. 1). The positive electrode holder was also mounted in a bearing and provided with a fiber hand-wheel, so that the whole electrode could be rotated through

[1] *Astrophysical Journal*, 43, 161, 1916.

the water-jacket. In this manner the positive electrode was not only cooled, but as the convection currents carried the arc to the upper side of the positive terminal face the electrode could be rotated to prevent this, and as a result the whole face of the electrode was burned away evenly. This simple device worked a transformation upon a very unsteady arc and since its installation flaring has not been observed. The practical constancy of the arc under these conditions is also well shown by the argreement between the measures from different plates.

Considerable difficulty was at first experienced with mechanical shift, since, on account of the astigmatism of the concave grating, the occulter for producing the comparison spectrum must be

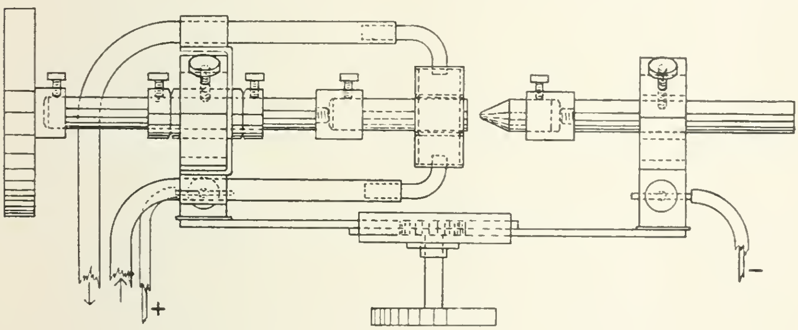


FIG. 1

mounted before the plate and in this case was supported by the camera box which also supported the plate carriage. In this way it was impossible to shift from the central to the comparison spectrum without introducing relatively large displacements of varying sign and magnitude.

A new occulter was therefore constructed (Fig. 2), which consisted of a brass cylinder 53 cm long and 10 cm in diameter. Slots the full length of the photographic plate were cut in the side of the cylinder at *c* and *a*. A wire was drawn through a die to give it the cross-section indicated, and stretched in the middle of the aperture at *c*. This double slot was used for the comparison spectrum. The ends of the cylinder were capped, and the whole was mounted in bearings so as to rotate about a horizontal axis. This

rotation was, however, restricted by the stop-arm which in moving from the stop S_1 to stop S_2 brought the aperture at a into the position formerly occupied by the wire in the aperture at c. A small slider d could be placed between the arm and stop S_2, so that a fourth spectrum might be photographed above the others. This was found to be a great aid in identifying lines due to impurities. A large section was removed from the side of the cylinder toward the grating to permit the light to pass unhindered to the occulter slits and plate. The strips a and b were made adjustable so that

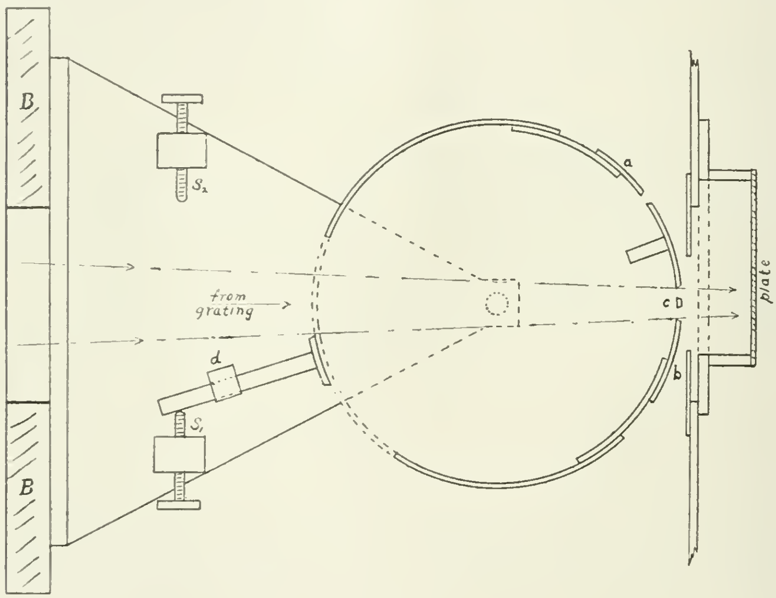


FIG. 2

different widths of spectra could be employed. In this investigation the central and comparison spectra were about 1.5 mm wide and separated by about 0.5 mm. The mounting board BB was supported upon clamp stands directly from the cement floor of the grating room, and in this way the whole occulter was mounted independent of the camera, yet much nearer the plate than was the case with the former arrangement.

In order, however, to detect any small instrumental displacements which might occur at times, the following device was

employed. The light from an incandescent bulb was dispersed by means of a small direct-vision spectroscope, and a second slit was placed in the position of the eyepiece. This second slit could be regulated in width, and the color and intensity of the light falling upon it could be chosen at will. An image of it was projected upon the plate, generally near one end, by a good long-focus lens. This furnished a very sharp line which could be accurately measured for plate-shift. All the apparatus involved was mounted directly upon the cement floor and independent of the spectrograph.

It was also found desirable to take all the plates between midnight and morning as at other times the mechanical jars of the building were sufficient to cause considerable plate-shift during exposures. In this manner and with the aid of the new occulter, spectra were obtained which were practically free from plate-shift.

Considerable care was taken to avoid overexposure. This is especially necessary in the case of unsymmetrical lines, as otherwise false photographic shifts of the center of maximum density are easily obtained. In all cases the effort was made, in measuring, to place the cross-hair of the comparator microscope upon the position of maximum density in the photographic image.

The spectra were measured on a small Gaertner comparator provided with a screw of 0.5 mm pitch, the smallest division on the large graduated head reading 0.001 mm. By using the narrow spectra described above, it was possible to increase the magnification as far as the plate grain would permit, and in this way to increase somewhat the accuracy of measurement.

Table I contains the data from the water-cooled arc. The first column gives the wave-lengths and series designation, if any. Saunders'[1] notation has been followed with the exception that in the cases of the two subordinate series and the narrow triplet series a subscript has been added to indicate the number of the group in the series formula. It has not been possible to detect any systematic difference in the pole-effect for the different lines of any single triplet, and these measures have been averaged and the mean taken as the shift for the group. Column two gives the intensity and character of the lines; "h" indicates a hazy line and "hR" or "hV"

[1] *Astrophysical Journal*, 32, 152, 1910.

TABLE I

Pole-Effect for Calcium. Atmospheric Pressure (75 cm, 20° C.)

λ Series	Int.	Pos. − Center	No.	Av. Diff.	Neg. − Center	No.	Av. Diff.
6499.8 p.	3	+0.005Å	4	0.001Å	0.000Å	4	0.001Å
93.9 p.	7	+ .003	4	.002	.000	4	.002
71.8	4	+ .004	4	.001	.000	4	.001
62.7	8	+ .003	4	.001	− .002	4	.001
49.9 p.	4	+ .003	4	.001	− .003	4	.001
39.3	8	.000	4	.001	− .001	4	.001
6169.8	3	+ .017	4	.001	.000	4	.001
62.4 T_{23}	9						
22.4 T_{23}	8	+ .017	9	.001	− .001	9	.001
02.9 T_{23}	7						
5857.7	8hR	+ .032	4	.003	+ .001	4	.001
5603.1	6	+ .011	4	.003	− .003	4	.003
01.5	6	+ .011	4	.004	− .003	4	.003
5598.6	6	+ .008	4	.001	− .003	4	.001
94.6	8	+ .011	4	.004	− .001	4	.003
90.3	6	+ .009	4	.003	− .003	4	.003
88.9	8	+ .008	4	.003	− .004	4	.003
82.1	9	+ .009	4	.001	− .003	4	.003
13.1	6hV	− .032	4	.001	− .003	4	.001
5349.6	8	+ .007	4	.001	− .003	4	.001
5270.4	8	+ .018	4	.001	.000	4	.001
65.7	8	+ .016	4	.001	.000	4	.001
64.4	6	+ .014	4	.003	.000	4	.001
62.4	6	+ .016	4	.003	.000	4	.001
61.9	6	+ .016	4	.003	.000	4	.001
5189.0	7	− .007	4	.001	+ .003	4	.001
5041.9 SL2	7hR	+ .024	6	.004	− .003	6	.001
4878.3 SL3	8hR	+ .045	4	.004	.000	4	.001
4685.4	4hV	− .012	1	....	+ .002	2	.001
4586.1 t_4	6hR						
81.6 t_4	5hR	+ .040	6	.005	− .004	9	.001
78.8 t_4	4hR						
4527.1 SL2	4hR	+ .036	6	.003	− .008	6	.001
4456.1 $T1_4$	6						
54.9 $T1_4$	9						
35.8 $T1_4$	6	− .003	30	.003	+ .001	30	.003
35.1 $T1_4$	8						
25.6 $T1_4$	7						
4355.4*	3hR	....	......	....	....	......	....
4318.8 T	6hV	− .003	6	.001	+ .001	6	.001
07.9	6	− .001	6	.001	.000	6	.003
02.6 p.	7	− .001	6	.001	.000	6	.003
4299.1 T	5hV	− .003	6	.001	.000	6	.001
89.5 T	6hV	− .001	6	.003	+ .001	6	.003
83.1 p.	6	− .003	6	.003	.000	6	.003
4240.5†	3h	....	......	....	....	......	....
4226.9	15	+ .005	6	.001	.000	6	.001
4098.6 t_5	5hR						
95.0 t_5	4hR	+ .055	4	.007	− .013	6	.004
92.7 t_5	3hR						
3973.8 T_{24}	7hR						
57.2 T_{24}	6hR	+ .024	12	.005	− .003	12	.003
49.1 T_{24}	5hR						

* λ 4355 was too diffuse for measurement.

† λ 4240 was masked by λ 4226.

TABLE I—*Continued*

λ Series	Int.	Pos.—Center	No.	Av. Diff.	Neg.—Center	No.	Av. Diff.
H 3968.6 PH.	20	+0.002A	10	0.003A	0.000A	10	0.004A
K 33.8 PH.	25	+ .002	10	.001	.000	10	.003
3737.1 P2..	8	+ .012	10	.004	+ .001	10	.003
06.1 P2..	9	+ .012	10	.005	+ .001	10	.004
3644.8 T15.	5hV						
44.5 T15.	8hV						
31.1 T15.	4hV	− .037	15	.009	+ .005	15	.003
30.8 T15.	7hV						
24.1 T15.	6hV						
3487.7 T25.	5hR						
74.9 T25.	4hR	+ .030	16	.005	− .003	9	.001
68.6 T25.	3hR						
3361.9 T16.	7hV						
50.2 T16.	6hV	− .050	11	.010	+ .011	12	.004
44.4 T16.	5hV						
3286.2 T26.	4hR						
74.8 T26.	3hR	+ .034	4	.005	− .005	4	.001
69.3 T26.	2hR						
3181.4.....	7	+ .008	6	.003	− .003	4	.003
79.4 P1..	8	+ .008	8	.004	+ .001	8	.003
58.9 P1..	9	+ .010	8	.003	.000	6	.003

The lower members of the subordinate series groups were too diffuse for measurement.

one unsymmetrically widened toward the red or violet. Column three contains the measured pole-effect between center and positive. The sign has been chosen such that + indicates a greater wave-length at the pole. Column four gives the number of observations and column five the average difference from the mean. The last three columns contain the data for the negative pole.

It is to be noted from Table I that in general the shift at the negative pole was less than that at the positive pole and of opposite sign. This fact is very striking, in connection with the subordinate series lines where the shifts are relatively large, and is in marked contrast to the observations of Goos and of St. John and Babcock in the iron arc. These observers found that for the lines of the *d* and *e* groups in the spectrum of iron the shift in going from the center of the arc to the negative pole was in the same direction and greater than that found in going from the center of the arc to the positive pole. St. John and Babcock state that a longer exposure was required at the center of the arc than at either pole in order to secure lines of the same strength. In the calcium arc a longer exposure was required at the negative pole than at the center of the

arc, and a longer exposure at the center than at the positive pole. At the positive pole practically all lines are about three times as strong as in the center of the arc. More variation, however, was observed at the negative pole. Here the triplets and single-line series have about one-half the intensity shown in the center of the arc. Many of the non-series lines show no difference in intensity between center and negative pole, while the pair series lines show a greater intensity at the negative than in the center of the arc. All lines are broadened and hazy at the positive pole, and in the case of the pair series lines, although they are stronger at the negative pole than in the center of the arc, they are nevertheless sharper at the pole.

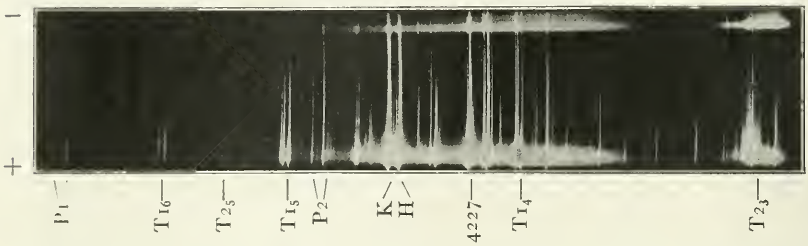


FIG. 3

To show these changes of intensity from pole to pole, the image of the arc was projected upon the slit of a quartz spectrograph and the intensity-gradients along the lines were observed as shown in Fig. 3. The scale of the quartz plates toward the red end of the spectrum was rather small, hence the arc was also photographed in the first order of a 30-foot Littrow spectrograph. These plates showed that intensity-gradients existed throughout the spectrum similar to those indicated by the quartz spectrograms for the violet region.

The pole-effects for the groups of the first and second subordinate series are collected in Table II.

It will be observed that these values for positive minus center are somewhat larger than those reported in a previous paper,[1] while the values for negative minus center are less. This difference is no doubt due to the use of the water-cooled arc in the present

[1] *Astrophysical Journal*, 43, 161, 1916.

investigation, since on account of the greatly increased steadiness it was possible to keep the slit of the spectrograph upon the bright spot at the pole of the arc. In this way a greater difference of intensity was introduced between the positive pole and the center of the arc and the greater pole-effect observed supports the hypothesis advanced in the previous article.

TABLE II

Pole-Effect for First and Second Subordinate Series

Series	Pos.—Center	Neg.—Center	Series	Pos.—Center	Neg.—Center
T14.........	−0.003A	+0.001A	T23........	+0.017A	−0.001A
T15.........	− .037	+ .005	T24........	+ .024	− .003
T16.........	− .050	+ .011	T25........	+ .030	− .003
			T26........	+ .034	− .005

At the negative pole, furthermore, the photographs of the intensity-gradients, made in the larger scale of the Littrow spectrograph, showed that the intensity of these lines diminished continuously as the pole was approached, but that a small increase in intensity was experienced as the line traversed the bright spot which terminated the arc. In this case, then, the difference of intensity between the center of the arc and the negative pole was less and the corresponding pole-effect was also less than that observed before.

The relations between pole-effect and wave-length have been investigated for the first and second subordinate series, the narrow triplet series, the single-line series SL2, and also for three groups of so-called non-series lines which appear at λλ 6499–6449, λλ 5603–5582, and λλ 5270–5261. The average wave-length of a group, or triplet, was compared with the average total pole-effect or positive minus negative as obtained in Table I. The number of determinations available for any one sequence, however, was necessarily restricted and but little weight can be attached to the results obtained.

The data for the first subordinate series are of some interest, as Royds[1] has called attention to the peculiar appearance of the lines of

[1] *Ibid.*, 41, 154, 1915.

this series in the spectrum of barium together with the somewhat similar case for calcium, and from Table II it is seen that the pole-effect for the group $T1_4$ was small as compared with that observed for the other two groups of this series. This would seem to indicate, in accordance with Royds's suggestion, that the pole-effect for the group $T1_3$ in the infra-red might be of the opposite sign to that observed for the other members of the series. The difficulty attending the measurement of these very unsymmetrical lines is quite well known. The departure here, however, could hardly have resulted from the unsymmetrical widening which appeared with the fifth and sixth groups only, since the third group in the second subordinate series at λ 6160 was quite as symmetrical as the fourth group of the first subordinate series, but no similar peculiarity in the pole-effect and wave-length relations was observed. It is of interest to note in this connection that the lines in the group at λ 6160 were at times unsymmetrically reversed, but that whether reversed or not the measured pole-effects were identical.

The pole-effect and wave-length data for the second subordinate series were more systematic and indicated that here the pole-effect was inversely proportional to the cube root of the wave-length. This is shown by the curves of Fig. 4. The abscissae are wave-lengths and the ordinates for curve *a* are $1/P$, for curve *b*, $1/P^2$, for curve *c*, $1/P^3$, and for curve *d*, $1/P^4$, where P is the total pole-effect, or positive minus negative. The values of the ordinates were so adjusted that the first and last points on the four curves were coincident. It is of considerable interest to note that curve *c* intersects the wave-length axis at very near λ 2938, the convergence wave-length of the series. Swaim found the pressure-shift of the second subordinate series lines of zinc to be inversely proportional to the first power of the wave-length.

A similar treatment of the data for the non-series groups showed their pole-effect to be inversely proportional to about the seventh or eighth power of the wave-length, while that for the narrow triplet series and the series SL2 seemed to be inversely proportional to the fourth and fifth power of the wave-length. No great weight should be attached to these results, however, as the data are altogether too meager.

In conclusion it may be said that the additional data brought forward in this investigation confirm very closely the conclusions stated in the first paper, namely, that the pole-effect depends in general upon the existence of an intensity-gradient along the arc

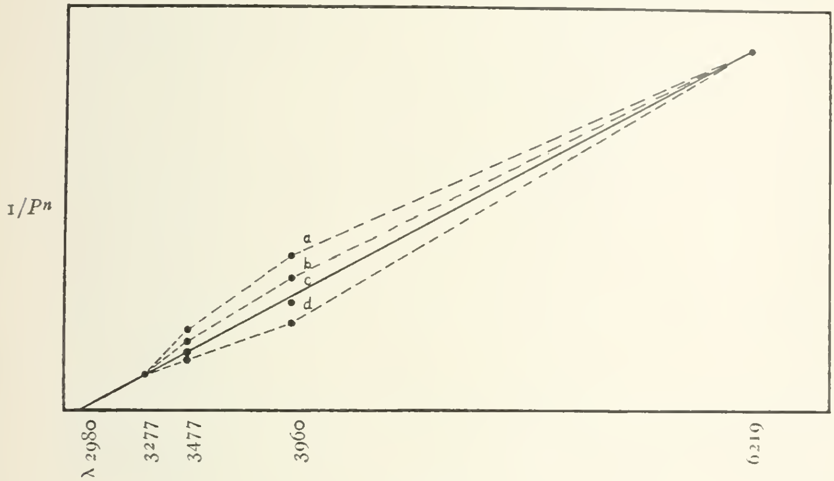


FIG. 4

and the hypothesis that the amplitude of vibration of the electrons is the determining factor in the pole-effect is thereby strengthened.

I wish here to express my thanks to the members of the Department of Physics and especially to Professor Gale, who suggested the problem, for his continued interest and assistance.

RYERSON PHYSICAL LABORATORY
June 1916

ON SPECTROSCOPIC RESOLVING POWER

By C. M. SPARROW

If a spectroscope is just able to separate two monochromatic lines of equal intensity and wave-lengths λ and $\lambda+\Delta\lambda$, the ratio $\frac{\lambda}{\Delta\lambda}$ is called the resolving power of the instrument for the wave-length λ. This is the *definition* of resolving power, and if we can determine by actual measurement the value of $\Delta\lambda$ for some particular instrument, we can obtain the resolving power of that instrument. If, however, our problem is to calculate the resolving power from the optical theory of the instrument, the *definition* must be supplemented by a *criterion* of some sort which will enable us to say when the two lines are to be considered as just resolved. In the case of a prism without absorption, or of a grating with many lines, the criterion proposed by Rayleigh[1] has hitherto been universally adopted. The intensity in a single line being given by

$$I=I_0\,\frac{\sin^2 x}{x^2}\,, \tag{1}$$

and that due to two lines by

$$I=I_0\left\{\frac{\sin^2(x-a)}{(x-a)^2}+\frac{\sin^2(x+a)}{(x+a)^2}\right\}, \tag{2}$$

the two lines are considered as just resolved when $a=\frac{\pi}{2}$, that is, when the maximum of one line coincides with the first minimum of the other. Under these conditions the composite diffraction pattern has a distribution of intensity given by the familiar curve 6 of Fig. 1. The ratio $\frac{I_{\text{min}}}{I_{\text{max}}}$ is in this case $\frac{8}{\pi^2}$ or about 0.81.

As originally proposed, the Rayleigh criterion was not intended as a measure of the actual limit of resolution, but rather as an index of the relative merit of different instruments. In the form in which it is stated above, the criterion is applicable only to instruments

[1] *Philosophical Magazine* (4), **47**, 193, 1874; (5), **9**, 266, 1879; also article on "Wave Theory" in the *Encyclopaedia Britannica.*

whose diffraction pattern is of the form (1). For such instruments it is as good an index as any other, and leads to simple formulae for the prism and grating. For instruments such as an absorbing prism or a Fabry and Perot interferometer it ceases to be immediately applicable. For such instruments we may, it is true, express the criterion in the form

$$\frac{I_{\text{min}}}{I_{\text{max}}}=0.81, \tag{3}$$

and this course has been generally adopted heretofore.[1] But now the criterion has lost its simple theoretical significance, and the choice of the value 0.81 for the right-hand side of (3) has become an arbitrary one. Moreover, the relative merit of different instruments will vary with our choice of the right-hand member of (3). Thus suppose that a grating and a Fabry and Perot interferometer have equal resolving power on the basis of (3). On a 90 per cent basis the interferometer would be superior, while on a 70 per cent basis the advantage would lie with the grating. If we should follow Schuster's proposal[2] and take complete separation as a basis, an infinitely thick prism with finite absorption would have zero resolving power.

It should be clear from the foregoing that the only fair basis on which such different instruments can be compared involves the adoption of a criterion which gives a measure of the actual limit of resolution. It has hitherto been assumed by many that the Rayleigh criterion does this, but the basis of fact on which this assumption rests is small and inconclusive; and, as we shall see, the true limit is quite different.

In the present paper we shall present the results of an empirical study of the actual appearance, visual or photographic, of different doublets. In this way the actual limits of resolution are determined. The results of these observations lead to the formulation of a criterion with a simple theoretical basis and applicable to a great variety of instruments. In addition the limits of resolution

[1] See, for example, Wadsworth, *Philosophical Magazine* (6), 5, 355, 1903, where the effect on resolving power of absorption in a prism system is calculated.

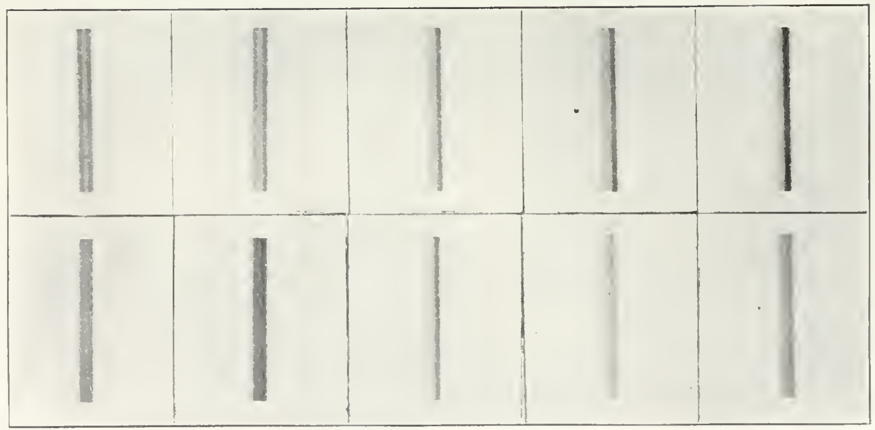
[2] *Theory of Optics* (London, 1909), p. 158.

for two lines of different intensities have been determined—a case for which the theory gives no inkling as to what we may expect.

The experimental method is simple, and by no means new, being the same as that used by Langley for the conversion of bolographs into spectrographs. The form of a diffraction pattern was calculated and the intensity-curve, in rectangular co-ordinates, was drawn carefully on black paper. The area between the curve and the x-axis was then cut out, making a screen with an aperture of the required form. This screen was placed against a uniformly illuminated background and viewed or photographed with a cylindric lens having its axis of curvature parallel to the y-axis of the curve. In this way "artificial doublets" of any form and separation could be produced. The screens were about the size of a lantern slide, and the lens, of about 15 cm focus, was about 10 m from the screen. The camera was fitted with a multiplying back, so that six exposures could be taken on one plate. In order to test the focus, one exposure on each plate was made of a screen with a pair of narrow parallel slits. Hammer lantern plates (white label) were used; they were developed with hydrochinon. Visual observations on the lines led in all cases to the same results as the photographs; hence they are not specially mentioned in what follows.

Grating with infinitely narrow slit.—The actual appearance of a doublet whose separation is that given by the Rayleigh criterion is shown in the first row (1–5) of photographs in Plate II for different relative intensities of the two components. Considering for the moment only the case of equal intensity, it is obvious that the lines are quite distinctly resolved. On the plates the effect is so much more pronounced that most spectroscopists would call the separation measurable. The numbers which give the separation are the values of $2a$ in (2). They are thus half the phase difference in radians between the maximum of either line and the position on its diffraction pattern where the other maximum falls. The corresponding intensity-curves for each line are given in the first row of Fig. 1. In the second row of photographs (6–10) the components are closer, but are still clearly resolved. (The intensity-curves bear the corresponding numbers in Fig. 1.) As the lines are brought

PLATE II



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still closer, the central minimum becomes shallower, until it finally disappears. To find the value of $2a$ corresponding to this condition

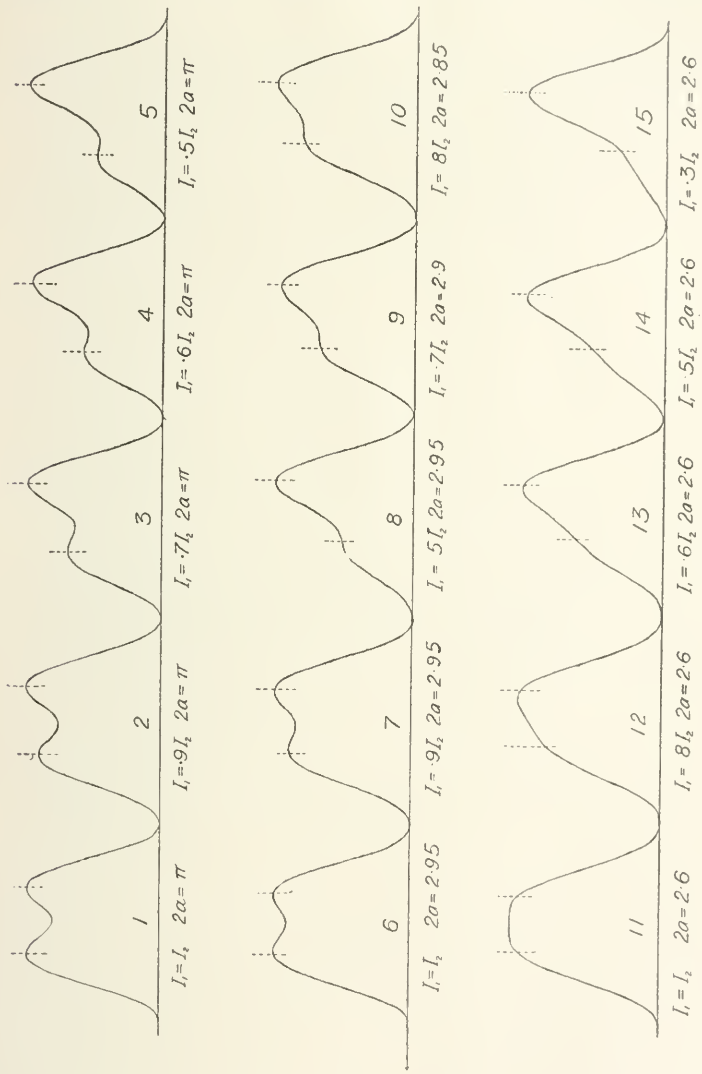


Fig. 1

(for equal intensities) we may differentiate (2) twice with respect to x and put

$$\frac{\delta^2 I}{\delta x^2}=0 \text{ when } x=0, \tag{4}$$

and solve for a. Since the two curves are symmetrical, the odd derivatives necessarily vanish at the origin, and thus (4) gives the composite curve an "undulation" at the origin. I shall refer to (4) hereafter as the "undulation condition."

It is obvious that the undulation condition should set an upper limit to the resolving power. The surprising fact is that this limit is *apparently actually attained*, and that the doublet still appears resolved, the effect of contrast so intensifying the edges that the eye supplies a minimum where none exists. The effect is observable both in positives and in negatives, as well as by direct vision. It cannot be seen in prints because of insufficient illumination. I have therefore not attempted to reproduce it here, but have given only the forms of the intensity-curves (11–15, Fig. 1). My own observations on this point have been checked by a number of my friends and colleagues. The same phenomenon has been noted by Wood in connection with the apparent reversal of a broad spectral line. A very slight further diminution of the separation rounds the top of the intensity-curve so that there is no resolution; the undulation condition thus defines the limit of resolution quite sharply.

A solution of (4) by successive approximations leads to the value $2a = 2.606$. Thus the actual resolving power of a perfect grating of n lines in the Nth order is

$$\frac{\pi}{2.606} \cdot Nn = 1.26\, Nn, \tag{5}$$

and similarly for a prism of thickness t and refractive index μ,

$$1.26\, t\, \frac{d\mu}{d\lambda}. \tag{6}$$

When the intensities are unequal, the form of the intensity-curve is of course completely altered. Nevertheless the actual observations show the remarkable fact that the *limit of resolution remains about the same*. In this case it is of course impossible to say definitely where resolution stops: a line which one observer would call resolved would perhaps be regarded by another observer as a single line shaded on one side. Nevertheless the form of the intensity-curve is quite sensitive to small changes in separation,

the result being that the actual limits given by different observers who have examined the plates vary by only a small percentage. When the ratio of intensities is greater than 10:2 the appearance of the doublet is complicated by the greater relative prominence of the secondary maxima; it has therefore been found difficult to draw definite conclusions for intensity-ratios greater than this. The form of the intensity-curves is rather noteworthy; the curves, 8, 9, for example, show no actual minimum, but slope away continuously from the vertex, while those (11–15) which correspond to the limiting case hardly suggest a doublet by their shape.

Instruments with diffraction patterns other than the normal one.—Besides imperfections in the optical surfaces (which we shall not consider here) there are two principal causes for the deviation of the diffraction pattern from the form (1); namely, the use of a slit of finite width, and absorption or loss of light by reflection. The previous work with the normal pattern having shown that the resolving power varies at most only slightly with the relative intensity, it was found possible in subsequent work to simplify the experimental method. Instead of making a separate screen for each doublet (about a hundred such were used for the observations described above), one screen was made having the form of the diffraction pattern of a single line. This screen was mounted so that it could be displaced parallel to the x-axis through any required distance, and the doublets were made by superposing two exposures of the screen in different positions on the same plate, intensities being regulated by the time of exposure.

Two cases were studied in detail: that of a grating with "4-normal" slit-width, and that of an infinitely thick prism with finite absorption. Detached instances were studied for other cases. The general results may be summed up by the same criterion as that found for the grating with a narrow slit; namely, that *the limit of resolution is given by the undulation condition.* Since this was found to hold for a narrow slit and a wide slit, it seemed safe to assume that it would hold for intermediate slit-widths. Since it holds for an infinitely thick absorbing prism and for a perfectly transparent prism, it may be assumed to hold for all cases which are intermediate between these two, or which approximate them very closely.

As will be seen below, these cases include a finite absorbing prism, an echelon grating, a Lummer-Gehrcke plate, and a Fabry and Perot interferometer. As this list includes most of the important forms of spectroscopic apparatus, it may be concluded that the undulation condition furnishes a criterion of very general applicability. There thus remains only the task of formulating this criterion for the different types of instrument. This formulation is best expressed by the use of factors which indicate the relative resolving power of such instruments with respect to the more perfect instruments. We thus have two sets of factors, slit-width factors and absorption factors. In order to calculate, for instance, the resolving power of an echelon grating, taking account of absorption and slit-width, we have only to multiply the resolving power for the ideal instrument by a suitable factor which depends only on the absorption and slit-width, not on the type of instrument.

The slit-width factors.—The intensity pattern of a doublet may be written in the form

$$I=I_0\left\{\int_{x-d}^{x+d}\frac{\sin^2(x+a)}{(x+a)^2}\,dx+\int_{x-d}^{x+d}\frac{\sin^2(x-a)}{(x-a)^2}\,dx\right\}. \tag{7}$$

The analytic expression for the undulation condition is not in this case easy to apply. The values of $2a$ were therefore obtained by a

TABLE I

Slit-Width $\times f/d$	$2a$	Slit-Width Factor (C.M.S.)	Purity Factor (Schuster)
0	2.606	1.00	1.00
0.25	2.64	0.99	0.986
0.5	2.72	0.96	0.943
0.75	2.91	0.90	....
1.00	3.14	0.83	0.780
1.25	3.77	0.69	....
1.50	4.75	0.55	0.579
1.75	5.61	0.46	....
2.00	6.30	0.41	0.450

combination of graphical and numerical methods. The results are given in Table I. The "purity factors" of Schuster,[1] which

[1] *Astrophysical Journal*, 21, 197, 1905; *Theory of Optics* (London, 1909), p. 163.

were calculated for the same purpose, but with a different theoretical basis, are given for comparison in the fourth column. The difference in the two sets of factors is not great, and either would probably prove sufficiently accurate for most practical purposes.

Absorption factors.—The problem of finding the diffraction pattern is here one of combining n disturbances with amplitudes in geometric progression and phases in arithmetic progression. The summation leads to the well-known formula of Airy which we may write in the form

$$I = s_0^2 \frac{1 - 2r^n \cos n\phi + r^{2n}}{1 - 2r \cos \phi + r^2}, \tag{8}$$

where r is the ratio of the $(p+1)$th to the pth amplitude, s_0 the initial amplitude, and ϕ the phase difference between successive disturbances. The equation which expresses the undulation condition is here quite complicated unless n is infinite. For most practical purposes an approximate formula will do as well. We may obtain such a formula by making n infinite, while the total phase change $n\phi$ and the total absorption r^n, as well as ns_0, approach finite limits. Writing $r = e^{-k}$ and multiplying and dividing numerator and denominator in (8) by n^2,

$$I = \frac{n^2 s_0^2 (1 - 2e^{-nk} \cos n\phi + e^{-2nk})}{n^2(1 - e^{-k})^2 + 4e^{-k} \cdot n^2 \sin^2 \frac{\phi}{2}} \tag{9}$$

$$= I_0 \frac{(1 - 2e^{-k} \cos \Phi + e^{-2k})}{k^2 + \Phi^2}. \tag{10}$$

Here I_0 is the maximum intensity which we should have without absorption, Φ is the total phase difference between the two extreme disturbances, and k is the logarithm of the ratio of the final to the initial disturbance. The approximation amounts to this: if we represent the disturbances by vectors, the vector sum (9) is a polygon inscribed in a logarithmic spiral; in (10) we pass from the polygon to the limiting spiral. Equation (10) is a rigorous expression for an absorbing prism[1] and an approximate one for the

[1] See Wadsworth, *op. cit.*, ... , where essentially the same formula is derived.

case of an echelon grating, or a Lummer-Gehrcke plate. To form some idea of the degree of approximation the value of $2a$ for $n=5$, $k=1$ was computed by both (8) and (10). The value from the exact formula was about 2 per cent greater than that from the approximate formula. As this value of n is very small for an actual instrument, and as the accuracy of (10) increases very rapidly with increasing n, we may consider (10) a sufficient approximation for most purposes.

The undulation condition for equation (10) was solved for different values of k by successive approximations, giving the absorption factors listed in Table II. The first column gives the values of k, the second the corresponding values of $e^{-k}(=r^n$, see (8)), the third gives the values of $2a$, and the fourth the absorption factors.

TABLE II

k	e^{-k}	$2a$	Absorption Factor
0	1.0000	2.606	1.00
0.5	0.6065	2.611	0.998
1.0	0.3679	2.637	0.988
1.5	0.2231	2.662	0.979
2.0	0.1353	2.710	0.962
4.0	0.0183	3.041	0.857

For infinite values of k the expression (10) becomes indeterminate, since I_0 also becomes infinite. By returning to (9) we may obtain an expression for the intensity in this case, which is the case of an infinitely thick prism with finite absorption. The expression here reduces to the simple form

$$I=\frac{I_1}{\Phi_1^2+k_1^2}, \tag{11}$$

where I_1, k_1, and Φ_1 have the same meaning as the corresponding quantities in (10) except that they refer to any finite portion of the prism. If we make I_1 the intensity of the incident light, k_1 the logarithmic decrement due to loss by reflection and absorption, and Φ_1 the phase difference between two successive interfering beams, the expression (11) is an approximate expression for the intensity-curve of the Fabry and Perot interferometer. The undu-

lation condition obtained from (11) leads to an extremely simple formula for the resolving power of this instrument. Writing

$$I=\frac{s_1^2}{k_1^2+(\Phi_1+\Phi_0)^2}+\frac{s_1^2}{k_1^2+(\Phi_1-\Phi_0)^2},$$

differentiating twice as to Φ_1 and putting $\Phi_1=0$, we obtain

$$4\Phi_0^2=k_1^2+\Phi_0^2 \text{ or } \Phi_0=\frac{k_1}{\sqrt{3}}. \tag{12}$$

If D is the distance between the plates, this gives for the resolving power

$$\frac{2\pi D}{\lambda}\,\frac{\sqrt{3}}{k_1}=\frac{10.9\,D}{\lambda k_1}. \tag{13}$$

It is worth while to compare this result with that obtained from the exact formula, which may be obtained from (8) by making n infinite. The undulation condition leads to a quadratic in $\cos\Phi$, the solution of which gives

$$\cos\Phi_0=-\frac{1+r^2}{4r}+\sqrt{\frac{(1+r^2)^2}{16r^2}+2}\quad (r=e^{-k_1}). \tag{14}$$

For $k_1=0.1$ we obtain from (12) $\Phi_0=0.1155$, and from (14) $\Phi_0=0.1158$, thus showing that (11) represents the form of the Fabry and Perot fringes in the neighborhood of a maximum with a high degree of approximation.

There is one further advantage of the criterion furnished by the undulation condition, namely, that it is independent of any particular photographic process; for contrast can be enhanced by photography only where it exists, so that we should expect the appearance of a pair of lines at the limiting separation to undergo little change with any variation of the photographic process.

Visual resolving power.—It is obvious that the undulation criterion should apply equally to the calculation of the visual (telescopic) resolving power of a rectangular aperture. For apertures of other shapes we should not a priori expect it to apply. The problems presented are of far less practical importance than those furnished by the spectroscopic case, and it has not seemed worth while to carry the investigation farther in this direction.

SUMMARY

1. The actual limit of the resolving power of a perfect grating or prism has been determined experimentally. It is found that this limit is given, for equal intensities of the two lines, by the "undulation condition," that is, by the condition that the central minimum shall just disappear. This gives a theoretical resolving power about 26 per cent greater than that obtained by the Rayleigh criterion.

2. The limit given by the undulation condition has been found to hold for unsymmetrical doublets when the ratio of intensities of the two components is less than 10:3.

3. The undulation condition gives the limit for all cases in which the diffraction pattern is modified by finite slit-width, or by a decrease in geometric progression of the intensities of the interfering beams, whether this is due to absorption or to loss of light by reflection. These cases include most of the important forms of spectroscopic apparatus.

4. The effect of slit-width and absorption can be introduced by the use of suitable factors. These factors have been calculated for various values of the slit-width and absorption.

5. A simple approximate formula has been given for the resolving power of the Fabry and Perot interferometer.

The foregoing work was begun during the last Christmas vacation in the Physical Laboratory of the Johns Hopkins University. I am indebted to the Department of Physics there for the facilities so freely placed at my disposal during the beginning of the work, and for the loan of the cylindric lens with which I have continued the work here. I am also especially indebted to Dr. J. A. Anderson for his valuable advice and assistance.

Rouss Physical Laboratory
University of Virginia
July 1916

THE EFFECT OF AN ELECTRIC FIELD ON THE LINES OF CALCIUM AND LITHIUM[1]

By JANET T. HOWELL

INTRODUCTION

After the discovery of the Zeeman effect the analogous decomposition of spectral lines in an electric field was looked for by many investigators. In 1913 the effect was discovered in the diffuse series of hydrogen by J. Stark[2] and A. Lo Surdo[3] under entirely different experimental conditions. Since then it has been studied in hydrogen and helium by both methods and Stark has investigated the transverse effect for lithium, mercury, and a number of other elements.

Although a very large number of data have been accumulated since the discovery of the new effect, the work in this important field is still in its infancy. The results obtained by Stark and Lo Surdo differ markedly for the elements investigated, and the number as yet uninvestigated is large. The results have a most important bearing on atomic theory,[4] but, so far, they seem to have led to new complications rather than to new solutions. The contradictions between the results given by Stark and Lo Surdo suggest the possibility that the electric effect may vary under different conditions, but no definite conclusion on this subject has been reached. To make a comprehensive theory of the effect possible, much more investigation is necessary. Both the methods employed thus far have serious limitations, hence new ways of attacking the problem are important. It is especially important, from the point of view of solar work, to find a method adapted to the investigation of the heavy elements of the reversing layer. Although no new

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 121.

[2] *Berichte der K. Preuss. Akad. der Wiss.*, **47**, 932, 1913.

[3] *Rendiconti d. Lincei*, **22**, 2d sem., 664, 1913.

[4] A. Garbasso, *ibid.*, **22**, 2d sem., 635, 1913; *Il Nuovo Cimento*, **7**, 354, 1914; *ibid.*, **9**, 376, 1915. W. Wien, *Berichte der K. Preuss. Akad. der Wiss.*, **48**, 70, 1914; W. Voigt, *Annalen der Physik*, **4**, 197, 1901; *Göttingen Nachr. Ges. Wiss.*, 1914; E. Gehrcke, *Verh. der deutschen phys. Ges.*, **16**, 431, 1914; N. Bohr, *Phil. Mag.*, **27**, 506, 1914.

method has been perfected as yet, a preliminary survey of a large number of elements under low dispersion has brought out some new and interesting results in calcium and lithium which are reported here.

The results obtained in previous work can be summarized briefly as follows.

STARK'S METHOD AND RESULTS

The great number of data contributed by Stark and his co-workers have been collected and discussed in Stark's *Elektrische Spektralanalyse chemischer Atome.*[1] They used the luminous canal rays behind the cathode in a discharge tube as the source of light, and submitted them to an auxiliary electric field of 28,500–74,000 volts per centimeter. The source of potential for this auxiliary field was a dynamo of 4,500 volts and a storage battery of 3800 volts. In his earlier work Stark used a small concave grating of 1.5 m radius giving a dispersion in the first order of 1 mm = 9 A. Later, in photographing the fine division of the hydrogen lines, he decreased the dispersion somewhat, but more than doubled the electric field. He used two methods for obtaining the spectra of elements in the electric field: (1) a luminous gas, (2) the bombardment of the salts of alkali metals by the canal rays. He also mentioned the possibility of producing metallic lines by the bombardment of metal electrodes by canal rays.

Stark investigated the transverse effect for H, He, Li, Hg, Al, C, Ca, Mg, Na, and Th, and the longitudinal effect for H and He.[2] In both cases he observed in a direction perpendicular to the direction of motion of the canal rays in order to avoid complications due to the Doppler effect. The general conclusions derived from his results have been summarized by Fulcher[3] and may be put briefly as follows:

1. The diffuse series of H, He, and Li show a separation directly proportional to the field-intensity and of an order of magnitude of 3–18 A for a field of 28,500 volts per cm.

[1] Leipzig: S. Hirzel, 1914.

[2] *Annalen der Physik*, 43, 965, 1914; J. Stark and G. Wendt, *ibid.*, 43, 983, 1914; J. Stark and H. Kirschbaum, *ibid.*, 43, 991, 1017, 1914; J. Stark, *ibid.*, 48, 193, 1915.

[3] *Astrophysical Journal*, 41, 359, 1915.

2. The hydrogen components are symmetrical as to the displacement from the original line and probably as to intensity in sources at rest. The red components are more intense when the field and the motion of the luminous particles are in the same direction, and the violet components when they are in the opposite direction.

3. Helium and lithium show asymmetry both as to intensity and as to displacement.

4. Unlike the magnetic effect, the electric effect varies from line to line of the same series. The number of components and the maximum displacement increase with the term-number.

5. The diffuse series shows large effects, while the separation in the sharp main and subordinate series of He and Li is less than 1 A except for the He lines λ 3613.8 and λ 4169.1.

6. Lines of Al, C, Ca, Hg, Mg, Na, and Th show practically no effect. The displacement in the diffuse series of Hg is the largest, being of the order of magnitude of 0.4 A.

7. In the transverse effect the components are polarized, the outer lines parallel to the field, the inner perpendicular.

8. In the longitudinal effect of hydrogen and helium the lines are unpolarized and agree in number and position with the perpendicular components of the transverse effect.

LO SURDO'S METHOD AND RESULTS

Lo Surdo observed the region immediately in front of the cathode in a discharge tube, where the luminosity of the negative glow and the sudden fall of potential fulfilled the conditions for electric decomposition. Investigating the conditions in the dark space,[1] he showed that the increase in the cathode fall at a certain pressure depends solely on the current-density. He therefore used very narrow tubes. He found, experimentally, that when the plain electrode completely fills the tube, the length of the dark space is independent of the diameter of the tube. It is essential, in any case, to have the electrode completely fill the tube so that the lines of force may remain parallel.

[1] *Il Nuovo Cimento*, **9**, 368, 1915.

In his earlier work Lo Surdo used a tube of 4 mm internal diameter and 20 cm long, but later decreased the diameter to 1.5 mm. The pressure was regulated in general to a 2 mm dark space and the tube was excited by storage batteries giving a potential difference of 5000–8000 volts. The image of the region in front of the cathode was polarized by a nicol and focused on the slit of a 4-prism spectroscope. In the transverse effect the varying electric field gives the lines the shape of a Y, making it easy to identify the components.

The hydrogen results given by Lo Surdo[1] and Puccianti[2] differ somewhat from those of Stark. Lo Surdo finds two parallel components in all the hydrogen lines investigated (α, β, γ, δ) and perpendicular components agreeing in number with the term-number of the series. Stark, even in his early papers, found more components than this, and his latest work shows the hydrogen lines to be very complex. Lo Surdo's examination of the longitudinal effect in $H\gamma$ agrees with Stark's result.

Brunetti[3] has recently published two papers on the electrical decomposition of helium lines. The work shows the presence of very interesting satellites which follow a different field law from the regular components. The components are, in general, somewhat different from Stark's, especially as regards polarization.

APPARATUS

The apparatus used in this work was essentially of the Lo Surdo form. Professor Stark was kind enough to prepare for the Mount Wilson Observatory a hydrogen tube, of the ingenious form devised by him. But as the life of such tubes is short, and as no glassblower having sufficient skill to make others was available, we were compelled to have recourse to the very simple form of tube used by Lo Surdo. Moreover, Stark has made a survey of most of the more promising elements with his apparatus, but the Lo Surdo tube has been applied only to hydrogen and helium. The discordant results of the two schools of investigators seemed to indicate that the

[1] *Rendiconti d. Lincei*, **23**, 1st sem., 82, 143, 252, 326, 1914.

[2] *Ibid.*, pp. 329, 331, 1914.

[3] *Il Nuovo Cimento*, **10**, 34, 41, 1915.

nature of the Stark effect is dependent on the mode of excitation. So the application of the Lo Surdo method to all the available elements was a hopeful point of departure. A diagram of the apparatus is shown in Fig. 1.

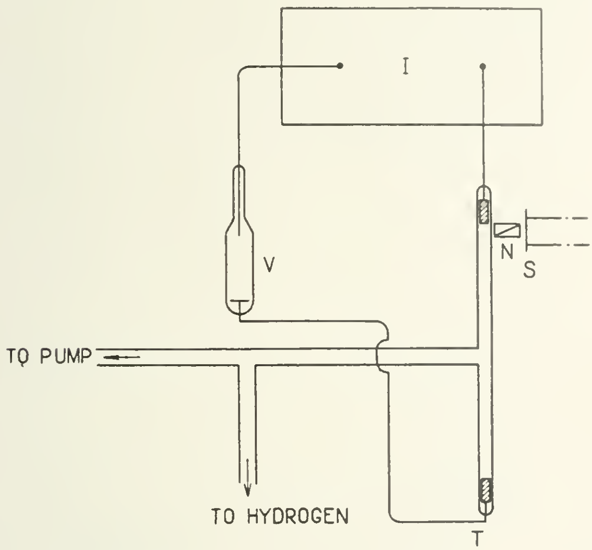


Fig. 1

T = Tube
V = Valve tube for rectifying the discharge
I = Induction coil
N = Nicol
S = Slit of 3-prism spectroscope

The spectroscope contained three prisms, two of ultra-violet glass, one of quartz (compound right- and left-handed), and gave a dispersion of 1 mm = 18 A at Hγ; 1 mm = 12 A at H and K. The source of light was placed directly in front of the slit with a small nicol interposed. This arrangement introduced some astigmatism, but, as only the maximum displacement was measured, the integrated effect along the line did not interfere. As the collimator was entirely filled, the arrangement was economical of light and also of space. An oil pump was used which held the pressure steadily at about 6 mm dark space. No value can be given for the current through the tube as no suitable instrument was available

for measuring it. The current in the primary of the coil was 11 amperes, the voltage 110, and a Wehnelt interrupter was used for which the maximum spark-gap of the coil was 17 cm. A valve tube in the circuit rectified the discharge.

As the tube cracked on overheating, it was run intermittently and not more than three minutes at a time. The interval covered by the exposure varied from 30 minutes to 6 hours, the actual exposure being about one-half this time. Seed "27" plates were used. The tube was a simple T-tube of heavy-walled glass tubing with electrodes sealed in at the ends. The internal diameter was 6 mm and the length 20 cm. The electrodes were solid cylinders of metal nearly filling the tube and were sealed in with fine platinum wires. Three types of tubes were used whose cathodes are shown in Fig. 2.

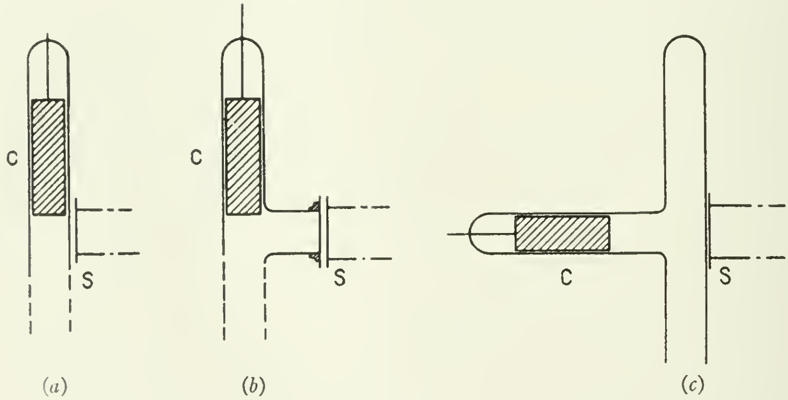


FIG. 2

S = Slit of spectroscope
C = Cathode

Types (*a*) and (*b*) were used in the transverse effect. When the electrodes were of aluminium, which did not sputter to any extent during the life of the tube, the simple form (*a*) was used. When the cathode was made of a metal that sputtered easily, or when a salt was used on the cathode that discolored the sides of the tube, type (*b*) was found to be very satisfactory indeed. Type (*c*) was used in the longitudinal effect and worked well except for its extreme fragility. Types (*a*) and (*b*) lasted for several days, if

used with care and caution, but in (*c*) the impact of the rays melted the glass directly opposite the cathode and the life of the tube was only from 30 minutes to an hour. A new tube could be made and set up in 20 minutes, but the fragility seriously limited the length of exposure possible. If any further work is done with tubes of this type it would be well to make them of quartz.

Photographs of the transverse effect showed electrical components in the Y-shape described by Lo Surdo. In the longitudinal effect the electrode did not entirely fill the tube, with the result that the field fell off at the edges and the components tapered inward and were easily identified.

METHODS OF OBTAINING SPECTRA

The discharge tube was always filled with hydrogen so as to have standard lines for field determination. The metallic cathode, bombarded by anode rays, gave the spectrum of the metal close to its surface, in just the proper region for observing the electric effect. This was true of all the metals investigated. Stark mentioned this method of obtaining spectra in a vacuum tube. Goldstein[1] investigated the phenomenon and found that the spectra appeared only when the gas in the tube was nitrogen, and were much strengthened at a liquid air temperature. Robinson[2] recently published a paper on the cathode spectra of metals. He was able to obtain spectra in H, CO, and, with special brilliance, in O, but found it necessary to have thin cathodes. No evidence of metallic lines was obtained with electrodes 1–2 mm thick. Since I had no difficulty in producing brilliant metallic spectra near the cathode with cylindrical electrodes of 5 mm diameter and from 1.5 to 2 cm long, it follows that the thin electrode is certainly not necessary. The spectra were obtained with about equal brilliance in air, oxygen, and hydrogen. Comparative photographs were taken of the cathode spectra of aluminium and iron at different pressures and there seems to be no particular connection between the cathode spectra and the sputtering of the metal. Although iron sputters easily, and aluminium almost none at all, the aluminium spectrum

[1] *Physikalische Zeitschrift*, 6, 14, 1905.

[2] *Astrophysical Journal*, 42, 473, 1915.

was obtained over a larger range of pressure than the iron spectrum. The aluminium lines λ 3962 and λ 3944 appeared before the pressure was low enough for the Crookes's dark space to appear and at 3–4 mm dark space they extended 3 cm from the electrode. The iron spectrum does not appear until the pressure is reduced to 3 mm dark space. The spectra of Fe, Ni, Al, Mg, Zn, and Ca were obtained by this method and, as found by Robinson, they were in general like the spark. The metallic lines are easily identified, as they are more intense near the cathode, and it seems likely that further study of them will bring out many interesting differences from arc and spark spectra. So far the only one that has been studied in detail is the copper[1] spectrum. In the cathode spectrum of magnesium there were three lines which clearly originate in the electrode and which do not agree with any known magnesium lines. The wave-lengths are 4155, 4113, and 4103. The last is probably the same as λ 4106.8 given by Fowler,[2] but the other two seem to be new.

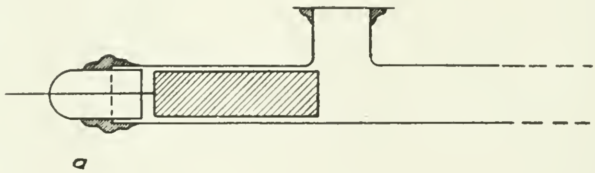


FIG. 3

When the cathode was covered with a thin coat of the chloride of a metal, carefully dried, the bombardment of the salt by the anode rays gave a very brilliant spectrum. Li, Ca, and Sr were used in this way. When using salts, the (*b*) or (*c*) type of tube was found best because the walls became discolored very fast. In order to renew the supply of salt between exposures the electrodes were made demountable as shown in Fig. 3. The electrode was sealed in at *a* with sealing wax.

METHOD OF MEASURING PLATES

The hydrogen lines Hβ and Hγ were used to determine the field-strength for each plate. Hβ gave a very clear separation when

[1] *Astrophysical Journal*, **42**, 473, 1915. [2] *Proc. R. S.*, **71**, 419, 1903.

polarized parallel, but was too indistinct for measurement when polarized perpendicular to the field. Hence for these plates only Hγ could be used. The red component of Hγ was usually confused by an air line. On some plates this line was absent, but whenever it was present only the violet component was measured. The field-strength was determined from Stark's measurements for the separation at 28,500 volts per centimeter. His later work showed the hydrogen separation to be symmetrical, and he claimed much greater accuracy for the total separation than for the displacement of the components from the center given in his earlier paper. It seemed best, therefore, to multiply the displacement of the violet component by two and use this value for the total separation when the red component could not be measured. A certain amount of inaccuracy is introduced here because the intensity of the components is slightly unsymmetrical for moving sources and this might shift the center of the middle component to one side. As no shift could be detected in the middle component, the error is apparently less than the error of measurement. There is, however, a large error in this method of determining the field-strength. Stark thought that his calculation of the field-strength from the potential difference and the distance between the electrodes was probably too large; hence the resultant separations for a given field-strength were probably too small. Also there is great likelihood that the magnitude of the separation may vary somewhat in going from the Stark to the Lo Surdo method where the conditions are different. This seems especially likely, since the field-strength often differed by 15 or 20 per cent in determinations from Hβ and Hγ on the same plate, while the determinations from Hγ on different plates, taken under the same circumstances, rarely varied more than 5 per cent. The general character of the components agreed with Stark's results, except that a single center component was always found in Hγ when polarized parallel, instead of the two faint ones given by Stark. This is probably due to the fact that the discharge was not absolutely unidirectional, and that a faint unseparated line is superimposed on the components. This indirect process of determining field-strengths undoubtedly introduces an error that may be as great as 10 per cent, but no better method was available.

Another uncertainty lies in the possibility of a complication from the Doppler effect when photographs are taken along the field. It is quite possible that this may have affected the measurements on $H\gamma$. The center line of $H\gamma$ in the longitudinal effect is composed mainly of light from in front of the dark space and appears very sharp; consequently there is certainly no Doppler effect present there. The components, however, are composed almost entirely of light from the negative glow on the surface of the cathode where the Doppler effect would be at a maximum. However, Stark always found an undisplaced line as well as a displaced line in his photographs of the Doppler effect in canal rays, and the displaced line varied in displacement and intensity with the velocity of the rays. At the presssure used in these experiments it seems safe to assume that the displaced line, if present, would be very faint and very close to the stationary one. As the components of $H\gamma$ are themselves faint, only the stationary line would show. Lo Surdo photographed the longitudinal effect for $H\gamma$ and found symmetrical unpolarized components agreeing in position and number with those observed by Stark. Apparently, therefore, there was no Doppler effect noticeable in his case. H and K and Li λ 4602 give components strong enough to be confused by a Doppler effect, but the components do not show two maxima of intensity. At any rate, these lines have not thus far been examined in canal rays for a Doppler displacement; nothing definite, therefore, can be said about them in this respect.

The maximum separation of components was selected for measurement and their displacement from the center was determined by reference to a neighboring unaffected line and to the unseparated line outside the field.

The accuracy of the results varies with the dispersion, and the character of the lines. Some idea of the degree of accuracy will be given under the results on each line.

RESULTS

Entirely negative results were found for Fe, Ni, Al, Mg, Zn, and Sr.

The aluminium lines λ 3962 and λ 3944 were very greatly strengthened toward the cathode—more than the increase in

intensity at the cathode could account for. Photographs of these lines with an echelon showed wings to the red, but they did not vary with the field-strength. The aluminium spectrum was examined from λ 4860 to λ 2350 with a single quartz prism spectroscope, but no electric effect was found. In Mg, Fe, Ni, Zn, and Sr the spectra were photographed from λ 4900 to λ 3500, and no electric effect large enough to be detected with the low dispersion used was observed.

TABLE I

LITHIUM

TRANSVERSE EFFECT FOR 20,000 VOLTS PER CENTIMETER (HOWELL)

λ	Comp. Polar. Parallel	Int.	Comp. Polar. Perpend.	Int.	Remarks
4602.37.......	+1.00	8	+0.48	8	
	−2.48	6	−2.00	6	
4132.93.......	+2.26	2	+1.78	2	
	−0.18	5	−0.18	5	Unpolarized
	−3.10	1	−2.24	1	

TRANSVERSE EFFECT FOR 20,000 VOLTS PER CENTIMETER (STARK)

λ	Comp. Polar. Parallel	Int.	Comp. Polar. Perpend.	Int.	Remarks
4602.37.......	+0.65	8	+0.56	8	
	−0.28	1	−0.28	1	Doubtful
	−1.81	6	−1.64	6	
4132.93.......	+1.87	3	+1.57	3	
	−0.37	4	−0.33	4	
	−2.62	2	−2.44	2	

LONGITUDINAL EFFECT FOR 20,000 VOLTS PER CENTIMETER (HOWELL)

λ	Comp. Polar. Parallel	Int.	Comp. Polar. Perpend.	Int.	Remarks
4602.37.......	+0.57	8	+0.34	6	
	−2.01	6	−1.53	3	
4132.93.......	+1.16	1	+0.77	1	
	−0.26	5	−0.26	5	Unpolarized
	−1.99	0	−1.50	0	

Results for lithium.—The results for lithium are given in Table I, together with Stark's results reduced to the same field-strength. The numbers marked + indicate components to the red, those marked −, components to the violet. The field close to the cathode was in the neighborhood of 25,000 volts per centimeter. In the

longitudinal effect, the displacement of the red component in λ 4602 was doubled because the photographed line was a blend of the red component and the unaffected line outside the field. For the same reason the displacement of the center component to the violet in λ 4132 (longitudinal) was doubled. As the discharge was not entirely unidirectional there is always a faint unseparated line superimposed on the electrical components, and this makes the displacement of the center component of λ 4132 too small in the transverse effect. But, as it was uncertain what percentage of the total intensity was due to this unseparated line, the measured displacement is given. Owing to the uncertain nature of the middle component of λ 4132 the total separation of the outer components is more accurate than their displacements from the center. The results for the transverse effect are in general larger than those found by Stark. As the error of the field-strength determinations is uncertain, the only idea of the precision of the results that can be given is the average deviation from the mean of the results on different plates. This deviation is given in Table II. The components of Hγ are more distinct when polarized perpendicular than when polarized parallel; consequently the error is greater for the parallel components. The deviation of λ 4602 refers to the total separation, that of λ 4132 to the separate components. Photographs of the lithium lines are given in Plate III.

TABLE II

AVERAGE DEVIATION IN A FOR LITHIUM RESULTS

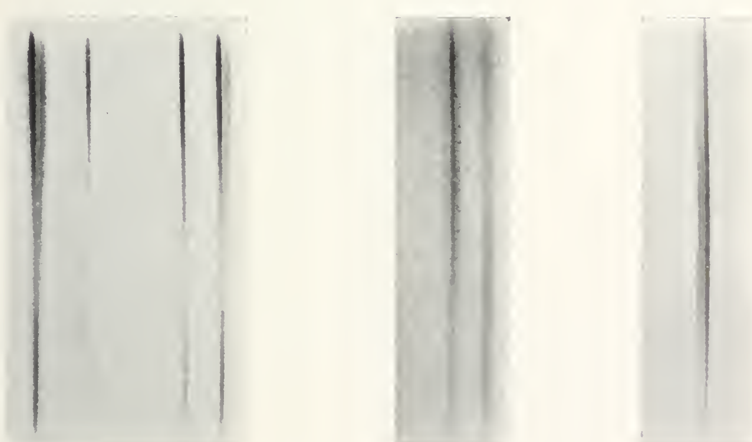
λ	Transverse Effect		Longitudinal Effect	
	Parallel Comp.	Perpend. Comp.	Parallel Comp.	Perpend. Comp.
4602.37......	0.32	0.04	0.14	0.04
4132.93......	0.28	0.02	0.12	0.12

Results for calcium.—The results for the H and K lines of calcium are given in Table III. In the earlier photographs a cathode of metallic calcium was used, but it had a decided tendency to rectify the discharge in the opposite direction and acted as an

PLATE III

λ 3933 (K) λ 3968 (H) λ 4132 (Li) λ 4602 (Li)

THE TRANSVERSE EFFECT IN CALCIUM AND LITHIUM



λ 3933 (K) λ 3968 (H) λ 4132 (Li) λ 4602 (Li)

THE LONGITUDINAL EFFECT IN CALCIUM AND LITHIUM

anode such a large part of the time that the photographs were very poor. Later, calcium chloride on an aluminium cathode was used and proved very satisfactory. The longitudinal effect gave very clear components, but in the transverse effect the lines were both broad and hazy, although the Y-shape proved the effect to be due to the electric field. The best photographs showed the lines to be composed of two very diffuse components, an intense one slightly displaced to the red, and a fainter one more displaced to the violet.

Table III contains measurements on the separation of these very indistinct components in the transverse effect. The actual separation of the components is small, but the width of the lines, owing to their diffuse character, is large. As the red component is more intense and more diffuse than the violet, the width is almost symmetrical with respect to the unseparated line. The total width of H is 2.38 A parallel and 2.16 A perpendicular; that of K is 2.64 A parallel and 2.42 A perpendicular.

TABLE III

CALCIUM H AND K

TRANSVERSE EFFECT FOR 20,000 VOLTS PER CENTIMETER

λ	Comp. Polar. Parallel	Int.	Comp. Polar. Perpend.	Int.	Remarks
3968.63	+0.22	6	+0.16	6	Unpolarized
	−0.86	2	−0.74	2	
3933.83	+0.22	9	+0.22	9	Unpolarized
	−0.92	3	−0.74	3	

LONGITUDINAL EFFECT FOR 20,000 VOLTS PER CENTIMETER

λ	Comp. Polar. Parallel	Int.	Comp. Polar. Perpend.	Int.	Remarks
3968.63	+1.27	3	+1.23	3	Unpolarized
	+0.01	8	−0.02	8	Unpolarized
	−1.17	0	−1.11	0	Unpolarized
3933.83	+1.42	4	+1.38	4	Unpolarized
	+0.06	9	−0.02	9	Unpolarized
	−1.30	1	−1.26	1	Unpolarized

The results of the transverse effect indicate pretty clearly that H and K are both composed of a strong unpolarized component 0.2 A to the red and of a fainter polarized component to the violet. The

longitudinal components are certainly unpolarized. The middle line shows evidence of polarization on the edges, but the polarized edges are such a small part of the total intensity of the line that the effect is scarcely measurable. The H and K lines were investigated for circular polarization, but no trace of it was found.

Photographs of H and K are shown in Plate III. The separation of H and K into components in the transverse photograph does not show in the reproduction, but the progressive widening with increasing field-strength proves the electrical nature of the effect. The violet component of H in the longitudinal photograph does not reproduce well, but is analogous to that of K which shows plainly. The Y-shape in the longitudinal components means that the discharge was passing from only a small section of the electrode. This happened frequently when calcium chloride was used. When the whole surface of the electrode was acting, the components extended across the field, tapering in at the edges.

The average deviation of the results is given in Table IV. It is calculated for the total separation in the transverse effect and for the separation of the components in the longitudinal.

TABLE IV

DEVIATION IN Å FOR H AND K COMPONENTS

λ	Transverse Effect		Longitudinal Effect	
	Parallel Comp.	Perpendicular Comp.	Red Comp.	Violet Comp.
3968.63......	0.12	0.05	0.04	0.03
3933.83......	0.10	0.09	0.05	0.01

DISCUSSION OF RESULTS

Instead of contributing new law and regularity to the electric decomposition of spectral lines, the results for lithium and calcium seem to contradict the laws already formulated. In former investigations the longitudinal effect has given only unpolarized components, but the lithium lines λ 4602 and λ 4132 are here shown to have very clearly polarized components along the field.

Previously only the diffuse series of different elements have shown any appreciable effect, but H and K belong to a principal-pair series of calcium, while the lines of the diffuse series at λλ 4457, 4435, and 4425 showed no effect at all. The lines at λ 3737 and λ 3706, which are analogous to H and K, were too faint on my plates for any examination, but I hope to investigate them in the near future.

It is still somewhat uncertain whether the electric effect is the same under different conditions or not. The difficulty in determining consistent field-strengths from the hydrogen lines is no very strong evidence on either side, because the difficulty of making accurate measurements on the components of Hγ was so great. The values found here for lithium in the transverse effect are larger than Stark's values, but the difference is not great enough to show any real dissimilarity. The general character of the components is the same. The chief reason for thinking that there may be a real difference between electrical components under different conditions is the fact that Stark[1] found no effect in the H and K lines of calcium (transverse). It was doubtful at first whether the production of spectra through the bombardment of metallic electrodes was favorable to an electrical effect. The fact that using metallic calcium as an electrode gave the same result as covering the electrode with calcium chloride proves this to be an entirely favorable method. The photographs with metallic calcium were unsatisfactory, owing to the tendency of calcium to act as an anode, but a few good ones obtained in this way showed the electric effect quite plainly.

The result most clearly shown by this work is the absolute necessity of increasing the intensity of the source so that high dispersion may be used. The effect in the majority of elements is so small that it will take high dispersion to bring it out.

GENERAL CONCLUSIONS

The spectra of H, Li, Ca, Fe, Ni, Mg, Al, Zn, and Sr were examined near the cathode in a discharge tube under low dispersion. Fe, Ni, Mg, Al, Zn, and Sr showed no electric effect. This

[1] *Annalen der Physik*, 43, 1017, 1914.

negative result brings out the absolute necessity of increasing the dispersion in dealing with the heavier elements. The effect, if it exists in these elements, is small and can be brought out only under much stronger fields and with higher dispersion. Both Stark's method and that of Lo Surdo give such faint luminosity that the present problem is to find a method for producing very brilliant spectra in a strong electric field.

New results were found in lithium and calcium which show most interesting variations from those previously obtained.

The longitudinal effect in lithium gives clearly polarized components, whereas all the longitudinal components in hydrogen and helium have been unpolarized.

The H and K lines of calcium show electrical components with a separation, especially in the longitudinal effect, which is comparable in magnitude with the separations in H, He, and Li. This is surprising, owing to the relatively large atomic weight of calcium and to the fact that H and K do not belong to a diffuse series.

In conclusion, I wish to express to Dr. Hale and the members of the Observatory staff my very deep appreciation for the opportunity of a year's work at the Observatory. The problem of the electric effect was undertaken at Dr. Hale's suggestion, and has been guided by his advice and encouragement throughout.

Mount Wilson Solar Observatory
July 1, 1916

SOME DETERMINATIONS OF THE APEX AND VELOCITY OF SOLAR MOTION FROM THE RADIAL VELOCITIES OF THE BRIGHTER STARS, INCLUDING AN APPARENT RELATION TO PROPER MOTION

By C. D. PERRINE

In a paper[1] on the residual radial velocities of the stars of the different spectral classes contained in Campbell's well-known catalogues of about 1300 of the brighter stars, I pointed out several peculiarities, among which were large and apparently systematic residuals in the regions at right angles to the axis of solar motion.

Those investigations were based upon Campbell's assumed apex of $A=270^\circ$, $D=+30^\circ$. This position is only 5° away from that found by him from the radial velocities of these same stars and is midway between the latter value and the value derived by Boss from the proper motions of a larger number of (chiefly) naked-eye stars. Positions of the solar apex differing but little from that assumed by Campbell have been used by other investigators of solar motions. At that time it seemed unlikely that the most logical apex could differ sufficiently from the one chosen to give any very large discordances due to that cause alone, and attempts were made to find explanations in systematic motions of the stars themselves. It was found that the large systematic residuals in many of the galactic groups approximately at right angles to the axis of the solar motion could be largely accounted for by the assumption of a component of motion for the stars toward the vertex of one of Kapteyn's streams at 7^h, $+64^\circ$.

In attempting to find the cause of the apparently systematic discordance of nearly 10° between the declinations of the solar apex as derived from radial velocities and proper motions, the first step was a determination of the apex from the proper motions of the stars of Class B. It was at once seen that there was a marked discordance between the mean proper motions of the groups in the

[1] *Astrophysical Journal*, 42, 305, 1915.

northern and southern hemispheres. This discordance was so marked and so suggestive that solutions were made for the apex and velocity of solar motion from the radial velocities of the northern and southern B stars separately. The results of these solutions and similar ones for some other classes yielded widely different positions of the solar apex.

At this point I received a letter from my friend, Professor Kapteyn, to whom I had written regarding the peculiarities found, in which he suggested that a different apex for the solar motion would remove the systematic residuals found in the regions of 0^h and 12^h of right ascension. This appears to be essentially the case. But I was now convinced that no single position of the apex would satisfy all of the discordances which I had found, not only from the radial velocities, but from the proper motions of the B stars as well. These discordances were believed to be something more than merely accidental. Separate solutions were made from the radial velocities of the north and south stars of all of the spectral classes. The separation into north and south groups divides the effect observed in the B stars on opposite sides of the axis of solar motion sufficiently at least for a preliminary investigation.

These solutions were first made for all of the stars of $3^M.0$ and fainter of each spectral class in each hemisphere. Such marked peculiarities were observed in the radial velocities of K-type stars within the same region, which had been separated according to size of proper-motion, and particularly in stars having contrary parallactic signs (i.e., negative signs in the first and fourth quadrants of right ascension and positive signs in the second and third quadrants), that ultimately the four classes A, F, G, and K were separated into four groups according to proper motion, viz., 0 to $0^s.0049$, $0^s.0050$ to $0^s.0099$, $0^s.0100$ and over, and those stars having "contrary" parallactic signs. The B stars practically all fell into the small proper motions, and the M-type stars and the 110 stars brighter than $3^M.0$ were not so classified, as they were thought to be too few in number.

The solutions were made by regions, each region being 2^h in right ascension by $30°$ in declination, commencing at 0^h and at the equator. The mean α, δ, and observed radial velocity of all of the

stars in each region were treated as one observation. The positions were rounded off to the nearest degree and the velocities to the nearest kilometer. In cases of but few stars observations were combined when they fell within an area differing but little from the assumed area, or rejected. Each region was given weight unity irrespective of the number of stars it contained or other conditions whose bearing for the present is unknown.

The equations of condition were of the well-known form

$$ax+by+cz+K-\text{Obs. }V=0$$

in which

$a=\sin\delta$,
$b=\cos\alpha\cos\delta$,
$c=\sin\alpha\cos\delta$,
$K=$ "constant error" term,
Obs. $V=$ observed radial velocity of star,
α and δ are the right ascension and declination of the star.

The solutions were made by the method of least squares. The A and D of the apex and the velocity $V_\odot$ were derived from the co-ordinates x, y, and z by the usual trigonometric relations, viz.,

$$V_\odot^2=x^2+y^2+z^2,$$

$$\tan A=\frac{z}{y},$$

$$\sin D=\frac{x}{V_\odot}.$$

In solutions of this nature, from small numbers of such different velocities as usually constitute each mean, a perfect representation of the spherical relations that exist among the coefficients is only accidental, even if there is no large systematic deviation. There is, therefore, generally an excess, sometimes considerable. This excess can be absorbed by the introduction of a constant-error term or other form of term which is independent of the spherical relations existing among the coefficients of the three unknown terms for determining the apex.

The term K, included largely for convenience in checking the solutions, was not used after that point. The values of the three unknowns x, y, and z are therefore independent of this excess (K).

If the K term were carried into the other unknowns, the values of y and z would have been considerably changed in only a few cases, but owing to the unsymmetrical nature of the coefficients in declination this co-ordinate of the solar apex is greatly changed by including a large K term. The evidently spurious values of such a term as resulted in many cases and their consequent effects on the declination of the apex caused me to discard entirely the effect of such a term in obtaining the position of the apex and the velocity.

In the earlier stages of the work the value of K was derived in the usual way from the solutions. It was often quite large and generally positive. The divisors for that term were, however, very small, owing to the unsymmetrical nature of the data, and it was evident that the effect of even one large discordant velocity was sufficient to yield an impossible value for K. It seems, therefore, that if we are to consider this as a constant error, we should, in place of the value derived for K (in these solutions at least), use the excess divided by the number of observations (regions in this instance). Values of K derived in this way are included in Table I with other data.

The stars with velocities of 50 km and over were generally omitted. The different classes of proper motion are indicated by large, medium, small, and contrary.

Several peculiarities of an apparently systematic nature are noticeable in these results, which are shown more clearly by the groupings in Tables II, III, IV, V, and VI. They may be summarized as follows:

A. There are such differences between the results from the northern and southern stars as to lead to the belief that there are radical structural differences in the two regions.

There appears to be greater consistency generally among the northern stars. All of the apices derived from the northern stars are north of the galactic plane, whereas five of the apices from the southern stars are south of it. Those from the southern stars are more widely scattered and show preferences for regions of sky somewhat different from that preferred by the apices derived from northern stars. The five groups with southern galactic latitudes all belong to the large and medium proper motions.

TABLE I

Spectral Type	$\mu\alpha$	North A	North D	North $V_\odot$	North K	North No. Stars	North No. Equations	South A	South D	South $V_\odot$	South K	South No. Stars	South No. Equations
				km	km					km	km		
$2^M.9$ and brighter	All	251°	+44°	−14.4	−1.2	50	17	258°	+40°	−24.5	+0.5	60	16
B $3^M.0$ and fainter	All	254	+27	−17.0	+0.8	70	13	288	+31	−24.8	+0.6	123	18
A $3^M.0$ and fainter	Large	242	+ 7	−14.2	+0.4	19	8	306	+14	−20.7	−0.1	10	7
	Medium	260	− 4	−22.6	0.0	19	8	305	+20	−14.5	−2.5	15	8
	Small	260	+35	−17.7	−0.5	59	17	261	+ 3	−20.2	+1.7	63	18
	Contrary	209	+37	−12.2	+0.6	32	13	240	+18	−18.9	+3.3	19	10
	All	258	+20	−17.7	+0.1	97	21	264	+11	−19.2	+0.8	88	18
F $3^M.0$ and fainter	Large	254	+ 5	−23.2	−0.1	40	10	238	−11	−13.1	−2.1	38	14
	Medium	253	−17	−25.5	+0.3	12	6	307	+19	−19.9	−3.2	21	9
	Small	270	+37	−15.0	0.0	27	10	264	+32	−20.7	+0.8	46	15
	Contrary	292	+58	−13.3	−0.2	29	11	204	+32	− 8.4	−0.2	39	12
	All	271	+ 9	−19.2	+0.2	79	20	264	+13	−18.7	−0.1	105	21
G $3^M.0$ and fainter	Large	268	− 4	−36.4	+4.0	19	11	287	− 6	−13.9	−7.5	17	10
	Medium	272	+ 7	−37.6	+2.6	9	8	246	− 6	−44.1	−0.3	5	5
	Small	271	+27	−14.6	−0.1	36	13	242	+28	−12.4	+0.3	45	17
	Contrary	207	+80	−19.0	+2.0	23	11	147	+ 1	− 6.4	−0.6	18	10
	All	267	+19	−20.9	+0.7	64	19	249	+23	−12.3	−1.0	67	19
K $3^M.0$ and fainter	Large	277	+18	−26.3	−0.2	34	14	298	+19	−30.0	+1.8	51	17
	Medium	262	−34	−25.2	+3.0	34	14	232	+59	−21.3	−1.2	51	19
	Small	265	+32	−19.6	+0.5	82	20	258	+39	−22.4	+1.2	138	25
	Contrary	288	+44	−14.2	−0.3	39	14	292	+60	−20.3	+0.7	57	17
	All	273	+ 8	−19.4	+2.3	151	28	268	+36	−22.2	+0.8	240	35
M $3^M.0$ and fainter	All	269	+15	−24.1	+1.1	24	11	272	+41	−22.5	+2.2	41	20

TABLE II

ACCORDING TO PROPER MOTION

$\mu\alpha$	Spectral Type	North						South					
		A	D	$V_{\odot}$	K	No. Stars	No. Equations	A	D	$V_{\odot}$	K	No. Stars	No. Equations
				km	km					km	km		
Large	A	242°	+ 7°	−14.2	+0.4	19	8	306°	+14°	−20.7	−0.1	10	7
	F	254	+ 5	−23.2	−0.1	40	10	238	−11	−13.1	−2.1	38	14
	G	268	− 4	−36.4	+4.0	19	11	287	− 6	−13.9	−7.5	17	10
	K	277	+18	−26.3	−0.2	34	14	298	+19	−30.0	+1.8	51	17
		260	+ 6	−25.0	+1.0	112		282	+ 4	−19.4	−2.0	116	
Medium	A	260	− 4	−22.6	0.0	19	8	305	+20	−14.5	−2.5	15	8
	F	253	−17	−25.5	+0.3	12	6	307	+19	−19.9	−3.2	21	9
	G	272	+ 7	−37.6	+2.6	9	8	246	− 6	−44.1	−0.3	5	5
	K	262	−34	−25.2	+3.0	34	14	232	+59	−21.3	−1.2	51	19
		262	−12	−27.7	+1.5	74		272	+23	−25.0	−1.8	92	
Small	A	260	+35	−17.7	−0.5	59	17	261	+ 3	−20.2	+1.7	63	18
	F	270	+37	−15.0	0.0	27	10	264	+32	−20.7	+0.8	46	15
	G	271	+27	−14.6	−0.1	36	13	242	+28	−12.4	+0.3	45	17
	K	265	+32	−19.6	+0.5	82	20	258	+39	−22.4	+1.2	138	25
		266	+33	−16.7	0.0	204		256	+26	−18.9	+1.0	292	
Contrary	A	209	+37	−12.2	+0.6	32	13	240	+18	−18.9	+3.3	19	10
	F	292	+58	−13.3	−0.2	29	11	204	+32	− 8.4	−0.2	39	12
	G	207	+80	−19.0	+2.0	23	11	147	+ 1	− 6.4	−0.6	18	10
	K	288	+44	−14.2	−0.3	39	14	292	+60	−20.3	+0.7	57	17
		249	+55	−14.7	+0.5	123		221	+28	−13.5	+0.8	133	
	Omitting G	263	+46	−13.2				245	+37	−15.9			

B. The position of the solar apex varies with the size and parallactic sign of the proper motions of the stars used in the determination, being for the northern sky progressive in *declination* from the large and medium values through the small values to those with

TABLE III

According to Declination of Apex

North					South				
Spectral Type	$\mu\alpha$	A	D	$V_{\odot}$	Spectral Type	$\mu\alpha$	A	D	$V_{\odot}$
				km					km
K..........	M	262°	−34°	−25.2	F........	L	238°	−11°	−13.1
F..........	M	253	−17	−25.5	G........	L	287	− 6	−13.9
A..........	M	260	− 4	−22.6	G........	M	246	− 6	−44.1
G..........	L	268	− 4	−36.4	A........	S	261	+ 3	−20.2
		261	−15	−27.4			258	− 5	−22.8
F..........	L	254	+ 5	−23.2	A........	L	306	+14	−20.7
A..........	L	242	+ 7	−14.2	A........	Con.	240	+18	−18.9
G..........	M	272	+ 7	−37.6	F........	M	307	+19	−19.9
M..........	All	269	+15	−24.1	K........	L	298	+19	−30.0
K..........	L	277	+18	−26.3	A........	M	305	+20	−14.5
		263	+10	−25.1			291	+18	−20.8
B..........	All S	254	+27	−17.0	G........	S	242	+28	−12.4
G..........	S	271	+27	−14.6	B........	S	288	+31	−24.8
K..........	S	265	+32	−19.6	F........	S	264	+32	−20.7
A..........	S	260	+35	−17.7	F........	Con.	204	+32	− 8.4
		262	+30	−17.2			250	+31	−16.6
F..........	S	270	+37	−15.0	K........	S	258	+39	−22.4
A..........	Con.	209	+37	−12.2	$2^{M}9$ and brighter	All	258	+40	−24.5
K..........	Con.	288	+44	−14.2	M........	All	272	+41	−22.5
F..........	Con.	292	+58	−13.3	K........	M	232	+59	−21.3
$2^{M}9$ and brighter		251	+44	−14.4	K........	Con.	292	+60	−20.3
		262	+44	−13.8			262	+48	−22.2

contrary parallactic signs. The progression for the northern stars is very consistent, not a single exception being observed to the condition that the large and medium proper-motion stars yield positions *south* and the apices from the contrary proper-motion stars *north* of that from the small proper-motion stars.

The most southerly positions of the apex appear to result from the medium-sized proper motions in the classes A, F, and K.

TABLE IV

ARRANGED ACCORDING TO A

NORTH

Type	A	D	$V_{\odot}$
			km
G.........	207°	+80°	−19.0
A.........	209	+37	−12.2
	208	+58	−15.6
A.........	242	+ 7	−14.2
$2\overset{\mathrm{M}}{.}9$ and brighter..	251	+44	−14.4
F.........	253	−17	−25.5
F.........	254	+ 5	−23.2
B.........	254	+27	−17.0
	251	+13	−18.9
A.........	260	− 4	−22.6
A.........	260	+35	−17.7
K.........	262	−34	−25.2
K.........	265	+32	−19.6
M.........	269	+15	−24.1
G.........	268	− 4	−36.4
	264	+ 7	−24.3
F.........	270	+37	−15.0
G.........	271	+27	−14.6
G.........	272	+ 7	−37.6
K.........	277	+18	−26.3
	272	+22	−23.4
K.........	288	+44	−14.2
F.........	292	+58	−13.3
	290	+51	−13.8

SOUTH

Type	A	D	$V_{\odot}$
			km
G.........	147°	+ 1°	− 6.4
F.........	204	+32	− 8.4
	176	+16	− 7.4
K.........	232	+59	−21.3
F.........	238	−11	−13.1
A.........	240	+18	−18.9
G.........	242	+28	−12.4
G.........	246	− 6	(−44.1)
	240	+18	−16.4
$2\overset{\mathrm{M}}{.}9$ and brighter..	258	+40	−24.5
K.........	258	+39	−22.4
A.........	261	+ 3	−20.2
F.........	264	+32	−20.7
	260	+28	−22.0
M.........	272	+41	−22.5
G.........	287	− 6	−13.9
B.........	288	+31	−24.8
K.........	292	+60	−20.3
K.........	298	+19	−30.0
A.........	305	+20	−14.5
A.........	306	+14	−20.7
F.........	307	+19	−19.9
	298	+22	−20.6

There is some reason to think that this is a real condition, especially when we consider the consistency within the K-type groups which contain the largest number of stars and the fact that, after omitting

a part of the stars (about the ellipsoidal vertex), we still find the same strongly marked effect.

In the G-type stars the most southerly declination for the apex results from the largest proper motions. In many ways the motions of these stars, which are so closely related to our sun in spectral class, appear to be different from all of the other spectral classes. One explanation of these peculiarities which suggests itself is that the general motion of the G-type stars may approximate more nearly to that of our sun than the other types.

TABLE V

APEX FROM SMALL PROPER MOTIONS

Type	North				South			
	A	*D*	$V_\odot$	No. Stars	*A*	*D*	$V_\odot$	No. Stars
			km				km	
B	254°	+27°	−17.0	70	288°	+31°	−24.8	123
A	260	+35	−17.7	59	261	+ 3	−20.2	63
F	270	+37	−15.0	27	264	+32	−20.7	46
G	271	+27	−14.6	36	242	+28	−12.4	45
K	265	+32	−19.6	82	258	+39	−22.4	138
	264	+32	−16.8	274	263	+27	−20.1	415
Omitting A	....	....	....	....	....	+32	....	....

For the southern stars this progression in declination is less marked, and there is also a progression in the right ascensions of the apex, decreasing from the large proper motions, through those of medium and small size, to those with contrary parallactic signs.

The positions of the apex from the four groups each of northern and southern stars given in Table II fall reasonably well on great circles, which make considerable angles with the galactic plane and with each other.

It should be remembered in this connection that to some extent the largest and smallest proper motions prefer essentially different regions of sky. The evidence from the stars of Class B, however, which have small proper motions in all regions where they are found, indicates that the variation of declination depends at least in part upon the size of the proper motions. While it seems not improbable that any dependence upon size of proper motion may be a

TABLE VI

EFFECT OF OMITTING STARS NEAR ELLIPSOIDAL VERTICES

STARS OF $3^{M}_{\cdot}0$ AND FAINTER	North						South					
	A	D	$V_{\odot}$	K	No. Stars	No. Equations	A	D	$V_{\odot}$	K	No. Stars	No. Equations
			km	km					km	km		
B	253°.9	+26°.5	−17.0	+0.8	70	13	287°.7	+30°.8	−24.8	+0.6	123	18
Omitting ellipsoidal vertices	253.5	+28.3	−17.5	+0.4	45	10	284.3	+49.4	−26.4	−3.6	84	15
A	256.9	+23.9	−19.6	+0.9	96	15	262.4	+13.5	−20.2	+0.7	85	16
Omitting ellipsoidal vertices	256.8	+34.8	−16.0	0.0	72	11	263.5	+11.6	−21.6	+1.2	71	12
F and G galactic	254.8	+22.0	−21.5	+0.2	66	12	274.2	+13.8	−12.0	−0.6	88	14
Omitting ellipsoidal vertices	257.5	+23.9	−21.1	+0.5	45	8	218.9	+24.3	−50.0	+1.4	53	9
K . . . Large	276.9	+17.8	−26.3	−0.2	34	14	297.8	+18.8	−30.0	+1.8	51	17
Omitting ellipsoidal vertices	288.5	+ 7.8	−28.1	−0.7	30	11	292.1	+ 6.8	−49.6	−0.1	40	13
Medium	262.2	−34.0	−25.2	+3.0	34	14	232.0	+58.6	−21.3	−1.2	51	19
Omitting ellipsoidal vertices	237.3	−42.4	−16.8	+1.2	25	11	220.0	+57.3	−23.3	−1.4	38	14
Small	264.7	+31.6	−19.6	+0.5	82	20	258.3	+39.4	−22.4	+1.2	138	25
Omitting ellipsoidal vertices	250.2	+43.8	−19.9	0.0	65	16	280.0	+18.2	−24.7	+0.7	98	18
Contrary	287.7	+44.1	−14.2	−0.3	39	14	292.2	+60.4	−20.3	+0.7	57	17
Omitting ellipsoidal vertices	Apparently no effect						305.5	+49.2	−20.2	+0.7	42	12
M	269.2	+15.0	−24.1	+1.1	24	11	272.2	+41.4	−22.5	+2.2	41	20
Omitting ellipsoidal vertices	275.0	+33.3	−19.0	+0.3	20	9	278.9	+ 7.8	−37.1	+1.5	26	13

dependence upon distance or other conditions, it seems almost necessary to conclude that the underlying cause is some form of rotary or spiral motion.

C. The solar velocity also varies in the northern sky with the size of the proper motions and the declination of the apex—the velocity decreasing from 27 km for the mean declination of apex -15° to 14 km for the mean declination of apex $+44^\circ$. This progression is fairly consistent, as shown in Table III. There is some indication that in the southern stars the *smaller* solar velocities go with the more southerly apices.

D. The solar velocity appears to vary in both hemispheres with the galactic latitude of the apex.

E. The declinations of the apex from the small proper motions of five of the spectral classes are with one exception quite accordant from both northern and southern stars, and do not differ greatly in the mean from that generally used by investigators of stellar motions. The conclusion is reached that the small declinations obtained for the solar apex from radial velocities have been due very largely to the influence of stars of medium and large proper motions.

F. With the exception of the stars of Class G, the velocity of the solar motion from the small proper-motion stars is consistently less for the northern than for the southern stars. Slightly the reverse appears to be true for the other classes of proper motions, but this conclusion is not very reliable.

If the stars about the ellipsoidal vertices are omitted, the solar velocities in the northern sky are, without exception in the eight general divisions of Table VI, smaller than in the southern sky.

G. The solar velocities from small proper-motion stars are less from the middle-type stars than from the early and late types. This may possibly be due to a larger admixture of stars with contrary parallactic components of motion.

H. The value of the solar velocity increases generally from the stars with contrary parallactic signs to those with large proper motions. Some such result was to be expected from a more or less accidental distribution of motions and distances.

I. The values of the constant error as found above from northern and southern stars separately are generally small and not very different in the different spectral types.

J. The systematic variation in the declination of the apex from the different spectral classes, which is shown in Table VII, appears to be largely due to the proportion of stars with large and medium-sized proper motions which they contain.

TABLE VII

FROM ALL REGIONS OF SKY

Spectral Type	A	D	$V_\odot$	K	No. Stars	No. Equations
			km	km		
$2^{M}_{.}9$ and brighter	258°0	+41°5	−18.9	+1.0	110	33
B $3^{M}_{.}0$ and fainter	276.0	+29.6	−20.3	+4.0	193	31
A $3^{M}_{.}0$ and fainter	260.9	+15.3	−18.3	−0.1	185	39
F $3^{M}_{.}0$ and fainter	267.9	+11.1	−18.9	+0.4	184	41
G $3^{M}_{.}0$ and fainter	257.4	+20.2	−16.4	−0.7	131	38
K $3^{M}_{.}0$ and fainter	274.2	+25.6	−20.7	+4.3	391	63
M $3^{M}_{.}0$ and fainter	269.7	+31.7	−22.9	+4.0	65	31

K. Marked excesses of positive velocities have been found in the K- and M-type stars in the regions of the ellipsoidal vertices. In some groups these excesses are as large as the solar velocity itself. There appears to be little or no such effect in the early and middle classes. Table VI contains the results of solutions including and omitting the groups within about 40° of the ellipsoidal vertices. Owing to the unsymmetrical nature of the data in the classes limited to galactic stars, the results omitting the vertices are rather uncertain. This is particularly true of the southern F and G stars. At the present time it does not seem necessary to try to strengthen those particular results.

These large excesses of positive velocities in the regions of the ellipsoidal vertices appear to be arranged in streams rather than symmetrically about an axis.

L. There are some preferences in the right ascensions of the apex, which if confirmed are undoubtedly significant. Those from the B stars may be taken as probably the most trustworthy. The right ascension of the apex from northern B stars differs over 30° from that derived from the southern stars, a difference which is not greatly changed by the omission of the stars about the ellipsoidal vertices, one-third of the total number. Something similar was found in the right ascensions of the apex of these stars derived from their proper motions.

It is perhaps suggestive that in the considerable region along which these positions of the apex are distributed we find the Milky Way divided into two well-defined streams of the same order of distance apart as these apices.

The progressive change in the position of the solar apex *toward the ellipsoidal axis* as we go from the stars of small to those of large proper motion is very significant.

The fact that any particular class of stars (in this case large and medium-sized proper motion) gives positions of the solar apex not far from the ellipsoidal axis would seem definitely to connect the phenomena observed with those of star-streaming as explained by the ellipsoidal hypothesis. The larger solar velocities from the northern stars giving such declinations for the apex appear also to confirm a close relation with ellipsoidal phenomena.

Discussion of these results is reserved until investigations have been made of other conditions which should also bear upon an explanation, such as proper motions, distances, the distributions of residual velocities and of the relations which appear to exist among the different conclusions themselves. The general conclusion may be stated, however, that the apparently systematic differences in the position of the apex and in the velocity of the solar motion strongly indicate, if they do not establish, variations of the general direction and velocity of motion of the stars themselves in different portions of our stellar system.

Systematic variation of the position of the apex of solar motion seems to be as definitely indicated as other conclusions which have been drawn from these same stars (1300 in number), such as increase

of velocity with increasing supposed age of the stars, constant-error term, magnitude-velocity equation, etc. If some such variation is confirmed, then we are confronted with the necessity of choosing some position of the apex as fundamental. Several bases for such a decision suggest themselves now, such as faint or very faint stars; small proper motions; large proper motions; the more distant stars; the nearer stars; stars which after a better knowledge of their motions have been obtained are known to be themselves nearly stationary; classes of stars having peculiar qualities (now unknown perhaps) which better fit them for this purpose than the condition mentioned. In any case a large number of objects is of course essential. If great difficulty is encountered in finding a suitable basis among the objects in our own stellar system, it might even be necessary or desirable to use the objects of external systems, as, for example, the spiral nebulae.

It would seem, however, that a better knowledge of the peculiarities of motions in our stellar system, together with a study of the structure of the Milky Way, should furnish a satisfactory basis for the choice of a fundamental apex.

The results of general solutions for all parts of sky for each spectral class separately are given in Table VII.

Observatorio Nacional Argentino, Córdoba
April 27, 1916

A PHOTOMETRIC STUDY OF THE ECLIPSING VARIABLE RZ CASSIOPEIAE

By R. S. DUGAN

The variability of RZ Cassiopeiae was discovered by Müller in 1906. He found it to be of the eclipsing type. The favorable position and brightness of the star, the short period of revolution and duration of eclipse made it an attractive object for observers. It has been quite extensively observed visually with and without photometers, and photographically in focus and out of focus, with and without color-screens.[1] RZ Cassiopeiae is B.D. $+69^\circ 179$, and its provisional designation as a variable was 77.1906. It is of type A.

Certain questions were, however, left unanswered by this mass of material. I started observing this star in 1911 with the hope of deciding whether we had at last an eclipsing variable without measurable secondary minimum, and to measure the minor effects of ellipticity and interchange of radiation. The 9792 measures necessary to answer these questions satisfactorily were made with the polarizing photometer in the usual manner. Corrections for atmospheric absorption have been applied. Two comparison stars were used. B.D. $+69^\circ 184 = a$, which was used in the early measures, proved to be variable through a small range. The later observations with B.D. $+69^\circ 181 = b$ are sufficiently numerous to

[1] Müller and Münch, *Astronomische Nachrichten*, **171**, 357, 1906; **183**, 76, 1909; Nijland, *ibid.*, **176**, 171, 1907; Lehnert, *ibid.*, **192**, 201, 1912; **194**, 165, 1913; Hoffmeister, *ibid.*, **202**, 42, 1916; **197**, 317, 1914; Parkhurst and Jordan, *Astrophysical Journal*, **26**, 251, 1902; Wendell, *Harvard Annals*, **69**, Part 2; Graff, *Mitteilungen der Hamburger Sternwarte*, No. 13; Beljawsky, *Mitteilungen der Nikolai-Hauptsternwarte zu Pulkow*, **3**, 31, 1910; Bemporad, *Memorie della Società degli Spettroscopisti italiani* (2), **2**, 153, 1913; Padova, *ibid.* (2), **2**, 57, 1913; Lazzarino, *ibid.* (2), **2**, 123, 1913; Jordan, *Publications of the Allegheny Observatory*, **3**, No. 16; Harvard photographic observations, received in manuscript; Yendell, *Astronomical Journal*, **28**, 126, 1914; Venturi Lacchini, *Riv. Astr.*, **7**, 241, 1913.

define the light-curve of RZ Cassiopeiae and hence the variations of star *a*.[1]

A careful study of all observations of RZ Cassiopeiae during eclipse, including the Harvard photographs, resulted in the clear detection of a variation of the period. A paper dealing with this matter is soon to appear in the *Monthly Notices*.

The observations were combined into normals, including, on the average, five sets of sixteen measures. The $a-v$ observations are reduced to $b-v$ by adding $0^{\mathrm{M}}.415$.

As the depth of secondary minimum (d) is comparable to the effects of ellipticity (c) and exchange of radiation or reflection (b), a least-square solution was made including all normals not observed during primary minimum. The light (a) of the system at longitude 90°, and the time of secondary mid-eclipse were two further unknowns.[2] The solution resulted in the following set of values:

$$a = 0.961 \pm 0.0034$$
$$b = 0.029 \pm 0.0038$$
$$c = 0.016 \pm 0.0069$$
$$d = 0.058 \pm 0.0068$$

$$\text{Time of sec.} = \text{time of primary} + \tfrac{1}{2}P - 4^{\mathrm{m}}.4 \pm 5^{\mathrm{m}}.6 .$$

The unit of intensity is $b-v = 3^{\mathrm{M}}.020$.

The shift of secondary minimum, found from the solution, was disregarded, because it evidently results from two low normals in the early part of secondary minimum. It is furthermore not at all comparable with the shift to be expected from the position of periastron and the eccentricity found from the spectrographic observations at Allegheny, and is in the opposite sense to the shift required by the interpretation of the change in the period as due to a revolution of the line of apsides. This assumption of a circular orbit makes no appreciable difference in the results.

The elements of the system RZ Cassiopeiae derived from the mean light-curve are shown in Table I.

The unit of light is the combined light of the brighter sides of the two stars at their maximum area, corresponding to $b-v = 3^{\mathrm{M}}.016$. The unit of length is the mean radius of the relative orbit.

[1] *Astronomical Journal*, **29**, 137, 1916.

[2] Russell, *Astrophysical Journal*, **39**, 407, 1914.

In combination with the spectrographic results[1] we obtain the following absolute elements, given in Table II.

TABLE I

	Uniform	Darkened
Maximum radius of larger star, a_f	0.2966	0.2886
Minimum radius of larger star, b_f	0.2917	0.2857
Maximum radius of smaller star, a_b	0.2278	0.2759
Minimum radius of smaller star, b_b	0.2240	0.2730
Ratio of the radii of the stars, k	0.768	0.956
Ratio of the axes of the spheroidal stars, $1+\frac{1}{2}z$	1.016	1.010
Least apparent distance of centers, cos i	0.153	0.147
Inclination of orbit plane, i	81°10′	81°34′
Eccentricity of orbit, e	0	0
Maximum fraction of light of smaller star obscured during primary eclipse, α_0	0.806	0.770
*Difference of light of sides of larger star, $2b_1$	0.063	0.063
Difference of light of sides of smaller star, $2b_2$	0.006	0.006
Light of brighter side of smaller star, L_b	0.877	0.918
Light of brighter side of larger star, L_f	0.123	0.082
Ratio of surface brightness:		
Of the bright sides of the two stars, J_b/J_f	12.1	12.3
Of the sides of the fainter star	1.51	1.77

* The quantity b in the least-squares solution is the difference of the reflection effects b_1-b_2 on the two stars. This separation assumes that the quantities b are proportional to the energy received by the surface of each star from the radiation of its companion.

TABLE II

	$m_b=2\ m_f$		$m_b=2.7\ m_f$	
	Uniform	Darkened	Uniform	Darkened
Max. radius larger star, in km, a_f	1,024,000	996,000	1,263,000	1,229,000
Max. radius smaller star, in km, a_b	786,000	953,000	970,000	1,175,000
Mass of larger star, m_f	0.38 ☉		0.58 ☉	
Mass of smaller star, m_b	0.77 ☉		1.58 ☉	
Density of larger star, ρ_f	0.13 ☉	0.14 ☉	0.10 ☉	0.11 ☉
Density of smaller star, ρ_b	0.56 ☉	0.31 ☉	0.61 ☉	0.34 ☉
Radius of orbit, in km	3,452,000		4,258,000	

The second assumption of the mass-ratio is based on the conclusions reached by Shapley in his study of relative mass and relative brightness in spectroscopic and visual binaries.[2] Some

[1] *Publications of the Allegheny Observatory*, 3, No. 16.

[2] *Contributions from the Princeton University Observatory*, No. 3, p. 121.

assumption of the mass-ratio is necessitated by the faintness of the large star and the consequent failure to observe the lines of its spectrum with the spectrograph. The two assumptions made above are regarded as reasonable on several grounds.

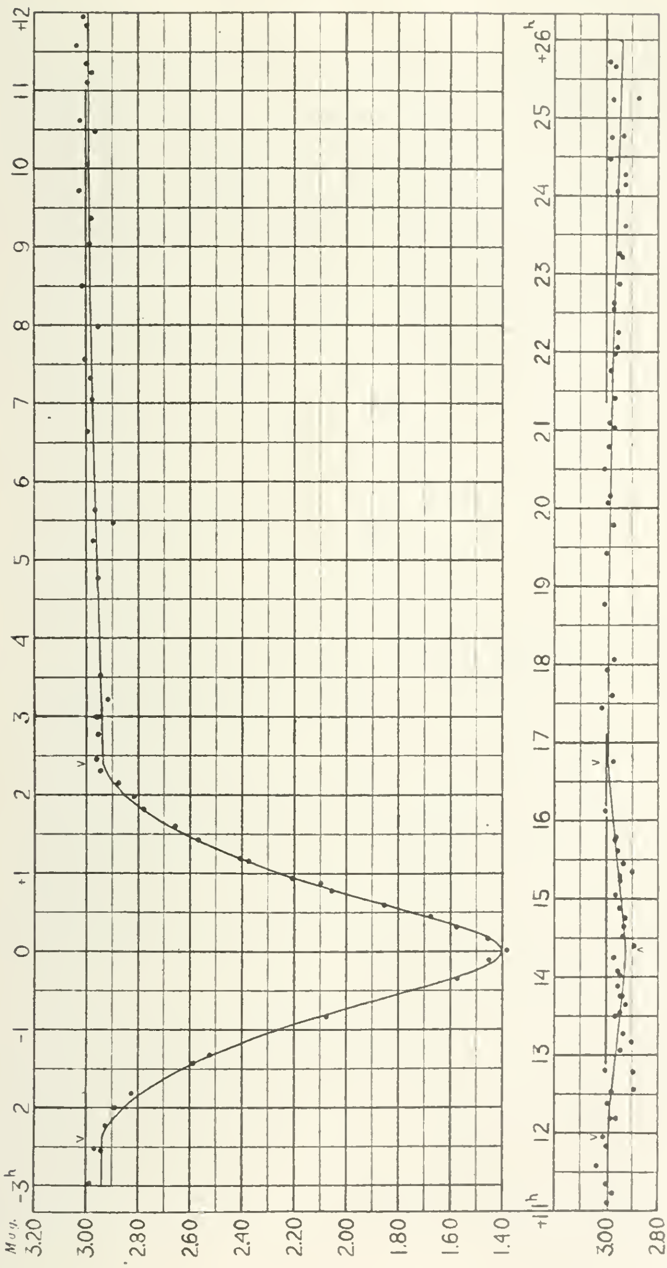
The residuals of the darkened solution are identical with those of the uniform solution with the exception of the regions $-2^h38^m.7$ to -1^h30^m and $+1^h30^m$ to $+2^h38^m.7$. In these regions the darkened curve lies below the uniform, the maximum difference being $0^M.026$. The uniform curve is apparently the better fit.

The probable error of a single set of sixteen measures, from the residuals of the uniform solution, is $\pm 0^M.037$. The corresponding figures for stars previously investigated by the writer were: RT Persei, $\pm 0^M.037$; Z Draconis, $\pm 0^M.039$; RV Ophiuchi, $\pm 0^M.051$. The last-mentioned star shows a decidedly asymmetrical light-curve, with the resulting large probable error.[1]

The Harvard photographic observations sent me, from which I formed a mean curve for the study of the period, are hardly numerous enough to warrant an independent solution. The photographic range of primary minimum is about $0^M.23$ greater than the visual range.

A comparison of Wendell's observations with my own should prove interesting, since they were made with the same kind of photometer. Wendell used star *b* for comparison, and made 312 sets of sixteen measures. Nearly 200 of the 312 were made during the five hours of primary minimum, leaving the other twenty-four hours of the curve not very well defined by the observations. No effects of ellipticity or reflection are apparent. There is some indication of a secondary minimum of about the depth shown by my curve, but displaced nearly two hours toward the following primary. The observations are not sufficiently numerous to determine these three quantities with much accuracy. The mean epoch of Wendell's observations of secondary minimum is about 900 periods earlier than mine. The brightness at primary minimum is practically identical in the two curves. The difference is at most one one-hundredth of a magnitude. The uniform curve computed from my observations follows the observations of Wendell perfectly

[1] *Astrophysical Journal*, 43, 130, 1916.



Computed Uniform Mean Light-Curve of RZ Cassiopeiae

from the bottom of primary minimum as far as phase $\pm 1^h 20^m$. From there on the observations of Wendell drop steadily more and more below my curve, the difference at the end of primary eclipse being $0^M.04$. The average of Wendell's observations outside primary eclipse is $b-v=2^M.90$, while the average of mine is $b-v=2^M.966$. As the difference in the two curves is probably due to different color-sensitiveness of the two observers and the two telescopes, it appears probable that the color of the variable approaches that of the comparison star at mid-eclipse. An independent solution of Wendell's primary minimum, using the ellipticity and reflection coefficients, and the depth of secondary minimum found from my observations, resulted in the following somewhat different set of uniform elements: $k=0.612$, $\alpha_0=0.844$, $i=79°28'$, $a_f=0.3105$, $a_b=0.1900$, $L_b=0.815$, $\frac{J_b}{J_f}=11.3$.

SUMMARY

1. The following new eclipse elements have been found for the system RZ Cassiopeiae:

J.D. $2417355.4208+1^d.19525\ E+0^d.007\ \sin\ (12°+0°.068\ E)=1906$ May $24^d 10^h 6^m.0$ G.H.M.T. $+1^d 4^h 41^m\ 9^s.6\ E+10^m \sin\ (12°+0°.068\ E)$.

2. At primary minimum, when eight-tenths of the smaller brighter component is covered by the larger fainter component, the star is $1^M.59$ fainter than at maximum. The loss of light at secondary minimum is $0^M.06$. Every eclipsing variable which has been observed with sufficient care and persistence shows a measurable secondary minimum.

3. The ellipticity and reflection effects detected in the curve are comparable with those found in the study of other systems.

4. The smaller star emits 7 times as much light as the larger, while its surface brightness is 12 times as great. The distance between centers is $3\frac{1}{2}$ times the radius of the fainter star, and probably between 5 and 6 times the radius of the sun. The brighter component is probably between $2\frac{1}{2}$ and 6 times as dense as the fainter component.

5. A comparison of my own observations with the Harvard photographic and Wendell's photometric observations shows that the star is redder than the comparison star B.D.$+69°184$ and approximates it in color more closely at minimum than at maximum light.

PRINCETON UNIVERSITY OBSERVATORY
May 13, 1916

Minor Contributions and Notes

THE MINIMUM RADIATION VISUALLY PERCEPTIBLE

A determination of the least quantity of radiant energy capable of exciting the sensation of light could and probably by preference should be made in the laboratory by a direct method. The existence of a commonly accepted standard of just visibility, namely, the sixth-magnitude star, permits a determination of this quantity (under certain limiting conditions) by a somewhat indirect method. Drude[1] in his *Lehrbuch der Optik* calculates this quantity in this way as 0.6×10^{-8} ergs per second, assuming a pupillary diameter of 3 millimeters. Unfortunately Drude's treatment of this problem suffers from the errors incident to the crude and inaccurate manner of handling the radiation-light relations which was in vogue when he wrote. His result is in error, for the size of pupil assumed, by a factor of about 10. The object of this note is the recalculation of this least-perceptible quantity of radiation, using methods free from objection, and at the same time taking advantage of the latest data on the relation of stellar magnitudes to terrestrial-light standards.

The steps followed by Drude are as follows: He first states the "mechanical equivalent" of the Hefner unit of light, meaning by this the radiation lying between the "visible" limits of the spectrum corresponding to 1 (Hefner) lumen. For this he takes the experimental figure of Ångström, namely, 0.8×10^{-5} ergs per second. From this he calculates that 1 (Hefner) meter-candle is equivalent to 8.1 ergs per second per square centimeter. He then notes that a sixth-magnitude star gives an illumination of 10^{-8} meter-candles since it appears as bright as a Hefner lamp at 11 kilometers distance. Taking the pupillary opening as 3 millimeters, he arrives by simple multiplication at his figure of 0.6×10^{-8} ergs per second as the radiation entering the eye.

[1] Drude, *Lehrbuch der Optik*, 2d ed., p. 471.

The fundamental error in this procedure is the assumption that all "visible" radiation has the same value as light, that is, as measured on a photometer. Actually equal amounts of "visible" radiation from an approximately white star and from the very red Hefner lamp would measure several times different on a photometer, while if the radiations were all concentrated in the most efficient part of the spectrum for light-production, the amount of "visible" radiation given out by the Hefner would yield nine to ten times the light it does. What Drude wished to determine was the *least* amount of radiation visible as light, for which he should have taken as his standard the most efficient possible radiation from the standpoint of light-production, while the Hefner is about the least efficient light-source finding any use at the present time.

In passing, it may be pointed out that no more complete proof of the inadequacy of the old purely physical definitions of "luminous efficiency" and the "mechanical equivalent of light" could be found than in this same chapter of Drude, where on this fundamental assumption that all visible radiation has the same light-value he proceeds to calculate the "luminous efficiency" of the arc lamp from its candles per watt, and the illumination due to sunlight from its "luminous efficiency" and the solar constant. In the one case he arrives at a figure much higher than any arrived at by experiments based on the same criterion of luminous efficiency; in the other case, for the same reason, he comes out with much less than the value obtained by direct measurements. It is to be hoped that later editions of this otherwise admirable textbook will have this chapter recast.

Without going into details, for which the reader is referred to previous papers of the writer,[1] suffice it to say that the process gone through by Drude is legitimate and exact, provided the crude definition of luminous flux as radiation lying between certain spectral limits is superseded by the definition that it is radiation evaluated according to its capacity to produce the sensation of light, that is, according to the luminosity-curve of the spectrum.

[1] Ives, "The Primary Standard of Light," *Astrophysical Journal*, **36**, 322, 1912; Ives, "The Establishment of Photometry on a Physical Basis," *Journal of the Franklin Institute*, **180**, 409, 1915.

Radiation thus evaluated *is* directly proportional to the photometric value of the light produced. The factor of proportionality is called, using the old misapplied term, the "mechanical equivalent of light." Experimentally this has been determined as 0.00159 watt per lumen.[1] This mechanical equivalent of light is the *least* quantity of radiation which can produce one lumen of luminous flux.

We are now in position to make the calculation which is the object of this paper. We first note that

1 meter-candle = 1 lumen per square meter = 0.0001 lumen per square centimeter = 0.000000159 watt per square centimeter = 1.59 ergs per second per square centimeter.

(On the basis of a 3-millimeter diameter pupil the amount of radiation entering the eye *from the most efficient unit light-source* at 1 meter would raise 1 gram of water 1° Centigrade in something over eleven years.)

In order to find the illumination due to a sixth-magnitude star it is necessary to know the relationship between the stellar-magnitude scale and the candle-power scale. This has recently been discussed by Russell,[2] who gives as the weighted mean of several determinations in which the comparisons were made at color-match, that is, as though at high illuminations, that

1 candle at 1 meter is of stellar magnitude -14.18.

By the ordinary formula for reducing stellar magnitudes to intensities we find that the brightness of a sixth-magnitude star is

$$0.849\times 10^{-8} \text{ of this;}$$

hence the least power corresponding to illumination from a light-source of this brightness is

$1.59\times 0.849\times 10^{-8}$ ergs per second per square centimeter $=1.35\times 10^{-8}$ ergs per second per square centimeter.

Drude assumed the diameter of the pupil to be 3 millimeters. This is probably too low, as under the conditions of nocturnal observation it would be fully dilated, probably to a diameter of

[1] Ives, Coblentz, and Kingsbury, "The Mechanical Equivalent of Light," *Physical Review*, **5**, 269, 1915; Ives and Kingsbury, "Physical Photometry with a Thermopile Artificial Eye," *Physical Review*, **6**, 319, 1915.

[2] Russell, "The Stellar Magnitudes of Sun, Moon, and Planets," *Astrophysical Journal*, **41**, 103, 1916.

6 or 7 millimeters. Taking 6 millimeters as a reasonable estimate, it follows that the radiation entering the eye from a light-source of maximum efficiency of the brightness of a sixth-magnitude star would be

$$0.38 \times 10^{-8} \quad \frac{\text{ergs}}{\text{sec. sq. cm}}.$$

This then is, on the assumptions made, the smallest amount of radiation perceivable by the eye. It is important to note, however, that this figure applies only to radiation from a distant-point source, e.g., a star. The energy is, of course, concentrated on the retina into an area of the size of the image formed, whereby the energy-density on the retina is greater than at the pupil by a factor of approximately 10^5. (A study of the visibility of large and small images of the same total intensity would be necessary in order to give the complete answer to the question under discussion.) The amount received by the retina is again reduced somewhat, owing to the absorption of the eye-media, which, however, are quite transparent for visible radiation.

No account has been taken in this calculation of the shift of the maximum of visual sensibility toward shorter wave-lengths at the low intensities of observation common in stargazing. This has been unnecessary because the connection between stellar and terrestrial magnitudes has been established, as stated, for high-illumination conditions, that is, for those by which the mechanical equivalent of light was determined. The only outstanding error then becomes the difference in area of the luminosity-curve of the spectrum of an observed star as the observing conditions are changed from high to low brightness. Since the average star is approximately white, the change of area of the luminosity-curve as its maximum shifts from $0.55\,\mu$ to $0.51\,\mu$ is slight, certainly much less than the uncertainty in choice of size of pupil.

It is of some interest to note that at the rate of energy-reception just calculated the eye receives through the pupil the elementary energy-quantum (6.585×10^{-27} erg. sec.$\times$frequency) in one-thousandth of a second.

Herbert E. Ives

Physical Laboratory
The United Gas Improvement Co.
Philadelphia, July 1916

THE PHOTOGRAPHIC BRIGHTNESS OF THE FULL MOON

Father Hagen has very kindly called the writer's attention to a series of observations of the brightness of the moon at different phases which was overlooked in preparing the summary of the subject recently published.

These observations were made by Scheller[1] at Prag in 1906–1907, by comparison of the darkening of photographic plates by known exposures to direct moonlight and to a Hefner lamp at a distance of one meter—the law of photographic action of light of different intensities and for different exposures being determined for each plate by supplementary exposures. The observations were made and reduced with much care, and the results should be of high precision, although suffering somewhat from the difficulties which beset photometric work in the midst of a smoky city. Scheller calls attention to these difficulties, which caused him to reject the results obtained on two of the twenty-two nights of observation.

TABLE I

Phase	Nights	Difference of Magnitude between		O−C
		Moon and Lamp	Moon and Full	
−88°........	3	$+1^{M}.49$	$+2^{M}.36$	$+0^{M}.09$
−75°........	2	+1.19	+2.06	+ .23
−58°........	4	+0.53	+1.40	+ .08
−24°........	3	−0.35	+0.52	− .01
+ 5°........	3	−0.88	−0.01	− .13
+18°........	2	−0.17	+0.70	+ .26
+68°........	3	+0.64	+1.51	−0.29

The results for the remaining twenty nights, transformed into the notation used in the writer's paper, are summarized in Table I. To save space, the observations at similar phases are grouped into normals, giving each night equal weight. The first column gives the phase angle (negative before the full), the second the number of nights combined in forming the normal, and the third the mean

[1] *Sitzungsberichte der Akad. der Wissenschaften in Wien*, Math.-Wiss. Kl., 120, II, 889, 1911.

difference in stellar magnitude between the photographic brightness of the moon at the given phase, reduced to mean distance, and the Hefner lamp at one meter. On reducing these to mean full moon with the aid of the light-curve given in the writer's paper,[1] the photographic brightness of the full moon is found, from all the observations, to be $0^{M}.87 \pm 0^{M}.05$ greater than that of the lamp at one meter. The resulting differences in magnitude between the moon at the different phases and at the full are given in the fourth column, and the residuals from the light-curve previously mentioned in the last column.

These residuals appear to be due to accidental error of observation. Their average value, regardless of sign, but reduced to equal weight, is $\pm 0^{M}.13$ for the mean of three observations, corresponding to $\pm 0^{M}.23$ for a single observation. This is almost exactly equal to the corresponding quantity in the case of King's photographic observations, which was found to be $\pm 0^{M}.09$ for the mean of 6.4 observations, to which corresponds $\pm 0^{M}.23$ for one observation.[2]

If Scheller's observations were added to those employed in forming the normals of the previous paper (p. 116), with weight equal to King's, the residuals of the normals affected would be modified as follows:

Phase	−80°	−55°	−18°	+5°	+23°	+63°
Old residuals.......	$+0^{M}.04$	+0.04	+0.01	−0.04	−0.04	+0.01
New residuals......	+0.06	+0.05	+0.01	−0.06	−0.01	−0.03

The mean light-curve from which these residuals are taken would not therefore be sensibly altered by the inclusion of the additional observations.

It is, however, of much interest to note that, photographically, the full moon gives 2.2 times as much light as a Hefner lamp at a distance of a meter, while visually, according to Graff,[3] the light of the moon is only 0.27 that of the lamp, or $1^{M}.43$ fainter, as against $0^{M}.87$ brighter photographically. Hence the actinic power of moonlight, for equal visual intensities, is 8.3 times that of the

[1] *Astrophysical Journal*, 43, 114, 1916.

[2] *Ibid.*, p. 116, 1916. [3] *Ibid.*, p. 128.

lamplight. (Scheller, with a different reduction-curve, gets a ratio of 10.)

This may seem at first glance in glaring contradiction with the result derived by the writer, that the color-index of moonlight is $+1^{M}_{.}18$, making its actinic intensity only one-third of the visual; but the apparent discrepancy is immediately explained when it is remembered that the standards of comparison in the two cases are very different, being in the first instance the yellowish-red flame, and in the second the stars of Class A, which are far whiter than any terrestrial sources of light.

From the foregoing data it appears that the color-index of the Hefner lamp is greater by $2^{M}_{.}3$ than that of moonlight, and is therefore about $+3^{M}_{.}5$ on the astronomical scale, so that, for equal visual intensities, the lamp has but $\frac{1}{25}$ the actinic power of the light of a star like Vega. This color-index is somewhat greater than that previously found[1] for the standard 2 c.-p. electric lamps studied by King which was $+2^{M}_{.}9$; but it is of the same order of magnitude and appears entirely credible.

It may be added that Scheller's conclusion that the moon is brighter at the third quarter than at the first, which depends upon only three discordant observations near the former phase, is negatived by the concordant results of more than fifty observations by six different observers, near the same phase, which are discussed in the previous paper, and show beyond a doubt that the brightness is greater at the first quarter. The explanation which he suggests for his result—that the lunar maria, which cover a larger portion of the visible surface at the third quarter, reflect more actinic light than the rest of the disk—is disproved by a glance at any photograph of the moon.

Henry Norris Russell

Princeton University Observatory
June 16, 1916

[1] *Astrophysical Journal*, 43, 129.

Reviews

Stereoskopbilder von Sternhimmel. 2. Serie. Von Max Wolf. Leipzig: J. A. Barth, 1915. M. 5.

The ordinary stereoscopic pictures—landscapes, etc.—are usually made by simultaneous exposures with two lenses several inches apart, thus obtaining the necessary parallactic effect which is given by the human eyes. If there are no moving objects in the view, the same result may be obtained with a single lens that is shifted a few inches between the two exposures. On account of the vast distances of the heavenly bodies the first of these methods is not possible when applied to the sky, except in the case of a meteor, which may be caught by two cameras some distance apart. The second method is applicable in a way for this purpose—the parallactic displacement being obtained either by the motion of the earth or of the object, or both, in the interval between the two photographs. To obtain the motion which will cause the required displacement, the time element in this method is a necessary factor. Beautiful results may thus be obtained in the case of a comet or a rapidly moving star, or the planets and satellites of the solar system. Though the result thus obtained seems in a manner to imitate the perspective given by the ordinary stereoscopic view, it is not strictly comparable with it in reality, for in the case of the rapidly moving star the appearance of relative distance is false. The star may really be farther away (though this is not probable) than its immediate neighbors, which appear as a distant background for it. The comet may have changed its form between the time of the two photographs, so that a false perspective of its relative parts may be shown. The perspective of the comet itself with respect to the stars gives no definite idea of the true distance and is always a false representation of the actual distance. The satellites of a planet are almost certain to be shown in a false perspective, and what appears to be a nearer satellite may really be a more distant one, the effect depending on the relative apparent motions of the satellites at the time, and not on their relative distances.

We must not, however, discourage the study of stereoscopic pictures of the celestial bodies; far from it, for they are not only beautiful but sometimes very instructive, and may lead us to correct conclusions not

otherwise obtainable. What we must do is to guard against the deceptions that will be produced by this method, for seeing is not always believing in such cases. Indeed, very serious errors may be promulgated by too much faith in what the stereoscope shows us in the sky.

The foregoing remarks will apply with more or less force to the subject we now have for review, Professor Wolf's second series of stereoscopic pictures of the heavens.

This consists of twelve stereoscopic views of certain celestial objects, and is issued in a neat cardboard case, for use in the ordinary stereoscope.

They are from photographs made by Dr. Max Wolf with reflecting and refracting telescopes at the observatory at Heidelberg, Germany, and are a continuation of a previous set of twelve such views. Following is the table of contents: Tafel 1, "Stern mit Eigenbewegung"; Tafel 2, "61 Cygni"; Tafel 3, "Mondkugel"; Tafel 4, "Mondlandschaft"; Tafel 5, "Patroclus"; Tafel 6, "Uranus"; Tafel 7, "Der Spiralnebel im Bären"; Tafel 8, "Der Spiralnebel in den Jagdhunden"; Tafel 9, "Der Nebel im Orion"; Tafel 10, "Komet Morehouse (1908, November 16)"; Tafel 11, "Komet Morehouse (1908, November 10)"; Tafel 12, "Blick in die Milchstrasse."

These pictures have been made by the second method which I have mentioned, but instead of a displacement of the camera of a few inches, its point of view has been changed by many thousands of miles, and generally with long intervals of time between the pictures. It is only necessary to describe a few of these photographs, though each one is of interest. The series consists of twelve stereoscopic views of comets, nebulae, proper-motion stars, planets and satellites, the moon, and the Milky Way. Apparently these are intended for the general public, as a popular explanation (in German) accompanies each plate.

The remarkable astronomical work of Dr. Max Wolf needs no word of comment here—and only praise of the highest order could be given it. It is pleasing to see that this effort to bring some of it to the attention of the public in a popular form has been so successful.

Plates 1 and 2 give views of proper-motion stars, one of these being the celebrated star 61 Cygni. These stars stand out strongly in perspective from the regular background, but in each case this is an effect wholly due to motion and has nothing whatever to do with the true distance, the effect of which would be inappreciable in these pictures. Plate 1 is very pleasing, the small star standing out beautifully between the observer and the other stars. The interval in this case is fourteen years.

Plate 3 shows the nearly full moon in plastic relief. The interval is about one month. This, like most stereoscopic views of the moon, does not show our satellite as a true sphere, but gives the impression of a distorted globe, some parts of which are rounder than others. The fault is due to the want of a perfect combination of phase and libration at the time of taking the pictures. A proper combination of these should give a perfectly spherical appearance to the moon.

Plate 4, which shows the lunar craters along part of the terminator, does not seem to be a success, as the relief is not satisfactory.

Plate 5 shows the small asteroid Patroclus. Allowance having been made for the motion of the planet, that object appears suspended in space as a small point, while the slight motion has caused the stars to trail somewhat. This picture is very successful and impressive and is well worth the making.

Plate 6 is a very interesting picture. It shows the planet Uranus and its two outer satellites in strong relief. The image of the planet is marred by the rays produced by the support of the flat in the reflecting telescope, but the picture is otherwise excellent. As these rays are artificial, their removal would have added much to the picture without injuring its scientific value.

The pictures of the spiral nebulae shown in Plates 7 and 8 are also very interesting; that of M 51 is especially pleasing. The coils of the spirals seem to stand out in relief from each other. This, however, cannot be due to either motion or parallax, or true perspective. One gets essentially the same effect with one picture and monocular vision.

Plate 9 is of the great nebula of Orion, and makes a beautiful and effective picture. It shows some perspective, both in the stars and in the nebula, but this does not seem to be real.

Plates 10 and 11 are views of Morehouse's comet. No. 10 is especially beautiful and the appearance of perspective is finely shown. It is the most effective picture of the set and is the more realistic from the fact that the exposures were so short that the motion of the comet during each exposure was not enough to elongate the star-images, so that they appear as points of light instead of trails. This photograph is to be highly commended.

Plate 12 is a fine view of the Milky Way in Aquila, made with a small lens. The perspective is good and the different distances of the various cloud-forms are well shown. One seems to be looking into the great depths of the Milky Way, although both pictures are made from

the same negative! It is an excellent example of how misleading stereoscopic views of the sky may sometimes be. With this fact in mind one must look again at the pictures of the nebulae shown in Plates 7, 8, and 9, and consider what the apparent perspective means. It shows how easily one can be misled by a false conclusion as to stellar displacements that do not exist except in the stereoscope itself. This is not intended as a criticism of the present work but as an explanation for those not familiar with the subject. As we have seen the case of the Milky Way photograph, two prints from the same negative may be used in the stereoscope with good effect. The writer has thus employed two prints from the negative of Saturn made with the 60-inch reflector at Mount Wilson, on 1911 November 19. Though there can be no possible true perspective in these pictures, they assume a pseudo perspective that greatly improves the view. When the two are thus seen in a stereoscope the result shows the planet as a ball suspended in the middle of the rings in excellent relief. It is a great improvement over the flatness that is so evident with the ordinary view.

If the limitations which have been pointed out are borne in mind, the photographs are to be commended, not only to the public, but also to astronomers.

While on the subject of stereoscopic vision it may not be out of place to call attention to some peculiarities of monocular vision that have come to the notice of the present writer. It is a subject with which very few seem to be familiar. Those who see the splendid perspective shown with the stereoscope are likely to express regret that one who has lost an eye is debarred from the use of these beautiful pictures. In a sense our sympathies are just. He who has lost an eye cannot use the stereoscope and hence loses some of their beautiful perspective. In reality, however, he is the only one who correctly sees a single photograph of a landscape or other subject—or a painting. The writer has been familiar with this fact for many years and has found much pleasure and instruction in it. The one who uses both eyes does not see such objects to the best advantage. He can put himself on an equal with the one-eyed man by closing one eye when he looks at most of the ordinary pictures.

Take any photograph, reproduction, or painting, where the perspective of distance is shown. View this at the ordinary distance; close one eye. The background will at once recede to its proper distance and the various objects will assume their regular perspective and you will have, in effect, a stereoscopic view. Open both eyes and the picture

at once becomes flat and the distances are in one plane—that of the photograph. Try this method on a picture where reflections are shown in a stream or pool of water. There will be a "sheen" on the water, which is such a beautiful feature in the stereoscope and which is so like what one sees in nature itself. Open both eyes and this "sheen" will disappear. I have often received additional pleasure by viewing with only one eye those exquisite photographs of mountain and other scenery so often given in the *National Geographic Magazine.* The difference between what is seen with one or two eyes is manifestly in favor of the one eye. Another point that strikes one in such examinations is that any particular object is more readily detected with one eye than with both. The one eye seems to pick out each individual object, so that when one is looking thus at a star photograph peculiarities not noticed with both eyes appear.

Space forbids my going further into the subject, but it is an interesting and important one. Of course we are doing here the same thing that we used to see done in art galleries, where the pictures were viewed with one eye, through a rolled-up pamphlet or tube of some kind. The tube is an added advantage because it excludes objects not intended as a part of the picture and which would otherwise detract from it.

E. E. Barnard

Yerkes Observatory, Williams Bay, Wis.
July 19, 1916

A Voyage in Space. By H. H. Turner. London: Society for Promoting Christian Knowledge, 1915. Pp. 304, figs. 96. Cloth, 6*s*. net.

Professor Turner has presented us in book form with the course of "Juvenile Lectures" which he delivered at the Royal Institution during the Christmas time of 1913. The book is written in the author's usual attractive manner, and although the presentation of the different topics is clear enough in most instances for a child to grasp their meaning, folk of more mature years will find the book delightful reading. Many of the recent discoveries in astronomy are discussed and the chief landmarks in the progress of the science are reviewed. One of the main attractions of the book lies in the apt elucidations of difficult points; at this sort of thing Professor Turner is a master-hand. There are six lectures. The first lecture deals with the starting-point, our earth.

In the following lecture the length of the voyage and the start through the air are considered. The third lecture discusses the means of conveyance, the telescope. In the fourth lecture a "visit" to the moon and the planets is made, and in the fifth and sixth lectures the voyage is concluded by a visit to the sun and the stars. The volume could well be used as an introduction to astronomy for lay readers.

C. C. Crump

THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XLIV OCTOBER 1916 NUMBER 3

THE REFLECTING POWER OF THE ALKALI METALS

By J. B. NATHANSON

I. INTRODUCTION

The alkali metals, by virtue of their great chemical activity and their interesting electrical and optical properties, afford interesting fields for research. The great difficulty in handling the metals has, however, limited their investigation. Further interest in these metals has been added by the comparatively recent study of the selective photo-electric effect which, it has been suggested, might be due to the peculiar optical properties of the alkali metals in the region of the selective effect. If such be the case, there ought to be a marked change in the reflecting powers of the alkali metals in this region. Consequently any knowledge of the reflecting powers of the alkali metals as a function of the wave-length and plane of polarization is highly desirable, and it is to this end that the present investigation has been carried out.

Up to the present no direct method has been employed in the study of the reflecting powers of the alkali metals. All our knowledge rests upon the comparatively few katopric measurements. The first investigation was made by Paul Drude[1] upon the single metal sodium. The reflecting surface was formed by melting the

[1] *Annalen der Physik*, 64, 159, 1898.

sodium in a vacuum, and the reflecting power obtained from a study of the nature of the reflected elliptically polarized light.

In 1913, R. W. and R. C. Duncan[1] made a more extended investigation of the optical properties of sodium and potassium as a function of the wave-length. Drude's method was employed. The metals were used in the form of mirrors, i.e., glass backed by metal. These authors found very high values for the reflecting powers of sodium and potassium, the former being the better of the two.

In the present investigation the reflecting powers have been obtained by a direct measurement of the incident and reflected light-intensities. A photo-electric cell was used as a photometer, the cell being previously calibrated in terms of known light-intensities. This use of the photo-electric cell has been anticipated by E. V. Hulburt,[2] whose work appeared during the progress of this investigation. Hulburt determined the reflecting powers of a large number of metals for ultra-violet light, the angle of incidence being kept constant. It appears that Hulburt assumed a linear relation between the photo-electric current and the light-intensity, an assumption which does not seem to be justified in view of this and other investigations.

II. WHITE, UNPOLARIZED LIGHT: RELATION BETWEEN THE PHOTO-ELECTRIC CURRENT AND THE LIGHT-INTENSITY

The apparatus.—In the present investigation of the reflecting power of the alkali metals, a photo-electric cell was employed as a photometer. The cell chosen was one of the most sensitive ones made by Dr. Jakob Kunz of this Laboratory. The cathode consisted of rubidium deposited by distillation upon a film of silver. The anode consisted of a loop of platinum wire. Between the anode and cathode is located a platinum guard ring which is usually earthed to avoid leakage across the glass between the electrodes.

Investigations on the relation between the photo-electric current and the light-intensity are not in good agreement with each other. While Elster and Geitel[3] and Richtmeyer[4] have shown

[1] *Physical Review*, **36**, 294, 1913.

[2] *Astrophysical Journal*, **42**, 205, 1915.

[3] *Annalen der Physik*, **48**, 625, 1893.

[4] *Physical Review*, **29**, 71 and 404, 1909.

that the photo-electric current is strictly proportional to the light-intensity, on the other hand Lenard[1] and quite recently Ives[2] have shown that the linear relationship does not strictly hold. Ives, in an extended investigation of the subject, wherein he subjected a great variety of cells to different conditions, showed that the relation is not a linear one, but that the photo-electric current is a complicated function of the voltage, electrode distance, and gas pressure in a cell.

Consequently, owing to the conflicting literature on this subject, it was decided, before using the cell above as a photometer, first to calibrate it in terms of known light-intensities by the aid of crossed nicol prisms. To this end, the arrangement of apparatus shown in Fig. 1 was employed.

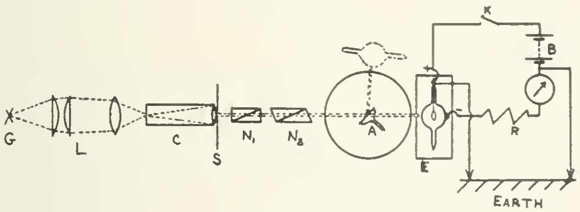


FIG. 1

A Nernst glower G was used as a source of light, both on account of its intensity and because of its reputation for constancy. The light after passing through several condensing lenses L is focused upon the circular aperture of the collimator lens. The light then passes through the nicols N_1 and N_2, the former having rectangular ends to eliminate rotation of the beam when N_1 is rotated.

The circular beam of parallel rays of light is then incident upon the photo-electric cell, which is inclosed in the earthed metallic box E. This box was air- and light-tight, and blackened on the inside. It served to eliminate any possible extraneous light. In this box was placed some phosphorous pentoxide to render the air dry and so to diminish leakage across the glass or across the hard rubber disks through which were conducted the wires leading to the electrodes of the cell. The beam was admitted through a glass

[1] *Annalen der Physik*, 8, 149, 1902.

[2] *Astrophysical Journal*, **39**, 428, 1914; **43**, 9, 1916.

window, before which was placed a shutter sliding on a groove, so that by means of a cord and pulleys the shutter could be easily raised or lowered by the observer at his observing station.

The voltage used across the photo-electric cell varied from 111 to 134 volts, this being furnished by a set of constant potential cells. As a detector of the photo-electric current, a sensitive galvanometer was employed, being loaned to the writer by the Department of Astronomy through the kindness of Professor Joel Stebbins. This galvanometer had a figure of merit of 5×10^{-10}. The terminal of the galvanometer next to the negative pole of the battery was earthed. This was found to be highly essential, serving to eliminate completely troublesome leakage currents in the circuit, and enabling one to take observations with the greatest accuracy in the most humid days of the summer.

Method of observation.—With the axes of the nicols parallel to each other, the cell was exposed to the light and the galvanometer deflection observed. Then the deflection was observed for some angle between the nicols. Finally, the nicols were made parallel again, and the deflection again observed. This last observation served as a check on any fluctuations of the light-intensity that might have occurred during the observations. As a matter of fact, the light-intensity was absolutely constant only on rare occasions, there being usually a small and slow variation. However, by continual checking of the deflection for zero angle between the nicols, proper corrections could be applied to the deflection for any angle between the nicols.

Several readings were always taken for each position of the nicols. The individual readings usually agreed to within two- or three-tenths of a millimeter for the larger deflections, and to a correspondingly smaller extent for the smaller deflections.

The results.—In Fig. 2 are given the deflections corresponding to various light-intensities, the latter being proportional to the squares of the cosines of the angles between the two nicols. Each deflection is the average of two to four observations.

Upon examination of the plotted results, it is evident that the current light-intensity relation is *not* a strictly linear one, but is in the form of a curve slightly concave toward the illumination-

axis. The results for the curve labeled 130 volts were obtained by using a different quadrant of the nicol prism, N_1; the light

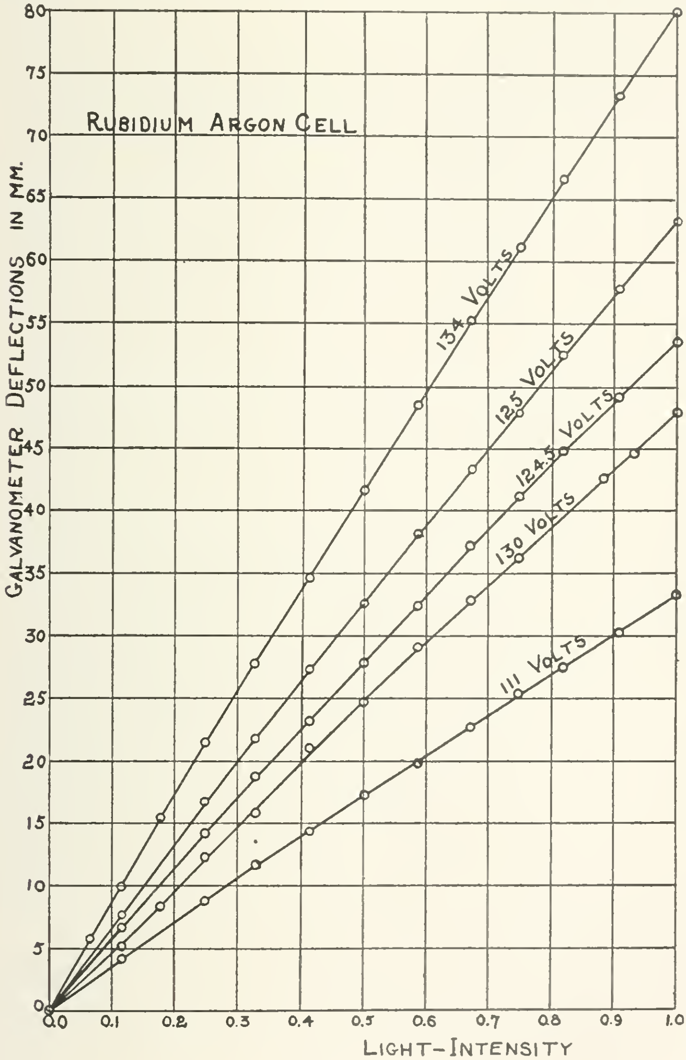


FIG. 2

also was somewhat weaker than in the other three cases, whence the smaller deflections. This serves to show that the concavity of the curves cannot be due to lack of symmetry of the nicol prism. This

concavity increases, the larger the range of the deflections. In the succeeding determinations of reflecting powers, proper corrections were always made in accordance with these curves.

III. PREPARATION OF THE ALKALI MIRRORS

The alkali mirrors were made either by distillation or by pouring the metal upon a square piece of plane glass about 2.5 cm on edge and 1.74 mm thick. The cell C was made by joining a small piece of glass tubing to a much wider piece. The latter was then cut off, leaving a bell-shaped opening. The edge of this bell was ground plane with emery and finally with rouge, the edge becoming highly polished. The glass plate, after being thoroughly cleaned with alcohol, potash, and nitric acid, was then clamped tightly against the polished edge of the bell, and "Rock Cement" applied thickly around the edge P. The bell was then placed in an electric oven, and baked at about 140°C. Usually three applications of cement were applied to the cell. The baking was continued until the cement turned to a brownish color. This cement served excellently in making the cell air-tight, and in enabling one to subject the cell to much heat during the distillation of the metal, without endangering the vacuum.

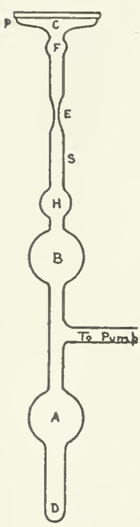


Fig. 3

The cell was then attached to the glass apparatus shown in Fig. 3. In the case of Na and K, the metal was introduced into A through D, which was then sealed off, and the tube evacuated. The metal was then melted down with an electric coil. Great care is necessary in this process, as the glass is likely to crack very easily when the molten metal bursts out of its oxide skin.

A portion of the metal was poured into B, and from there distilled to H. A small molten globule was then guided to the concavity F. In this process the tube was removed from the pump, and the globule guided to F by means of successive jars by the hand. The metal was then distilled against the glass plate which was placed in contact with a flat piece of ice. The metallic vapor thus deposited itself upon the plate in the form of a mirror,

the layer of metal being made thick enough to be entirely opaque to light. The mirror was then sealed off and removed at *E*.

In the case of one of the K mirrors (mirror No. 2), a large amount of the metal was collected in *H* by distillation, and the whole molten mass forced into the cell against the glass plate, making a mirror out of a solid cake of the metal.

Rubidium cannot be obtained on the market in metallic form, and so had to be obtained by reducing RbCl with Ca, the materials being placed in an iron boat inclosed in a hard glass tube attached to *D* of Fig. 3. The metallic vapor was condensed in *A* before it was redistilled and deposited upon the glass plate.

Success in making the mirrors was not always attainable, there being many opportunities for failure in the long process. Many of the mirrors after being formed were found to have thin oxide films on their surfaces, and so were discarded. Only those mirrors were picked for investigation which appeared perfect as viewed by the eye.

IV. ARRANGEMENT OF APPARATUS FOR THE DETERMINATION OF THE REFLECTING POWER

In the determination of reflecting powers the arrangement of apparatus shown in Fig. 1 was employed, the nicol prisms being, however, removed so that the light incident on the mirrors was unpolarized. The metallic box containing the photoelectric cell was mounted upon the telescope arm of the spectrometer to facilitate the movement of the cell for any angle of incidence desired.

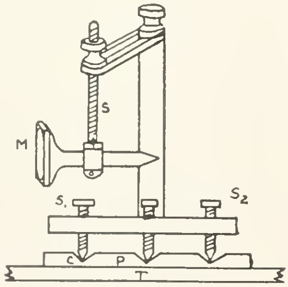


FIG. 4

The mirror *M*, Fig. 4, was mounted on a small tripod, whose legs fitted into cups *C* on a brass plate *P* which could be permanently clamped to the top of the spectrometer table *T*. The center of the reflecting face of the mirror was adjusted to lie along the axis of the spectrometer. After proper adjustment the plate *P* was clamped to the table *T*. By this means the mirror could be removed from the spectrometer table and quickly replaced in its original position.

This was found desirable, since, in the determination of reflecting powers, observations on the unreflected beam were alternated with observations on the reflected beam. Using this method of observation proper corrections could be made in case the light-intensity, or voltage across the photo-electric cell did not remain absolutely constant.

V. THE FORMULA

The ratio of the reflected light-intensity to the incident light-intensity gives the reflecting power of the whole mirror, i.e., metal plus glass. The greater interest lies in the reflecting power of the metal itself when in contact with the glass. Knowing this, it is possible to form a very close approximation as to what the reflecting powers of the metals would be in a vacuum.

Owing to the large number of internal reflections, the following mode of reasoning will be used to determine the reflecting power of the metal. Let

I = the light-intensity of the incident light
i = angle of incidence
r = reflecting power of the front face of the glass, i.e., the fraction of the incident light, incident on the glass, which is reflected back into the air
r' = reflecting power of the interior glass surface, i.e., for light internally reflected, e.g., as at C, E, and G
t = transmission power of the glass plate for a given thickness, i.e., the fraction of the light which, e.g., after penetrating the surface at A, reached the point B
R = reflecting power of the alkali metal, i.e., the fraction of the light incident on the metal glass boundary at B, for example, which is reflected by the metal.

The quantities r, r', t, and R are functions of the angle of incidence.

The sum of the components of reflected light-intensities, a, b, c, d, etc., are given by the series

$$O' = Ir + I(1-r)(1-r')t^2R(1+r't^2R+r'^2t^4R^2+r'^3t^6R^3+ \ldots).$$

The photo-electric cell registers the sum of all these components of the reflected light-intensity. If $O=\frac{O'}{I}$ is the reflecting power of the whole mirror (metal plus glass), then

$$O=r+\frac{(1-r)(1-r')t^2R}{1-r'Rt^2}.$$

Hence

$$R=\frac{O-r}{t^2(1+Or'-r-r')}. \tag{1}$$

This is the formula that was employed in the determination of the reflecting power of the alkali metals. It will be noted that in order to know R the optical properties of the glass plates used in these mirrors must be determined.

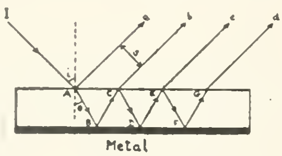


FIG. 5

Inspection of Fig. 5 shows that the beam after reflection is spread out. If S is the lateral displacement between two successive components, then applying Snell's law it follows that

$$S=\frac{t \sin 2i}{\sqrt{n^2-\sin^2 i}} \tag{2}$$

where t=thickness of the glass=1.74 mm
i=angle of incidence
n=1.5155=index of refraction

This was determined by means of the Abbe refractometer, using the method of grazing incidence.

Taking i as 60°, the value of S is 1.26 mm. Theoretically, an infinite number of internal reflections occur between the metal and the air-glass boundary. Practically, the components of the reflected beam after the fourth one are negligible. Hence, taking four components as effective, the displacement would be about 5 mm. Adding 3 mm for the width of the beam, the breadth of the reflected beam does not exceed 8 mm. The aperture to the photo-electric cell was almost 2 cm, hence all effective components of the reflected beam were certainly able to enter the cell.

VI. OPTICAL PROPERTIES OF THE GLASS PLATES

In order to determine r, the reflecting power of the front face of the glass plate, the back face was abraided with coarse emery and then coated with lamp-black. The results are shown in Fig. 7, where the continuous line represents the experimental values, while the dotted line gives the theoretical values as given by Fresnel's reflection equation for natural white light.

The experimental values, which at first are smaller than the theoretical values, are, however, larger than the latter for larger angles of incidence. This may be due to a possible slight specular reflection of the abraided rear surface which becomes more effective for the larger angles of incidence. In general, the agreement is quite satisfactory, and demonstrates the adaptability of the photo-electric cell as a photometer.

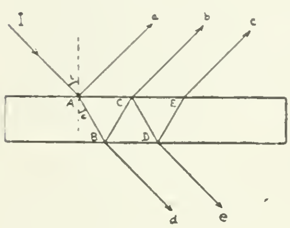


Fig. 6

Determination of t and r'.—The determination of the transmission power and the reflecting power of the internal surface of the glass was effected through a study of the light reflected and transmitted by the glass plate.

Following the method of reasoning employed previously, and adding up the components a, b, c, etc., of the reflected light, we have (see Fig. 6)

$$R'=r+(1-r)(1-r')r't^2+(1-r)(1-r')r'^3t^4+\ .\ .\ .\ .\ +\ .\ .\ .\ .$$

where R' is the reflecting power of the glass plate (both surfaces).

$$R'=r+(1-r)(1-r')r't^2(1+r'^2t^2+r'^4t^4+r'^6t^6+\ .\ .\ .\ .\ ).$$

Hence

$$R'=r+\frac{(1-r)(1-r')r't^2}{1-r'^2t^2}. \tag{3}$$

Considering the light transmitted by the plate, and summing up the components, d, e, etc., we have for the total fraction, T, of the light transmitted,

$$T=(1-r)(1-r')t(1+r'^2t^2+r'^4t^4+\ .\ .\ .\ .\ ).$$

Hence

$$T=\frac{(1-r)(1-r')t}{1-r'^2t^2}. \tag{4}$$

Combining equations (3) and (4)

$$\frac{R'-r}{T}=r't. \tag{5}$$

Solving for t and substituting in equation (4), and then solving for r',

$$r'=\frac{(R'-r)(1-r)}{T^2+(R'-r)(1-R')} \tag{6}$$

and

$$t=\frac{T^2+(R'-r)(1-R')}{T(1-r)}. \tag{7}$$

Experimentally, R' is obtained by determining the intensity of the reflected light. Keeping conditions the same, T is obtained by merely swinging the cell around on the spectrometer to catch the transmitted light.

The results for R', r', t, and r are plotted in Fig. 7. The values of r' are somewhat lower than those for r, except for values of the angles of incidence approaching the critical angle for the glass plate, when r' crosses r and would ultimately reach 100 per cent for values of the angle of incidence of 41°. It must be borne in mind that the values of the reflecting powers shown in Fig. 7, and in succeeding tables and curves for white light, have been corrected in accordance with the calibration-curves of Fig. 2. By setting up a simple proportion between observed values of the galvanometer-deflections for the unreflected and reflected beam, and values of galvanometer-deflections on the calibration-curves, it has been possible to obtain the corrected values of the reflected light-intensity directly from the calibration-curves. The corrections are of course rather small.

VII. REFLECTING POWERS OF POTASSIUM, SODIUM, AND RUBIDIUM

Three potassium mirrors were investigated. In Table I is given a summary of the results. Mirrors No. 1 and 3 were formed by distillation, while No. 2 was formed by pouring the molten metal

against the glass and allowing it to solidify. The latter method was rendered quite difficult by the tendency of the metal to crystallize upon solidification.

The agreement between the reflecting powers for the different mirrors is indeed quite satisfactory considering that the mirrors

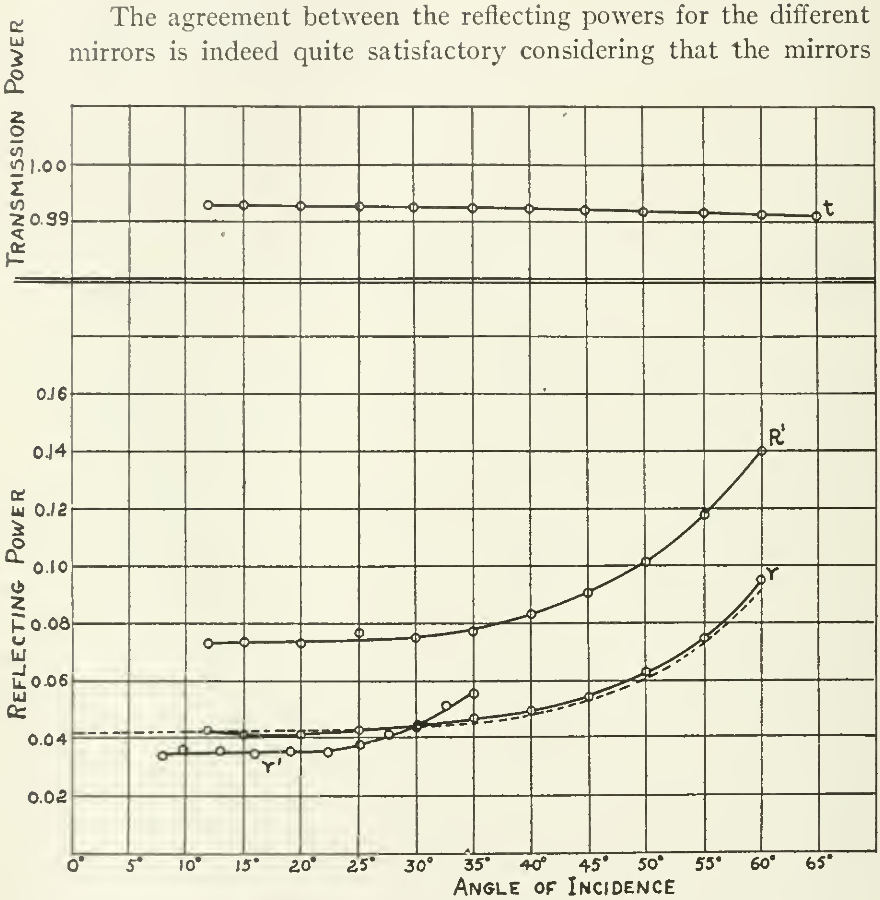


FIG. 7

were made at different times and under different circumstances. Especially is the agreement between mirrors Nos. 1 and 2 to be noted, the former being a distilled mirror, the latter being the "solid" metal mirror. This excludes any doubts that the metallic layers of the distilled mirrors were not thick enough.

The variation of the reflecting power with the angle of incidence is shown in Fig. 8. The reflecting powers increase very slowly

with the angles of incidence. Owing to refraction through the glass plates, the actual angles of incidence on the metal are of course less than those on the whole mirror, so that the curves for

TABLE I

POTASSIUM

O					*R*				
Angle of Incidence	No. 1	No. 2	No. 3	Mean	Angle of Incidence on Metal	No. 1	No. 2	No. 3	Mean
13°.5......	0.8675	0.869	......	0.868	8°52'......	0.876	0.878	......	0.877
15	.870	.8685	0.865	.868	9 50......	.8805	.8795	0.875	.8785
20	.869	.869	.8645	.8675	13 3......	.8805	.8805	.876	.879
25	.8675	.8805	.8685	.872	16 12......	.880	.8925	.8805	.8845
30	.888	.873	.875	.8785	19 16......	.900	.8845	.887	.8905
35	.8695	.8835	.863	.872	22 14......	.8815	.8965	.876	.8845
40	.885	.880	.858	.8745	25 6......	.898	.892	.869	.8865
45	.872	.880	.860	.8705	27 49......	.8845	.8925	.872	.883
50	.885	.887	.8685	.880	30 22......	.897	.899	.881	.8925
55	.879	.875	.8645	.873	32 43......	.890	.887	.877	.8845
60	0.874	0.880	0.8715	0.8755	34 51......	0.8825	0.890	0.8805	0.8845

R are "shrunk" to the left. The critical angle for the glass is the maximum that could ever be reached using glass plates.

Much greater difficulty was experienced in obtaining a good sodium mirror than in obtaining a potassium mirror, because of

TABLE II

Angle of Incidence	*O*		Angle of Incidence on Metal	*R*	
	Sodium	Rubidium		Sodium	Rubidium
13°.5............	0.8845	0.750	8°52'............	0.895	0.756
15............	.882	.757	9 50............	.895	.764
20............	.8825	.7575	13 3............	.8935	.765
25............	.8845	.7505	16 12............	.8975	.758
30............	.8785	.755	19 16............	.8915	.7625
35............	.8945	.755	22 14............	.9075	.762
40............	.8925	.7575	25 6............	.905	.7645
45............	.8975	.770	27 49............	.910	.7775
50............	.887	.7725	30 22............	.899	.7775
55............	.8975	.7705	32 43............	.909	.776
60............	0.900	0.775	34 51............	0.9115	0.7755

the higher distillation point of sodium. Most of the attempts resulted in failures due to the formation of slight impure films on the metal next to the glass. These were probably oxide films,

as great care was always taken to clean the glass thoroughly. A mirror was, however, finally obtained which showed none of these films and appeared very perfect as viewed with the eye. The results are given in Table II.

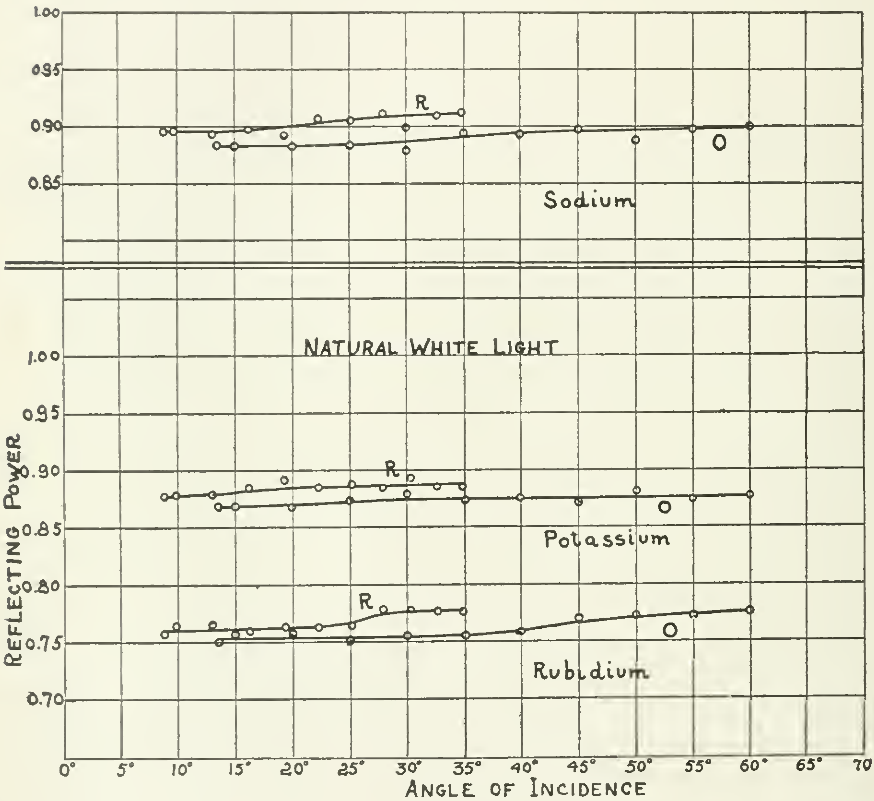


Fig. 8

No great difficulty was met with in the formation of rubidium mirrors, since the metal distills quite easily. The chief difficulty lies in obtaining the pure metal from its chloride. However, Rb is more easily oxidized than Na or K and hence a better vacuum must be insured. The values obtained with a mirror which showed no surface defects is given in Table II.

The values for Na are not plotted next to those for K in order to avoid confusion of contiguous points. In general, the reflecting

power rises slowly with the angle of incidence. The reflecting power is at first quite constant, then suffers a rather rapid rise, and then is nearly constant again. The values for the reflecting powers of the metals themselves are about 1 per cent higher than those for the whole mirror.

Na has the highest reflecting power, K being almost as good. Rb is less than K, so that the reflecting powers increase as the atomic weight decreases.

VIII. MONOCHROMATIC, POLARIZED LIGHT: ARRANGEMENTS OF APPARATUS

The source of light.—In this part of the work, it was proposed to investigate the reflecting powers of Na, K, and Rb for polarized

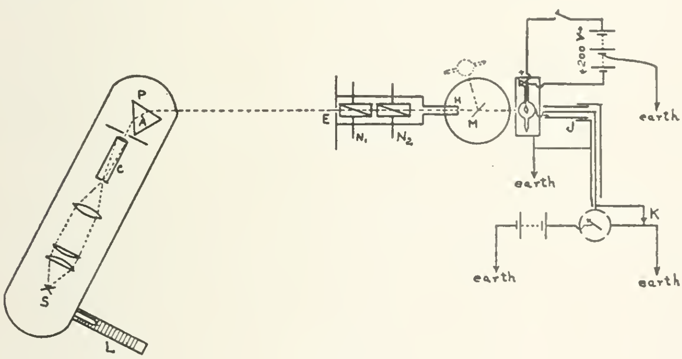


Fig. 9

monochromatic light. The arrangement of apparatus is shown in Fig. 9. In the case of white light, a Nernst glower was used as a source of light. Its use, however, was unsatisfactory, since it decreased so rapidly in efficiency owing to the polarization of the filament on direct current. The alternating current could not be used because it was too unsteady. Consequently for this part of the investigation, a 250-watt, nitrogen-filled, tungsten projection lamp was employed.

The optical system.—The light from one of the filaments S was focused upon the slit of the collimator C, after passing through the system of condensing lenses. The focal length of the collimator

was so adjusted that the emergent rays were just slightly convergent, the light being focused about 1.5 m away from the photo-electric cell. The central portion of the beam emerging from the collimating lens was allowed to pass through a good dispersing prism P, the spectrum being spread out over E. The source of light, condensing lenses, and prism were mounted on a movable table which could be rotated about an axis A through the center of the prism. Hence, by rotating this table any portion of the spectrum could be thrown upon the aperture E.

The light after passing through the rectangular nicols N_1 and N_2 and through the narrow slit H (7×1 mm), was incident directly upon the photo-electric cell, or else after reflection at the mirror M. The nicols served a double purpose, both for plane polarizing the light, and for calibration of the photo-electric current in terms of known light-intensities.

The photo-electric cell was used in connection with the spectrometer as in the previous work. The tube inclosing the nicols and containing the aperture H served to exclude extraneous light from the source. This tube was made purposely narrow toward the end in order to make the use of small angles of incidence possible when the photo-electric cell was swung around the spectrometer.

All light from extraneous sources was completely excluded by means of a double heavy cloth hung all around the apparatus, the room itself being partially darkened. That the screening was perfect was shown by the zero photo-electric current when the shutter of the cell was opened.

During observation upon the reflected light the nicols were arranged with their axes parallel to each other. The position of their planes of polarization was determined by the aid of a glass plate mounted so that the light was incident at the polarizing angle.

Calibration of apparatus in terms of wave-lengths.—The calibration of the scale L in terms of wave-lengths was effected by means of a Hilger wave-length spectrometer, the collimator of the instrument being placed in the position occupied by the photo-electric cell. It was found that the incident light was not purely monochromatic. A rapid test of the reflecting power of an alkali metal as a function of the wave-length showed this variation to

be very small. Hence it was unnecessary to procure purer monochromatic light.

The electrometer.—Since the light-intensities incident on the photo-electric cell are much smaller than in the case of white light, it was decided to use an electrometer to measure the photo-electric currents. The electrometer was of the Cambridge Scientific Co.'s type, giving a deflection of about 200 mm at a distance of 2.3 m for a difference of 1.5 volts between the two pairs of quadrants, there being 96 volts on the needle. The electrometer was placed in a fairly air-tight tin box which was well earthed to a water-pipe.

One pair of quadrants was always left earthed; the other pair (which could also be earthed) was connected to the cathode of the photo-electric cell. The latter connection was effected through a wire which was completely protected from outside disturbing influences by being run through a glass tube, the outside of which was wrapped in tin-foil and well earthed. This protection for the wire was found to be absolutely necessary. A loose joint *J* permitted the motion of the photo-electric cell about the spectrometer.

The needle of the electrometer was carefully insulated by means of amber, so that under ordinary working conditions no leakage could be observed when the quadrants were charged to a difference of potential. There was, however, a slow drift in the direction of an increasing deflection, which proved quite troublesome. This seemed to be clearly due to a "dark current" or a leakage current across the glass of the photo-electric cell, notwithstanding the presence of the earthed ring between anode and cathode. However, by putting this ring at a potential of about 200 volts below that of the anode, the drift was completely eliminated.

The voltage employed across the cell varied from 80 to 120 volts, the negative pole of the battery being earthed as shown in Fig. 9.

IX. METHOD OF OBSERVATION

The electrometer was used "ballistically," i.e., the photo-electric cell was exposed to the light for a definite short interval, and either the resulting steady deflection or the "first throw" of the needle noted. The time interval was given by a metronome

adjusted to beat (approximately) seconds. The shutter in front of the photo-electric cell could be operated by the observer at his observing-post at the telescope. At the beat of the ticker, the shutter could be "snapped up" and then smartly closed at the end of the chosen time-interval. Proper weighting of the cord insured the smooth working of the shutter, and, after some practice, the time-length of exposure could be made to a tenth of a second.

Toward the latter part of this investigation, the humidity of the spring air made it difficult for the electrometer to hold its charge, so that instead of waiting for the needle to come to rest and noting the steady deflection the observer noted the first throw of the needle. It was found that not only was the accuracy of observation not marred by this procedure, but that, furthermore, double the number of observations could be taken in a given time-interval.

Light of a known mean wave-length polarized either parallel or perpendicular to the plane of incidence was allowed to fall on the shutter of the photo-electric cell. The electrometer key was opened, and, when the needle had come to rest, the cell was exposed to the light for 10 or 15 seconds, and the steady deflection or else the first throw noted. The deflections for the reflected light were alternated with those for the unreflected beam as previously. Deflections varying from 50 to 200 mm were employed.

In order to calibrate the observed deflections in terms of known light-intensities, keeping conditions the same as before the nicol N_1 was rotated through various angles, observations were taken in adjacent quadrants and the mean taken to correct for any asymmetry of the nicols with respect to the axis of rotation. These mean deflections were then plotted against the squares of the cosines of the angles. The light-intensities corresponding to the observed deflections for the reflected light could then be read on the curve, giving the reflecting powers directly. A calibration-curve was thus obtained for every series of observations.

The true relation between the light-intensity and photo-electric current can be determined only under the imposition of proper experimental conditions. In the present investigation, the interest lay not so much in the relation between the current and the light-

intensity, as in the relation between the light-intensity and resulting electrometer deflections. Experimental conditions were always employed such as to give greater stability and accuracy to the observations. Consequently as low a voltage as possible was used on the needle. Also, in part of the work, as previously mentioned, the first throw of the needle was employed. For these reasons, it is not safe to assume that the photo-electric current is proportional to the electrometer deflections. Hence the curves between electrometer deflections and light-intensity are not to be assumed as being relations between photo-electric current and light-intensity. They are merely to be regarded as calibration-curves. In general the curves approximated to straight lines, being either slightly convex or concave to the illumination-axis, depending upon the experimental conditions imposed. No theoretical value is assigned to these curves, consequently they are omitted. It must be remembered, however, that the curves given under white light are true current-light-intensity curves.

X. OPTICAL PROPERTIES OF THE GLASS PLATES

In the case of white light it was shown that the reflecting power of the metal itself is only slightly different from that of the mirror as a whole, the former being only from a fraction to a little over 1 per cent greater than the latter.

It has been shown that within experimental errors, Fresnel's reflection equations hold for the front face of the glass. Hence the values for r were calculated from the equations

$$r=\frac{\sin^2(i-\theta)}{\sin^2(i+\theta)} \text{ and } r=\frac{\tan^2(i-\theta)}{\tan^2(i+\theta)}$$

for light polarized parallel and perpendicular to the plane of incidence, respectively.

The variation of r with the wave-length is rather small (0.2 per cent over the range employed), since the variation in index of refraction is small, the range of wave-lengths extending from about 450 to 650 $\mu\mu$. The values of r for yellow light are plotted in Fig. 10.

The range of wave-lengths used being in the visible portion of the spectrum, it could be assumed a priori that the transmission

power of the glass for the different colors would be the same as for white light, since no dispersion or color-effects are ever visible to the eye upon looking through the glass. Nevertheless it was decided to test out this point by a short method.

Light polarized perpendicular to the plane of incidence was incident on the glass plate at the polarizing angle. Very little

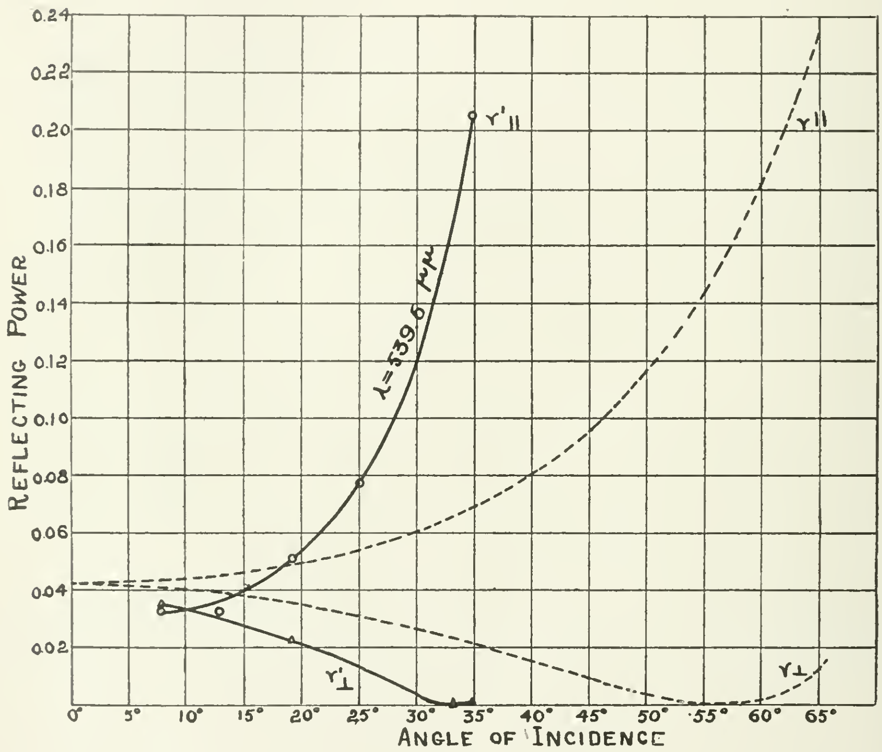


FIG. 10

light was thus reflected, practically all being transmitted. Hence the transmission power of the glass for that angle could be obtained by subtracting the transmitted from the incident light, allowing a small correction for the slight amount of reflected light.

Tests with various colors showed absence of any variation in the transmission power, the values for t being of the order given

in case of white light. As a result the values for t given in Fig. 7 were taken for this part of the work.

The values for r' (reflecting power of internal face of glass) for green light, taken for several angles of incidence, are shown in Fig. 10. These values for r' were used for all the colors. This use of r' is, however, not as radical as it might appear. In the first place, r varies very slowly with wave-length, hence the variation in r' must be likewise very small. Secondly, let us consider the equation for R in the form,

$$R=\frac{\frac{O}{I}-r}{t^2\left\{1-r'\left(1-\frac{O}{I}\right)-r\right\}}.$$

The values of $\frac{O}{I}$ vary from 0.8 to 0.95. $1-\frac{O}{I}$ is therefore a very small fraction, as well as r'. Consequently $r'\left(1-\frac{O}{I}\right)$ is very small compared to $1-r$, so that an error in r', even to the extreme extent of 5 per cent, could only result in an error of 0.1 or 0.2 per cent in the final value of R.

Inspection of the curves for r' shows that for small angles of incidence r' is less than r, but rises above r for angles of incidence approaching the critical angle at 41°. This rise is especially rapid in the case of light polarized parallel to the plane of incidence.

XI. RESULTS

The reflecting powers of potassium, sodium, and rubidium are given in the following tables and curves. The symbols $\parallel$ and $\perp$ on the curves indicate that the plane of polarization is parallel or perpendicular to the plane of incidence, respectively. In general, the reflecting powers are somewhat higher than those obtained for white, unpolarized light. The curves are similar to those obtained for a transparent medium, i.e., the reflecting power for light polarized parallel to the plane of incidence increases steadily with increased angle of incidence, while for light polarized perpendicular to the plane of incidence, the reflecting power decreases with increased angle of incidence. In the latter case the minimum value

reached would of course not be equal to zero as in the case, e.g., of glass.

The curves for K are not as smooth as those for Na and Rb owing to the fact that the steady deflection method was used in the case of K, while the method of first throw was used in the case of Na and Rb. Furthermore, nearly all of the points for Na and Rb are the mean of two values. Na has on the general average just

TABLE III

REFLECTING POWER (*O*) OF RB MIRROR NO. 2

Angle of Incidence	λ 640.9	λ 589.3	λ 539.6	λ 488.8	λ 454.6
PLANE OF POLARIZATION PARALLEL TO PLANE OF INCIDENCE					
12°	0.852	0.821	0.826	0.812	0.804
20	.844	.820	.820	.815	.805
25	.828	.828	.825	.824	.805
30	.870	.829	.820	.837	.809
40	.873	.845	.860	.833	.823
50	.875	.849	.828	.864	.845
60	.872	.868	.863	.875	.865
65	0.888	0.868	0.868	0.872	0.860
PLANE OF POLARIZATION PERPENDICULAR TO PLANE OF INCIDENCE					
12°	0.808	0.777	0.790	0.802	0.759
20	.813	.794	.793	.776	.753
25	.792	.788	.788	.783	.764
30	.822	.775	.800	.773	.748
40	.785	.769	.782	.770	.739
50	.797	.765	.760	.769	.750
60	.787	.762	.793	.780	.766
65	0.790	0.761	0.775	0.783	0.756

a slightly better reflecting power than K, Rb being less than either as for white light.

To the author's knowledge, no other investigation on Rb has ever been carried out, so that comparison is impossible. Great faith is, however, placed in the curves for Rb, since the results obtained for green light for two different mirrors check very well with each other (see Tables IV and V). The results of Tables III and IV are shown in Fig. 11. In these tables λ is given in $\mu\mu$.

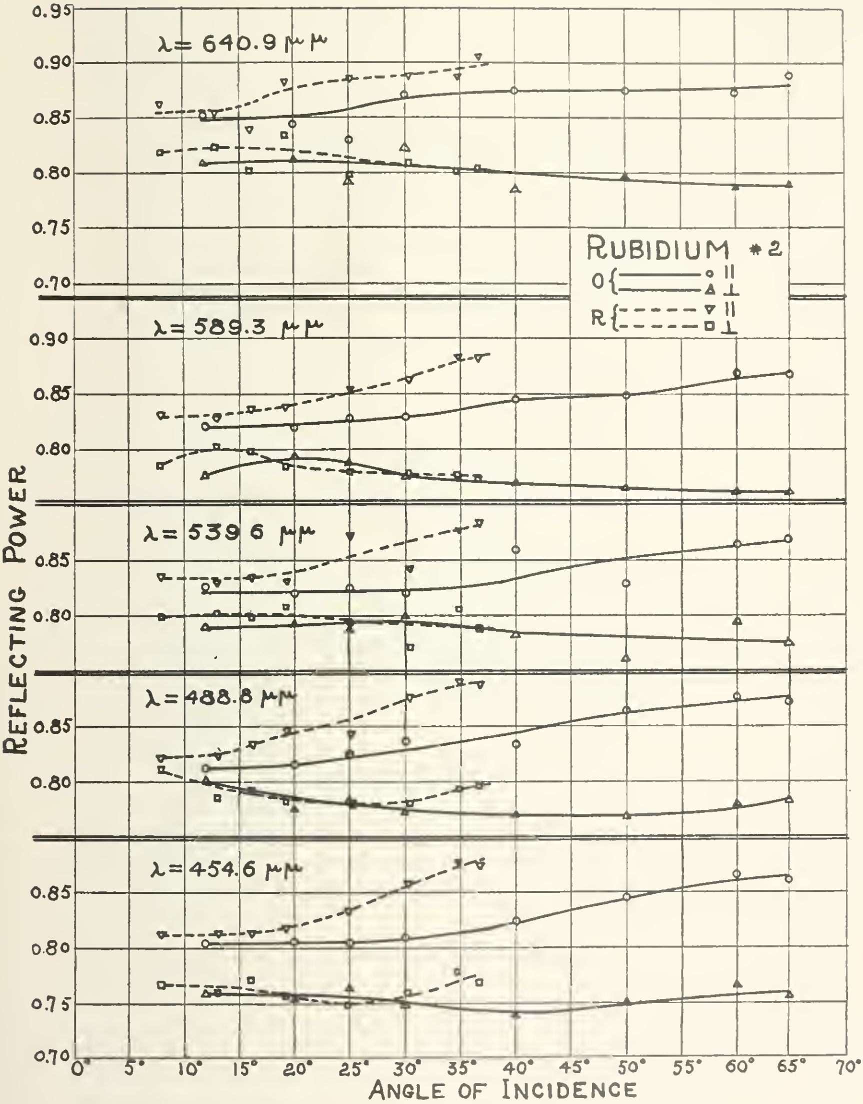

λ = 640.9 μμ
λ = 589.3 μμ
λ = 539 6 μμ
λ = 488.8 μμ
λ = 454.6 μμ
RUBIDIUM #2
O
R
REFLECTING POWER
ANGLE OF INCIDENCE
0.95
0.90
0.85
0.80
0.75
0.70
0°
5°
10°
15°
20°
25°
30°
35°
40°
45°
50°
55°
60°
65°
70°

FIG. 11

The results for Na and K are shown in Figs. 13 and 14.

The curves for Na for light polarized perpendicular to the plane of incidence are noteworthy. For small angles of incidence,

TABLE IV

REFLECTING POWER (R) OF Rb No. 2

Angle of Inc.	λ 640.9	Angle of Inc.	λ 589.3	Angle of Inc.	λ 539.6	Angle of Inc.	λ 488.8	Angle of Inc.	λ 454.6
PLANE OF POLARIZATION PARALLEL TO PLANE OF INCIDENCE									
7°54′...	0.862	7°53′..	0.831	7°53′..	0.835	7°52′..	0.821	7°50′..	0.812
13 4 ...	.854	13 3 ..	.829	13 2 ..	.830	13 0 ..	.824	12 58 ..	.813
16 13 ...	.838	16 12 ..	.837	16 10 ..	.835	16 8 ..	.834	16 5 ..	.813
19 18 ...	.881	19 16 ..	.838	19 14 ..	.831	19 11 ..	.847	19 8 ..	.818
25 8 ...	.885	25 6 ..	.855	25 3 ..	.872	25 0 ..	.843	24 56 ..	.833
30 25 ...	.888	30 22 ..	.863	30 18 ..	.841	30 14 ..	.875	30 9 ..	.857
34 55 ...	.887	34 51 ..	.883	34 47 ..	.877	34 42 ..	.891	34 36 ..	.878
36 48 ...	0.904	36 44 ..	0.882	36 39 ..	0.884	36 34 ..	0.888	36 27 ..	0.874
PLANE OF POLARIZATION PERPENDICULAR TO PLANE OF INCIDENCE									
7°54′...	0.818	7°53′..	0.785	7°53′..	0.799	7°52′..	0.811	7°50′..	0.766
13 4 ...	.823	13 3 ..	.803	13 2 ..	.802	13 0 ..	.785	12 58 ..	.760
16 13 ...	.802	16 12 ..	.798	16 10 ..	.798	16 8 ..	.792	16 5 ..	.771
19 18 ...	.833	19 16 ..	.785	19 14 ..	.810	19 11 ..	.783	19 8 ..	.756
25 8 ...	.797	25 6 ..	.780	25 3 ..	.793	25 0 ..	.780	24 56 ..	.748
30 25 ...	.809	30 22 ..	.778	30 18 ..	.771	30 14 ..	.780	30 9 ..	.760
34 55 ...	.801	34 51 ..	.776	34 47 ..	.805	34 42 ..	.794	34 36 ..	.778
36 48 ...	0.803	36 44 ..	0.773	36 39 ..	0.788	36 34 ..	0.796	36 27 ..	0.769

TABLE V

REFLECTING POWER FOR Rb MIRROR No. 1

FOR λ=539.6 μμ

PLANE OF POLARIZATION PARALLEL TO PLANE OF INCIDENCE				PLANE OF POLARIZATION PERPENDICULAR TO PLANE OF INCIDENCE			
O		R		O		R	
Angle of Incidence	λ 539.6	Angle of Incidence	λ 539.6	Angle of Incidence	λ 539.6	Angle of Incidence	λ 539.6
12°......	0.806	7°53′	0.814	12°......	0.805	7°53′	0.815
20.......	.828	13 2	.837	20.......	.802	13 2	.812
25.......	.837	16 10	.847	25.......	.813	16 10	.824
30.......	.838	19 14	.849	30.......	.808	19 14	.818
40.......	.843	25 3	.854	40.......	.801	25 3	.812
50.......	.850	30 18	.862	50.......	.789	30 18	.801
60.......	.863	34 47	.877	60.......	.795	34 47	.808
65.......	0.861	36 39	0.877	65.......	0.785	36 39	0.798

the reflecting power first increases and then decreases. This appears to become more marked as the wave-length decreases. This is the only evidence that has been obtained that may throw light on the selective photo-electric effect as depending upon the optical properties of the metals.

The Rb curves also show similar evidence, though not so marked as in Na.

The reflecting powers are given with reference to glass as the adjacent medium. Were the metals in contact with a vacuum, the results would have been slightly higher. The magnitude of

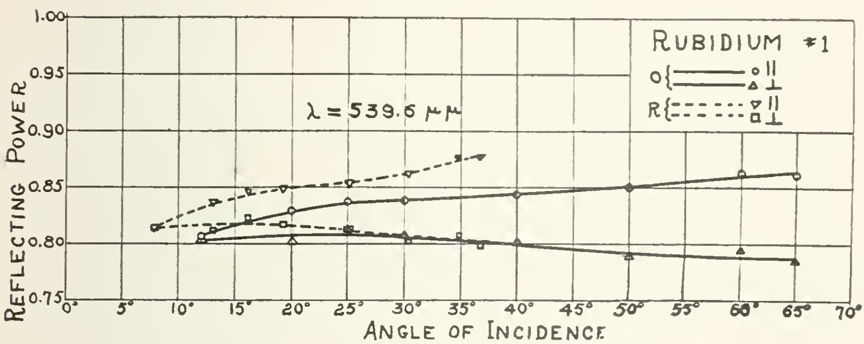


FIG. 12

this increase may be obtained as follows. Taking R. W. and R. C. Duncan's results for K for $\lambda = 589.3\ \mu\mu$, we have $n = 0.068$ and $\kappa = 22.1$. Hence, we obtain a value of 0.92 for the reflecting power of K by substituting in the formula for normal incidence, i.e.,

$$R = \frac{n^2(1+\kappa^2)+1-2n}{n^2(1+\kappa^2)+1+2n}.$$

It has been shown by Ingersoll[1] that the reflecting power of a metal in contact with a vacuum can be obtained from the reflecting power of the metal in contact with a medium of refractive index, m, by dividing n by m and substituting in the formula above for R. In our case, $m = 1.5155$, so that R would become in the case above, 0.914. The reflecting powers, as they are given in the

[1] *Physical Review*, 29, 392, 1903.

tables and curves, would thus be a fraction of a percentage *higher* were the metal in contact with a vacuum instead of with glass.

XII. RELATION BETWEEN OPTICAL AND ELECTRICAL PROPERTIES OF METALS

Starting with Maxwell's electro-magnetic equations, it can be shown that, in the case of very good conductors like metals, the relation between the reflecting power, conductivity, and wave-length is given by

$$R = 1 - \frac{2}{\sqrt{\sigma c \lambda}} \tag{8}$$

where σ is the electrical conductivity in c.g.s. electro-magnetic units, c is the velocity of light, and λ is the wave-length. This relation holds theoretically only for very long wave-lengths. Hence, as the wave-length λ increases, the second term diminishes, and R increases, becoming unity for $\lambda = \infty$.

The variation of R for the alkali metals as a function of λ is shown in Fig. 15. There is a distinct rise of the reflecting power with increasing wave-length, thus confirming theory. The points on the curves represent the mean of the reflecting powers for light polarized parallel and perpendicular to the plane of incidence, and for an angle of incidence of 12°. The mean therefore represents very closely the reflecting power at normal incidence, to which the equation (8) applies.

In Table VI are given the experimental values for the reflecting powers for normal incidence. The values of the reflecting powers as given by equation (8) have also been calculated and inclosed in this table.

Inspection of Table VI shows that the experimental values of the reflecting power of sodium is about 3 per cent less than that calculated by R. W. and R. C. Duncan from their katopric measurements. On the other hand, the values of potassium in this table seem to be somewhat larger than those of Duncan.

In general the theoretical values are lower than the experimental, though the agreement in the case of Rb is very close. The reflecting power of Rb is thus confirmed by Maxwell's equation

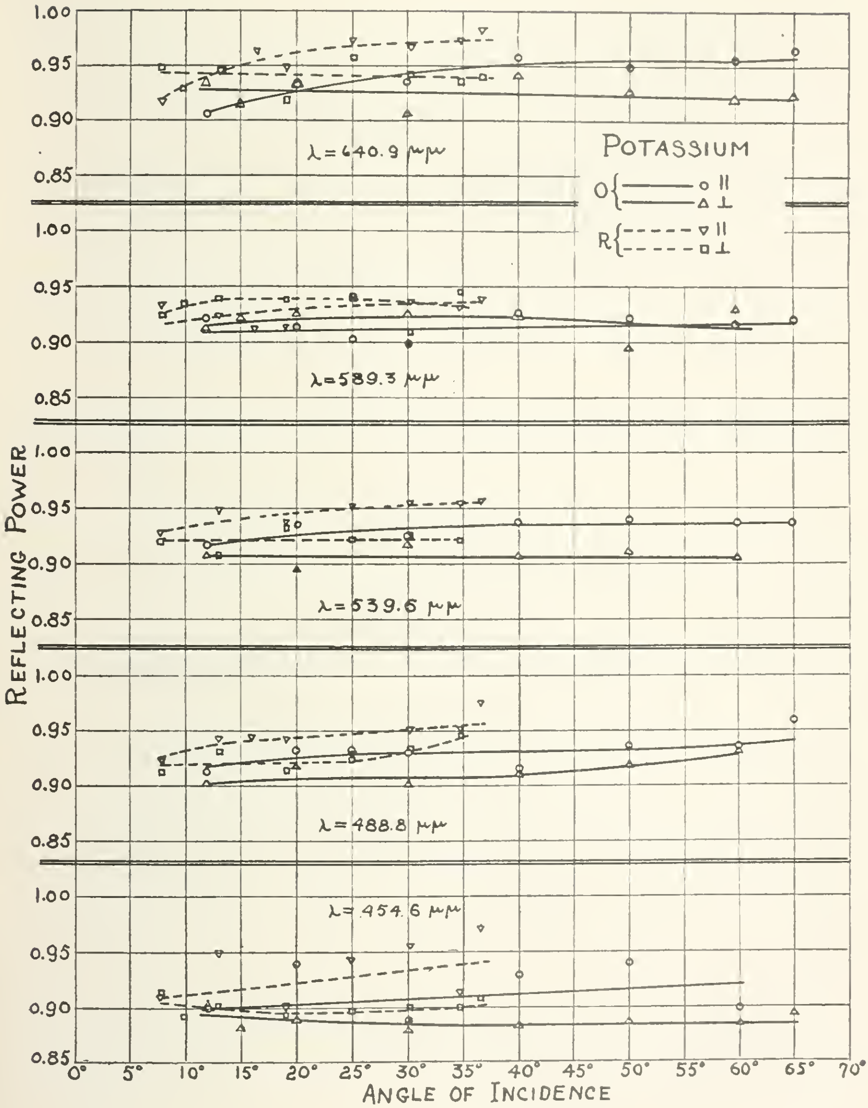

POTASSIUM
O ○ ‖
△ ⊥
R ▽ ‖
□ ⊥
λ = 640.9 μμ
λ = 589.3 μμ
λ = 539.6 μμ
λ = 488.8 μμ
λ = 454.6 μμ
REFLECTING POWER
ANGLE OF INCIDENCE
1.00
0.95
0.90
0.85
0° 5° 10° 15° 20° 25° 30° 35° 40° 45° 50° 55° 60° 65° 70°

FIG. 13

of reflection. The discrepancy between theory and experiment becomes more marked if we put equation (8) in the form

$$\frac{(1-R)}{2}\sqrt{\sigma_{e.m.}\,c}=\frac{1}{\sqrt{\lambda}}. \tag{9}$$

The value of the expression on the left-hand side of the equation appears to be only a function of the wave-length, and hence ought to be independent of the metal used. Calculations for $\lambda=640.9\ \mu\mu$ made with Na, K, and Rb show, however, that the values of the left member of equation (9) are respectively 66.9, 69.3, and 117; while $\frac{1}{\sqrt{\lambda}}=124.4$.

This discrepancy between theory and experiment shows that Maxwell's equations do not hold for the visible spectrum. In view

TABLE VI

NORMAL INCIDENCE

$\sigma_{e.m.}$ for Na $=\frac{1}{5072}$ e.m.u. (21°7)
$\sigma_{e.m.}$ for K $=\frac{1}{7010}$ e.m.u. (20°7) } Hornbeck[1]

$\sigma_{e.m.}$ for Rb $=71\times10^{-6}$ e.m.u. (19°3) Guntz and Broniewski[2]

WAVE-LENGTH $\mu\mu$	NA		K		RB	
	Experimental	Theor.	Experimental	Theor.	Experimental	Theor.
640.9	0.945	0.897	0.933	0.879	0.840	0.829
589.3	.926	.893	.930	.874	.808	.822
539.6	.938	.888	.825	.968	.817	.814
488.8	.924	.882	.918	.861	.816	.804
454.6	0.914	0.878	0.912	0.857	0.789	0.797

of the modern electron theory this is not at all surprising, for Maxwell's theory does not take into account the effect of the resonating electrons within the metals upon the incident electromagnetic waves. That Maxwell's equation holds very well for

[1] *Physical Review* (2), 2, 217, 1913.

[2] C. R., 147, 1474, 1908; 148, 204, 1909.

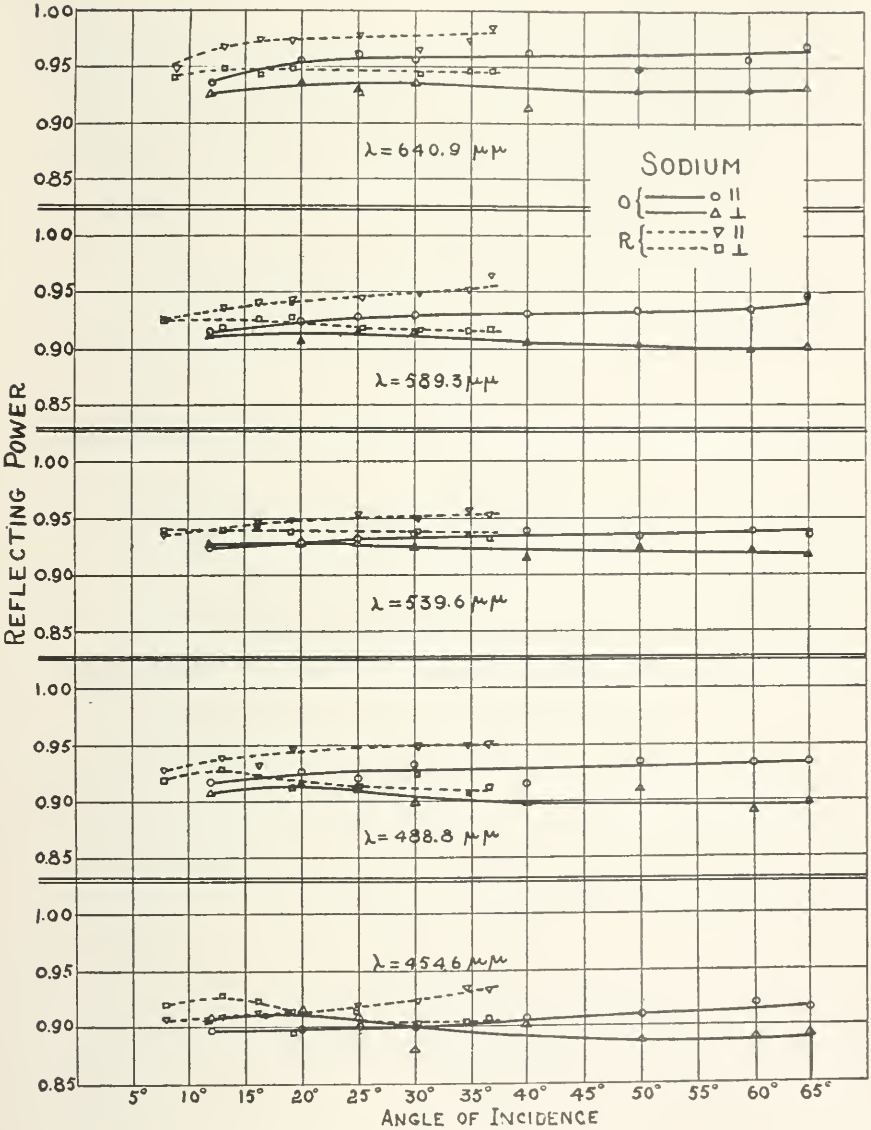

Sodium
O { ——— ○ ∥
——— △ ⊥
R { - - - - ▽ ∥
- - - - □ ⊥
λ = 640.9 μμ
λ = 589.3 μμ
λ = 539.6 μμ
λ = 488.8 μμ
λ = 454.6 μμ
Reflecting Power
1.00
0.95
0.90
0.85
5° 10° 15° 20° 25° 30° 35° 40° 45° 50° 55° 60° 65°
Angle of Incidence

Fig. 14

large wave-lengths was shown by Hagen and Rubens[1] for wave-lengths ranging from 8 to 15 μ.

Inspection of Fig. 15, for the case where the angle of incidence is 35°, shows that the reflecting powers of the alkali metals for light polarized perpendicular to the plane of incidence are always less than the reflecting powers for light polarized parallel to the plane

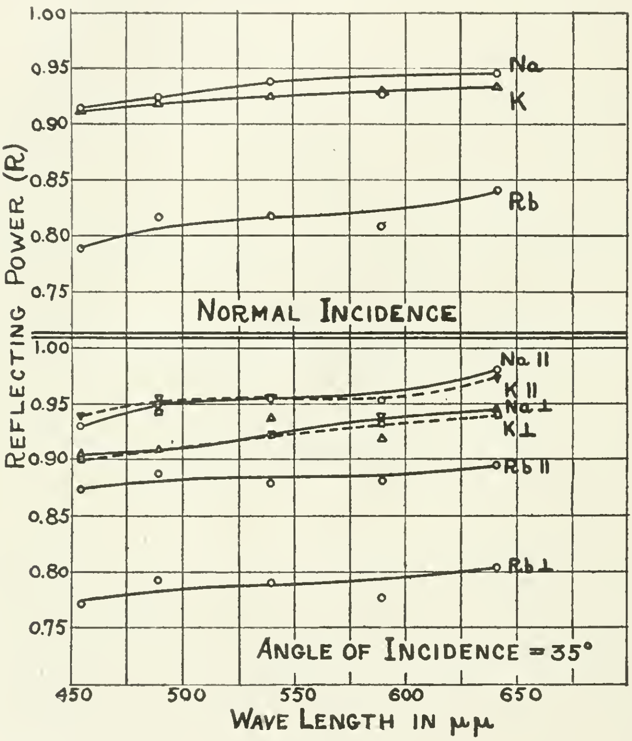


Fig. 15

of incidence. On the other hand, were the selective effect due to a change in the optical properties of the alkali metals, we should expect an abnormally high reflecting power for light polarized perpendicular to the plane of incidence. The range of wave-lengths employed in this investigation covers about one-half the

[1] *Annalen der Physik*, 11, 873, 1903.

selective region for potassium and about two-thirds that for rubidium. It therefore appears that the selective effect cannot be due to an abnormally high reflecting power in the region of the selective effect. This investigation must, however, be carried down to still smaller wave-lengths before the problem can be more definitely decided.

SUMMARY

1. The reflecting powers of sodium, potassium, and rubidium were determined for various angles of incidence, using white unpolarized light and also monochromatic light polarized parallel and perpendicular to the plane of incidence.

2. A rubidium-argon photo-electric cell was used as a photometer. It was calibrated in terms of known light-intensities by means of crossed nicols. The photo-electric current was found not to be strictly proportional to the light-intensity.

3. The alkali metals were used in the form of mirrors, which were made by distilling or pouring the metal on a glass plate forming a part of an evacuated cell.

4. Owing to reflection at the front and internal faces of the glass plate, the optical properties of the glass plate were determined in order to calculate the reflecting power of the metal itself. Fresnel's reflection equations for glass were verified.

5. The reflecting powers of Na, K, and Rb were found to decrease in the order named, i.e., as their atomic weights increased. The values for monochromatic light were found to be somewhat higher than those for white light. The results for Rb were found to be confirmed by Maxwell's equation for metallic reflection.

6. In the case of monochromatic polarized light, the reflecting power increased with increased angle of incidence for light polarized parallel to the plane of incidence, but decreased somewhat for light polarized perpendicular to the plane of incidence for the range of angles used.

7. The reflecting powers increased with increase of wave-length in accordance with Maxwell's theory.

8. The selective photo-electric effect does not seem to be due to any marked change in the reflecting powers of the alkali metals for light polarized perpendicular to the plane of incidence.

In conclusion, I wish to take this opportunity of expressing my thanks to Professor A. P. Carman for having so kindly placed the necessary facilities for research at my disposal, and to acknowledge my indebtedness to Professor Jakob Kunz for having suggested this problem, and for his many valuable and kind suggestions throughout this investigation.

Laboratory of Physics
University of Illinois
May 1916

THE STRUCTURE OF THE LITHIUM LINE λ6708 AND ITS PROBABLE OCCURRENCE IN SUN-SPOT SPECTRA[1]

By ARTHUR S. KING

In connection with an investigation of the electric furnace spectrum of calcium, it became highly desirable to obtain definite evidence as to the origin of the line λ 6708 which usually appears in calcium spectra. It has been uncertain whether calcium could under certain conditions give a line of this wave-length, or whether the line is in all cases the well-known red line of lithium, which may appear as an impurity in other spectra. The question is important also in connection with sun-spot spectra, in which Hale and Adams[2] found a strong line of the wave-length specified. They also observed it much intensified in the outer regions of calcium and other arcs. The writer[3] found the line very strong in the furnace spectrum of calcium. If in these cases it is produced by calcium, the fact is important chiefly as an indicator of reduced temperature in sun-spots, but if the line is always due to lithium, it furnishes the clearest evidence we have of the presence of lithium in the sun. This latter view was taken by Hemsalech and De Watteville,[4] who in their study of the calcium flame spectrum considered that λ 6708 is due to lithium impurity and that it is the line occurring in sun-spots. Holtz,[5] however, in measuring the calcium arc spectrum according to the international standards, ascribed λ 6708 to calcium.[6]

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 122.

[2] *Mt. Wilson Contr.*, No. 15; *Astrophysical Journal*, **25**, 75, 1907.

[3] *Mt. Wilson Contr.*, No. 35; *Astrophysical Journal*, **29**, 190, 1909.

[4] *Comptes Rendus*, **149**, 1369, 1909.

[5] *Zeitschrift für wissenschaftliche Photographie*, **12**, 101, 1913.

[6] After this manuscript was prepared, my attention was called to an article by H. G. Woodward (*Astrophysical Journal*, **41**, 169, 1915), in which the possibility of obtaining a calcium-arc spectrum free from the line λ 6708 is demonstrated in a very satisfactory manner. The portion of the present paper bearing on the origin of the line supplements that of Mr. Woodward with different evidence and reaches the same conclusion.

The relatively low dispersion used in the study of the calcium furnace spectrum showed λ 6708 as clearly double, though not resolved. It thus appeared possible that both calcium and lithium entered into the production of the line. Higher dispersion was then resorted to, the second order of the vertical plane grating spectrograph of 30-foot focus being employed, which gives a dispersion of approximately 1 mm=0.89 A. A large number of spectrograms were then taken for a wide variety of conditions of the arc and furnace, the line being studied both with calcium and with lithium in the source. A satisfactory resolution of what proved to be a highly complex line was obtained in this way. Measurement of the components appearing under various conditions were referred to the calcium line λ 6717.940 (Rowland). When lithium was used in the arc or furnace, λ 6718 was put on the plate by means of the occulting device above the slit or by screening out the lithium line during the calcium exposure by a strip of paper passed in front of the plate.

RESULTS

When the source is such that λ 6708 might be due either to calcium or to lithium impurity, that is, with metallic calcium in the arc or furnace, or with clean carbons in the arc, a narrow doublet is usually obtained, of which the violet component is about twice as strong as the red. When lithium chloride was used in the source, a reversed line appeared which was photographed on numerous plates in juxtaposition to the narrow doublet; but it was evident that the reversed line could not have sharpened down to either component of the doublet without decided shift of its center. It thus appeared that the doublet occurring in the calcium spectrum is distinct from the lithium reversed line and might belong to calcium. However, a very similar doublet was obtained by Zeeman[1] in absorption when white light was passed through an exhausted vessel containing lithium vapor, the relative intensity of the components being the same in the two cases, while the interval of 0.144 A measured by Zeeman is in fair agreement with the mean value of 0.152 A

[1] *Proceedings of the Amsterdam Academy*, 15 (II), 1130, 1913; 16 (I), 155, 1913.

obtained from six arc and furnace photographs. Kent[1] observed the same doublet in emission from a vacuum tube containing lithium, and measured the separation as 0.151 A. This close agreement, together with other observations to be described, indicates that the doublet is in no case due to calcium.

A remarkable condition became evident when the quantity of lithium in the arc was such as to give a rather narrowly reversed line. A bright line then appeared inside of the reversal and slightly to the red of its center. Visual observations showed the same appearance as that recorded on the plates. As the quantity of lithium in the arc was increased, the bright line was gradually suppressed by the absorption of the vapor producing the reversal of the main line. Turning the spectrograph so as to bring the axis of the arc-image parallel to the slit showed the emission line weakening toward the upper pole, where the denser absorbing vapor appeared as a red hood beneath the arc terminal.

Further experiments were then made with a very small quantity of lithium in the arc, a drop of dilute solution of the chloride being placed on either carbon or copper poles. By projecting the image on a long slit, the spectrum line was made to register the condition in the arc from pole to pole. Near the upper terminal and extending to the middle of the arc, the line consisted of an unsymmetrical triplet. The bright line usually appearing within the reversal was present as the middle component, while the sides of the reversal had become two sharp lines, sometimes with incipient wings. The red component of the triplet had a close satellite on its red side. The triplet blurred in the middle of the arc, and from here to the lower (positive) pole appeared a gradual development of the two components of the close doublet which it had been thought might belong to calcium. Putting the calcium line λ 6718 on the plate by a second exposure to serve as a standard, it was found that the doublet in the lithium arc agreed closely as to wave-length, separation, and relative intensity of components with the doublet given by calcium in the arc or furnace. Unless it was added by separate exposure, the photographs showed no trace of the calcium line λ 6718, which is good evidence that the doublet was not due to

[1] *Astrophysical Journal*, **40**, 337, 1914.

calcium mixed with the lithium, since in the calcium arc λ 6718 is much stronger than λ 6708.

The components of the lithium triplet were studied to best advantage in the furnace spectrum, especially with regard to the separation of the side components. In the vacuum furnace, with a small quantity of lithium vapor present, the components of the triplet were sharp, with quite the appearance of Zeeman components, the phenomenon being clearly distinct from the regular reversal effect.

Measurement of a number of photographs proved that the interval between the side components was decidedly variable, this seeming to be controlled by the amount of vapor present. Nine furnace photographs gave intervals ranging from 0.25 A to 0.36 A, a difference much greater than the error of measurement for components as sharp as these. As the separation became wider, the middle component became weaker and the satellite by the red component strengthened. With a larger quantity of vapor, the side components lost their sharpness and gave the regular reversed line with widely shaded wings. A description of a succession of furnace runs at about 2100° C. without fresh supply of lithium will serve to illustrate the changes and the transition to the doublet stage. Starting with a small supply of vapor, the components came closer together on successive plates with a strengthening of the central line until the three were no longer resolved. The next photograph showed the doublet with the usual separation of components. The interval of this doublet seems to be constant, as the measurements on good plates have never differed by more than 0.007 A.

The variation of the lithium components is thus shown to be the same in arc and furnace, and the controlling agency in each has seemed to be the amount of vapor in the source. The appearance of the two sets of components in the arc near the positive and negative poles respectively can scarcely be dependent upon the discharge conditions, since either set can be produced in the furnace without material temperature change. The whole effect agrees with the observations of Nutting,[1] who studied line structure for a number of elements by means of the echelon spectroscope. He

[1] *Astrophysical Journal*, **23**, 64, 1906.

observed only a broadening of the red lithium line, but in other spectra frequently recorded a "twinning" of sharp components which spread apart with increasing material in the arc and changed into the winged structure of regular reversal. In some cases a central component was present, as is the case for the lithium line. The variable interval of the components suggests electrical resolution, and Stark[1] has recently developed a theory of the connection between electrical resolution and the widening of spectrum lines. The production of the effects in the furnace shows that they are not due to an external electric field, but it would seem that there are strong possibilities in the action of interatomic fields, in the variation of which the vapor-density might well be the controlling agency.

An observation of the structure of λ 6708 by Jewell in an early paper by Rowland[2] is explained by the foregoing results. Jewell notes: "With but little material in the arc this is a difficult triplet. The violet component is very strong, the red component about half as strong, and between them but nearer the red component is a very narrow line much weaker than either of the others." This appearance would result from the use of a concave grating and the projection of most of the length of the arc on the slit. The strong violet component of the doublet would then blend with the violet component of the triplet, producing an unsymmetrical triplet of which the central and red components seen by Jewell correspond with those on my photographs.

In Table I, are given measurements of those members of the group at λ 6708 whose wave-lengths seem to be invariable. The mean wave-length of the variable side components of the triplet appears to be that of the center of the reversed line into which they gradually develop. The measurements were made in arc spectra from the sharp calcium line λ 6717.940 (Rowland), the wave-length of which in the international system is given by Holtz as 6717.70.

The mean wave-length of the close doublet in the international system is 6707.81, which agrees exactly with the value given by

[1] *Jahrbuch der Radioaktivität und Elektronik*, 12, 349, 1915.

[2] *Astronomy and Astrophysics*, 12, 344, 1893.

Holtz, who measured it as an unresolved line ascribed to calcium. This mean wave-length of the doublet agrees closely with the wave-length of the reversed line; but the agreement would cease if the doublet were faint, as when produced by a small impurity, so that the measured wave-length would be that of the strong violet component. A discrepancy as large as 0.07 A might thus result. The possibility of variable wave-lengths due to complex structure has been recognized by other observers, and we may have here the key to the condition which Burns[1] notes: that "lines of impurities are not as yet recommended for standards," since a small impurity may give a distinct set of components.

TABLE I

WAVE-LENGTHS OF COMPONENTS OF λ 6708

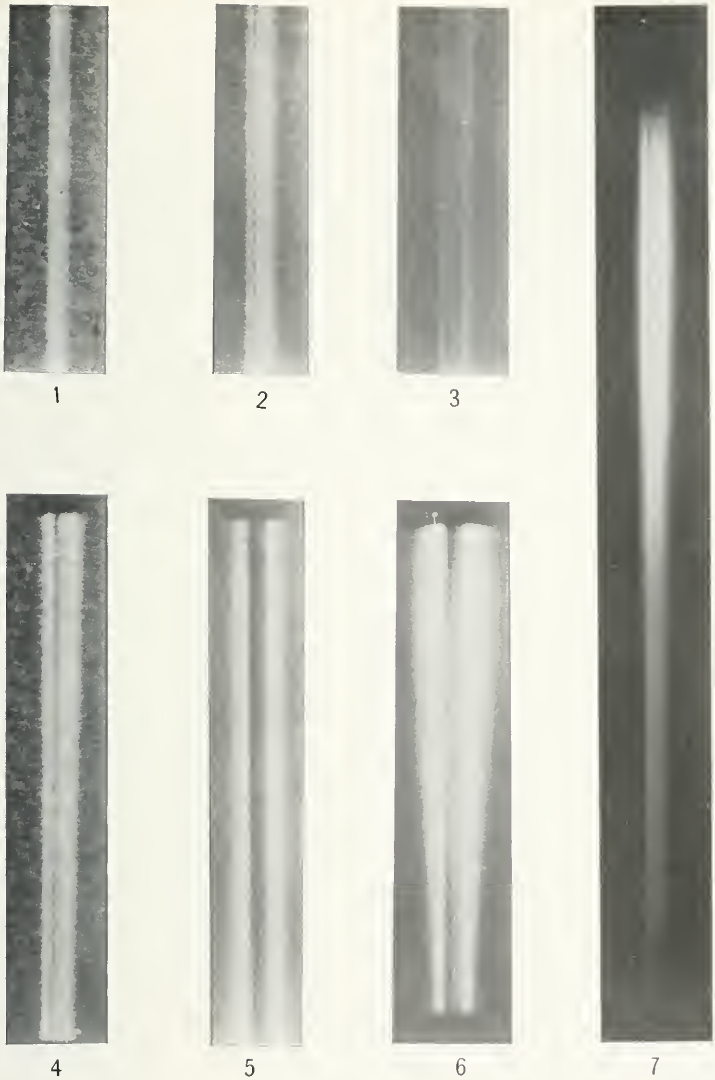
	No. Plates	λ (Rowland)	λ (I. A.)
Violet component of doublet	7	6707.977	6707.74
Red component of doublet	3	6708.125	6707.88
Center of reversed line	8	6708.053	6707.81
Bright line inside of reversal	6	6708.072	6707.83

Various conditions of the lithium components are shown in Plate IV. The first three enlargements are of the narrow doublet and two states of the triplet in the furnace, the strength of the red satellite in No. 3 giving a quadruple structure. The next three are for the arc with increasing quantities of vapor, the side components being almost sharp in No. 4, while Nos. 5 and 6 show the regular reversal with the bright line inside. No. 6 was taken with the arc-image parallel to the slit, showing the extinction of the bright line near the upper terminal. No. 7 shows the simultaneous production of both sets of components, the one near the positive, the other near the negative pole.

COMPARISON WITH λ 6708 IN THE SUN-SPOT SPECTRUM

Sun-spot spectra showing λ 6708 have been photographed by Mr. Ellerman and Mr. Nicholson, using the first and second orders of the 75-foot spectrograph on Mount Wilson, and kindly placed at

[1] *Bulletin of the Bureau of Standards*, 12, 179, 1915.



λ 6708 in Furnace and Arc Spectra

1. Doublet appearing in furnace or arc with trace of lithium vapor.
2. Triplet which takes the place of the doublet when slightly more vapor is present.
3. Appearance of line in furnace with more vapor than for No. 2.
4. Triplet in arc, with side components beginning to show the widening characteristic of reversal.
5. Reversed line in arc with inner component still visible.
6. Reversed line in arc showing fading of inner component near upper electrode where absorption is strongest.
7. Combination of triplet and doublet in arc when vapor density gradually increases from lower to upper terminal.

my disposal for comparison with the laboratory plates. The line seems to be absent from the regular solar spectrum, the faint line in Rowland at λ 6708.176 apparently being too far to the red. In the spot spectrum, the line is rather wide and of low density, so that the first-order plates, with a scale of 1 mm = 0.713 A, proved much better for measurement. The mean value from four plates is λ 6708.065. Considering the character of the line, which, exclusive of wings, is about 0.2 A wide, this is in fair agreement with λ 6708.053 for the reversed lithium line and is very close to the mean value, λ 6708.062, of the center of this line and of the bright line appearing within the reversal. Such dissymmetry as is present in the sun-spot line is toward the red, so that the condition of the vapor is probably not that giving the close doublet with a strong violet component. Second-order plates taken with nicol prism and quarter-wave plate show a decided Zeeman effect for the sun-spot line. This, together with the strength of the line, makes it probable that λ 6708 is not one of the fluting lines which are plentiful in this region, though possibly it may be blended with such a line.

The material here presented appears to furnish a strong probability that the sun-spot line is due to lithium, and we have the remarkable condition that an element of very wide distribution in terrestrial substances is apparently unable to show its spectrum in the solar atmosphere except over sun-spots.

The relative intensities of λ 6708 and the strong orange line λ 6104 were compared in a set of furnace spectra of lithium at various temperatures. At 1650° C., λ 6708 was very strong and reversed, while λ 6104 was barely visible. Above 2000°, the latter line gained in relative intensity, though still much weaker than λ 6708. This behavior of λ 6708 justifies its use as a criterion of low temperature. λ 6104, if present in the spot spectrum, is extremely faint. Other lithium lines will be searched for when spot photographs of the more refrangible region are available, but if they are not found their absence may be fairly ascribed to a low-temperature condition, since λ 6708 is by far the most sensitive to a weak excitation in laboratory sources.

SUMMARY

1. The complex lithium line λ 6708 may appear under different conditions with two distinct sets of components, either as an unsymmetrical doublet or as a triplet of variable separation.

2. In a third stage, the side components of the triplet change into the ordinary reversal, within which the central component still can be seen.

3. All three conditions of the line may be produced in either arc or furnace, the doublet and triplet sometimes appearing at the same time in different parts of the arc.

4. The observations show that the line at λ 6708, often found in calcium and other spectra, has the same structure and wave-length of components as the lithium line when produced by a source containing but little of the substance. The line is probably always due to lithium, and if low-temperature conditions are present in the source, may attain a very high relative intensity.

5. Measurements of the strong sun-spot line λ 6708, which is probably absent from the photosphere, agree so closely with the wave-length of the arc and furnace line as to leave little question regarding the presence of lithium in the solar atmosphere; while the high intensity of the line at low furnace temperatures is evidence of the correspondingly low temperature of sun-spots.

Mount Wilson Solar Observatory
July 7, 1916

THE EFFECT OF HAZE ON SPECTROSCOPIC MEASURES OF THE SOLAR ROTATION

By RALPH E. De LURY

Spectroscopic determinations of the rate of the sun's rotation by different observers at various times present remarkable and puzzling differences. Certain sources of error have been proved to be present; but other explanations of some of the differences are of a hypothetical nature and are veiled in doubt. It is the object of this note to present with supporting evidence a new interpretation which seems to clear up much of the uncertainty attached to the problem of the solar rotation.

DIFFERENCES IN MEASURES OF SOLAR ROTATION

Notable observed differences are:

(1) *The measurements of the solar rotation made by different observers exhibit a large range of values.*—For example, the values of the equatorial solar velocity, derived from about twenty groups of determinations, range from 2.11 to 1.86 km per sec. Furthermore, *measurements by the same observer of a series of plates taken over a short interval of time frequently show a considerable range in their values.*

(2) *Some observers have found a difference in velocity for different spectral lines, while others have not.*—The determinations by Adams and Miss Lasby at Mount Wilson in 1906–1908 show such differences,[1] and this is confirmed[2] in 1915 by St. John, Adams, and Miss Ware, and also in Ottawa by the writer in 1915, a summary of these measurements being given later in this paper. On the other hand, all other observers (the writer included) find no serious difference for different lines in the interval 1909–1913. Hence

[1] Adams, *Mt. Wilson Contr.*, Nos. 20, 24, 29; *Astrophysical Journal*, **26**, 203, 1907; **27**, 213, 1908; **29**, 110, 1909; Adams and Lasby, *Publication No. 38, Carnegie Institution of Washington.*

[2] St. John, Adams, and Ware, *Popular Astronomy*, **23**, 641, 1915.

the conclusion: *The difference in velocity for different spectral lines is a variable, being present in some observations and apparently absent from others.*

(3) *One observer found that the northern and southern hemispheres of the sun rotated at different rates.*—The observations of Hubrecht at Cambridge[1] alone give information on this point. In 1911 the writer suggested the method of using simultaneous exposures from the center of the solar disk and from the limbs for determining the rates of rotation in the two hemispheres independently,[2] and since the apparatus was received in 1913 he has been making such observations; in 1915 a similar method was started at Mount Wilson,[3] so that more evidence on this point will soon be at hand.

(4) *Some observations show a value of the rate of rotation progressively increasing with wave-length over the small range of wave-lengths covered by a plate; a great many more observations do not exhibit this effect.*—The 1906–1907 series[4] of Mount Wilson measurements show this effect, while the 1908 series[5] does not. Some measurements by Schlesinger in 1909, and those by Hubrecht[6] in 1911 show the effect. All other observations appear to be free from this effect.

MECHANICAL EXPLANATION OF DIFFERENCES

The following explanations of the foregoing results have been or may be offered:

(*a*) *Instrumental errors.*—Such instrumental errors as would be caused by uneven illumination of the prism or grating, combined with observations of the spectrum out of focus, may account for part of results (1), (3), and (4).

(*b*) *Observational errors.*—Small inaccuracies in determining the points observed are possible, but it is unlikely that these could ever equal 0.5 per cent.

[1] Hubrecht, *Monthly Notices*, 73, 5, 1912.

[2] De Lury, *Report of the Chief Astronomer, Ottawa*, 1911, p. 290.

[3] St. John, Adams, and Ware, *loc. cit.*

[4] See footnote 1, p. 177.

[5] *Ibid.*

[6] Hubrecht, *loc. cit.*

(*c*) *Errors of measurement.*—In 1910 the writer suggested that errors of measurement might account for (1) and (2). He tested this explanation of (2) by mechanically introducing displacements of the spectral lines the same for all lines and of configurations and magnitudes of displacement similar to actual observations; a slight tendency to systematic difference for different lines was found in a series of twelve of these "imitation" rotation plates, as well as a systematic difference depending on the direction of the plate.[1] These plates were taken in the region of λ 4250, where Adams and Miss Lasby found the differences for different lines,[2] and the plates were sent to them in the hope that their measures of the same lines mechanically shifted would settle the question as to whether the differences for different lines in their original measurements were due to personal errors. Unfortunately they did not have time for the measurements, hence the part played by systematic error of measurement in their 1906–1908 determinations remained unsettled. This explanation of (1) was tested by having various observers measure the same lines on the same plates. J. S. Plaskett kindly offered to co-operate with the writer in measuring the above-mentioned twelve plates of the mechanical shifts, with the result that a systematic difference between the two measures of about 2 per cent was discovered. This difference persisted throughout the measurements of the solar rotation in 1910–1913. These suggestions of the writer followed by the comparative measurements led to these recommendations made at the meeting of the International Solar Union held at Bonn, in 1913: "It is highly desirable to trace to their source the systematic differences that are found in the values of the solar rotation by different observers. . . . . Investigation should also be made into the personal differences that are found in measures of the same plates by different observers." (In this connection it would seem advisable to have a series of plates, say one or two from each observer, measured by the automicrophotometer at Mount Wilson, and then passed around among the various observers for measurement.)

[1] De Lury, *op. cit.*, p. 264; *Journal of the Royal Astronomical Society of Canada*, 5, 384, 1911.

[2] See footnote 1, p. 177.

PHYSICAL EXPLANATIONS OF DIFFERENCES

The foregoing explanations are based upon the possibility of instrumental, observational, or personal errors; those which follow are based on physical considerations:

(*d*) *Convection in the solar atmosphere.*—Local convection currents undoubtedly account for some of the differences obtained by the same observer under apparently similar conditions (1), and it is quite possible that in small series of observations the mean may be considerably distorted by this cause. Adams found instances of such local motions in the neighborhood of spots.[1] The writer found in one case a difference of 8 per cent between the top and bottom of a spectrum 1 mm wide, the lines being quite visibly bent from their normal straightness.

(*e*) *Periodic variation in the rate of the solar rotation.*—From variations in the visual measurements of Dunér (at Upsala, 1887–1889 and 1899–1901) and of Halm (Edinburgh, 1901–1906), the latter suggested that there was a periodic change in the sun's rate of rotation. If such is the case (1) could be accounted for, partially at least; and since there is periodicity in sun-spots and asymmetry in the spottedness of the northern and southern hemispheres, (3) might result from such periodic variation; and possibly result (2) could be explained by such periodicity, for the evidence on this point seems to bear some relation to the sun-spot variation.

(*f*) *Variation in the angular rate of rotation depending on level in the solar atmosphere.*—When Adams discovered differences in angular velocity for different lines of the spectrum (2), he suggested[2] that it was due to the fact that the gases producing the different lines existed at different levels (an assumption apparently supported by other lines of evidence) and that the angular rate of rotation increased with elevation. To account for the additional facts mentioned in (2) above, this explanation would have to be modified by adding: *and such variation in the angular rate of rotation varies periodically.*

[1] See footnote, 1, p. 177.

[2] *Ibid.*

(*g*) *Sky spectrum.*—Halm noted the possibility of error caused by the sky spectrum blending with the displaced spectrum of the limb, and observers have for the most part been careful to select the clearest days for observation. However, there seems to be error due to this source in some of the observations. The writer made, in 1911, some tests of the effect of sky spectrum in lessening the rotation displacement, with the result that for the very clearest days there seemed to be little error from this source.[1] This work led to the consideration of the general question of blended spectra, and in 1912 measurements of blends of spectra of limb and center were made which showed a striking, though predicted, relationship between measured displacements and line-intensity, owing to the fact that the difference in intensity for a line at limb and at center increases in general with decrease in intensity of the line.[2] These results led to the following explanation (*h*), though mentioned previously, presented only now because recent results in the measurement of the solar rotation at Ottawa by the writer, and at Mount Wilson[3] by St. John, Adams, and Miss Ware are strikingly well explained by it.

(*h*) *Spectrum of haze.*—It has been shown[4] (see also later) that a variable haze, between the observer and the sun, causing to be blended on the spectrum of the limb a spectrum of variable intensity and of character somewhat similar to that of the center of the solar disk in regard to intensity and wave-length of the spectral lines, causes: (i) the spectroscopic determinations of the solar rotation to vary, and (ii) the velocities of rotation from the different lines to decrease in general with decrease in intensity of the lines, the amount of the decrease in velocity for a given line depending on the strength of the continuous spectrum due to the haze relative to the continuous spectrum of the limb and on the ratio of the intensities of the line in the spectrum of the haze and in the spectrum of the limb. Observations already made make it seem probable that the variable terrestrial atmosphere and its

[1] De Lury, *Report of the Chief Astronomer, Ottawa*, 1911, p. 281.

[2] De Lury, *Journal of the Royal Astronomical Society of Canada*, **10**, 201, 1916.

[3] St. John, Adams, and Ware, *loc. cit.*

[4] See footnote 2.

clouds and hazes are sufficient to account for differences (1) and (2), after eliminating the systematic and accidental errors mentioned above. But if in any series of observations the spectrum of terrestrial haze can be proved of insufficient strength, then we may introduce the idea of haze existing between the earth and the sun, near the sun, or even in the solar atmosphere (such as produced by matter falling in variable amounts into the sun and requiring an interval of time before being swept along in the general rotation). Such a variable haze possibly could account for the differences in the solar radiation observed by Abbot and others; it would be interesting to make simultaneous observations of solar rotation and radiation to see whether the changes in their values synchronize.

That explanation (*h*) is the true explanation of the residual differences in (1) and (2) above, after due allowance has been made for the other known sources of error, seems established from the similarity of the following three series of results, dealing with measurements of blended spectra, measurements of the solar rotation at Ottawa on plates made through different amounts of haze, and measurements of the solar rotation made at Mount Wilson.

MEASUREMENTS OF BLENDED SPECTRA

In the paper cited, it has been shown that the measured rotational displacements of the lines from the limb when blended with the lines—undisplaced by rotation—from the center of the solar disk decrease progressively with decrease in the intensity of the lines; and this was explained as due to the fact that the difference in intensity between lines in the spectra of center and limb decreases, in general, with increase in intensity of the lines. There are exceptions to this latter generalization which serve to test the various theories (see later). The accompanying summary (Table I) of the first table in the paper quoted will suffice to illustrate the general results. It is thus seen that the lessening of the displacement due to rotation in the blend with the spectrum of the center is greater progressively with decrease in intensity of the line, which, in turn, is accompanied by steadily increasing values

of the ratio of intensity from center to limb, and decreasing values of the ratio of width at center to limb. (Thus decreasing intensity at the limb seems to be accompanied by increasing width. To explain this the writer has advanced the hypothesis that the

TABLE I

Blended Spectra, λ 5600

Mean displacements of equatorial limb lines blended with five different blends with center spectrum in which the ratios of the densities of deposit on the photographic plate from the continuous spectrum of the limb to the total of continuous spectrum were 0.89, 0.83, 0.74, 0.62 and 0.54; mean ratio, 0.72.

Plate, L854, September 20, 1911

Line-intensity, center...	1	2	3	4	5	6	8
Line-intensity, limb....	0	1−	2	3	5−	6−	7
Line-width, center.....	4.8	4.2	5.4	6.8	7.8	7.2	9.2 km per sec.
Line-width, limb.......	6.2	5.6	5.8	6.6	8.2	8.0	8.6 " "
No. of lines...........	3	6	5	2	1	1	1
Mean velocity from blends.............	1.553	1.573	1.575	1.594	1.633	1.648	1.651 " "
Equatorial velocity not blended............	2.026	2.053	2.014	2.053	2.085	2.085	1.978 " "
Mean............							2.042

widening and weakening of the lines at the limb are due to convections similar to those in the penumbral regions of spots. Other factors come in to play and account for many exceptions. The question will be discussed soon in another communication.)

MEASUREMENTS OF THE SOLAR ROTATION AT OTTAWA ON HAZY DAYS

The results from the measurements of the solar rotation on hazy days for varying degrees of haze show a striking similarity to those from the measures of the artificial blends as shown in Tables II, III, and IV. It will be thus seen that the differences of percentage between the values for intensity 1 and 22 are: Table II, 1.2; Table III, 4.6; Table IV, 8.2. After the observations of Table IV were made, a photographic comparison of the intensity of the spectrum of the haze relatively to the spectrum of the limb was secured; however, the haze was continually varying so that only a rough approximation could be arrived at, and from this it would seem that the average ratio of intensity of the continuous

spectrum of the haze to the continuous spectrum of limb and haze for the observations of Table IV was 12± per cent. This would involve the assumption that the haze in Table II was about 2 per cent, while the haze for observations of Table III was about 7 per

TABLE II

SOLAR ROTATION, λ 5200

March 11, 1:30 P.M., 1916, very slightly hazy, 6 *double* observations, i.e., 2 strips of spectrum from each limb

	Mean	Mean	Mean	Mean	Mean
Intensity	1	2	5.3	22	5.3
Number of lines	3	11	7	3	24
Equatorial velocity	1.956	1.972	1.972	1.968	1.967

TABLE III

SOLAR ROTATION, λ 5200

June 16, 4:15 P.M., 1915, slightly hazy, 6 double observations

	Mean	Mean	Mean	Mean	Mean
Intensity	1	2	5.3	22	5.3
Number of lines	3	11	7	3	24
Equatorial velocity	1.808	1.842	1.845	1.883	1.843

TABLE IV

SOLAR ROTATION, λ 5200

March 3, 12:55 P.M., 1916, very hazy, haze varying, 3 double observations

	Mean	Mean	Mean	Mean	Mean
Intensity	1	2	5.3	22	5.3
Number of lines	3	11	7	3	24
Equatorial velocity	1.738	1.760	1.814	1.887	1.816

cent. These are of course only rough estimates, but they serve to point out the necessity of very accurate measures of the relative strengths of the spectrum of the haze and the spectrum of the limb. When such are made and accurately correlated with measurements of solar rotation for groups of lines of different intensities, it will be possible to eliminate the effect of the spectrum of the haze from any similar series of measurements of rotation. Such being the

case, it should be possible to estimate the strength of haze present during the Mount Wilson observations.[1]

MEASUREMENTS OF THE SOLAR ROTATION AT MOUNT WILSON

TABLE V

SOLAR ROTATION, λ 5200

1914–1915 measurements (St. John, Adams, and Ware, *Popular Astronomy*, 23, 641, 1915)

	Mean	Mean	Mean	Mean	Mean
Intensity	1	2	4.9	22	6.4
Number of lines	2	5	9	3	19
Equatorial velocity	1.924	1.933	1.945	2.043	1.950

It is seen from Table V that the difference between the values for lines of intensity 1 and 22 is 6.1 per cent. It would seem that, if this is altogether ascribable to haze, there was an overlapping spectrum of the haze of about 9± per cent in these observations. The three lines of average intensity 22 were the same as in the Ottawa observations, namely, the three strong Mg lines in the *b* group, λ 5167 to λ 5184, but the lines of intensity 1 could easily yield different results in the two series, 3 in the Ottawa observations and 2 in the Mount Wilson observations. However, it seems likely that there must have been a considerable effect of haze during the latter observations. The large difference between the values of the rotation in the two series is probably accounted for by some of the other sources of error, though the Ottawa values in Table II (very slight haze) are nearly the same in the mean, the strongest lines being, however, exceptionally high in the Mount Wilson measures. That is a question which can best be attacked after the influence of the spectrum of light scattered from haze or optical parts has been accurately eliminated.

MEASUREMENTS TO TEST THE LEVEL HYPOTHESIS

The measurements of the solar rotation given in Tables VI and VII seem to support the haze explanation and to disprove the level hypothesis, unless the latter be assumed to be variable, as pointed out above.

[1] St. John, Adams, and Ware, *loc. cit.*

In Table VI are given the measurements of 6 lines of intensities 0 and 1 paired off with 6 lines of intensities 4–15, giving a great difference in penumbral displacements in spots, interpreted as

TABLE VI

SOLAR ROTATION, λ 4500

Ottawa, June 30–July 25, 1910, 32 observations

	Mean	Mean	Mean
Intensity	0.7	8.5	4.6
Number of lines	6	6	12
Penumbral displacement	+0.028 A	−0.001 A	+0.015
Equatorial velocity	1.968±0.003	1.972±0.007	1.970±0.004 (lines) ±0.010 (plates)

Seven of the foregoing plates taken on cloudy or hazy days, yield:

Equatorial velocity	1.909	1.939	1.924 km per sec.

TABLE VII

SOLAR ROTATION, λ 5600

Ottawa, December 6–12, 1910, 32 observations

	Mean	Mean	Mean
Intensity	1.6	6.2	3.9
Number of lines	5	5	10
Equatorial velocity	1.930±0.006	1.936±0.002	1.933±0.003 (lines) ±0.005 (plates)

indicating range in level in the reversing layer (Evershed and St. John).[1] If Adams' hypothesis of increasing angular velocity for increasing elevation in the sun be true, there should be a considerable difference between the velocities of rotation from these two groups of lines. There is no appreciable difference, however, and the results of Table VII show this also. We are thus forced to abandon the level hypothesis, or else to modify it by adding the idea of variability. From the seven plates of Table VI taken on days when the spectrum of the haze was stronger than for the other

[1] Evershed, *Kodaikanal Observatory Bulletin*, **15**, 1909.

St. John, *Mt. Wilson Contr.*, Nos. 69, 74; *Astrophysical Journal*, **37**, 322, 1913; **38**, 341, 1913.

plates, it is seen that there is a difference between the determinations of velocity of the two groups of lines, of 0.030 km per sec.—a difference explainable by the spectrum of the haze blending with the spectrum of the limb. The mean value of the December determinations (Table VII) is smaller by 2 per cent than the value from the July determinations (Table V), possibly owing to the lower declination of the sun in December and to the lower mean intensity of the lines, as well as to the probably greater relative strength of the sky spectrum in December than in July.

SOME GENERAL DISCUSSIONS

It has been mentioned that observers during 1909–1913 found little difference for different lines. Can this be due to the fact that at sun-spot minimum there is less danger from the error due to haze than during sun-spot maximum, pointing either to the presence of varying quantities of matter about the sun or to varying haziness in the terrestrial atmosphere caused by the variation in its ionization accompanying the spot-activities? In most of these 1909–1913 observations the lines were not considered in groups as to difference in intensity, but rather with regard to the element producing the line—in accordance with the recommendations of the Solar Union in 1910—the important relationship between penumbral displacements and intensity and level[1] not having been fully developed at that time. It would seem advisable to investigate the published results from this point of view. This has been done in a preliminary way by the writer. Some results show no appreciable relationship of velocity with line-intensity, some show evidence of this, and some seem to indicate the reverse of what would be expected from Adams' level hypothesis, i.e., a lower rate of rotation with increasing level, a physically possible and quite probable state of affairs. Some exceptions to the level hypothesis are readily explained on assumption of blended spectrum of haze, e.g., λ 4287.566 of intensity 1 at the center of the solar disk is strengthened and widened at the limb, and it has a penumbral displacement of 0.026 A; if this is interpreted as meaning low level, it is to be expected on the level hypothesis that this line

[1] *Ibid.*

should give a lower rotational value than the mean. Adams and Lasby find[1] in 1908 that this line has an equatorial velocity 0.004 km per sec. above the mean; this is explainable by the fact that this line is strengthened, not weakened, at the limb and therefore should yield a larger value than the mean of the other lines which are for the most part weakened at the limb, if the spectrum of the haze is of sufficient strength. In those measurements the lines that are weakened at the limb show a mean residual of −0.003, while the lines that are strengthened at the limb show a residual of +0.005 in the mean, indicating a slight effect of sky spectrum. Similar means, −0.002 and +0.005, occur in the 1906–1907 series. All published results should be discussed fully from this point of view so that a correction can be made in the absolute values. A knowledge of the behavior of the lines at the limb is essential. Is it possible that the results (4) can be due to chance selection of the lines, so that at one end of the plate the lines will yield a smaller value of the rotation than do the lines at the other end? A cursory examination of Hubrecht's results would make this seem a possible explanation. It is assuredly not a physical effect depending on wave-length, for, if it were, there should be profound differences between series taken at widely different parts of the spectrum, and this is not the case. It may possibly be due to uneven illumination of the grating and one end of the plate being slightly out of focus. It is possible, too, that Hubrecht's result (3) may also be due to blended spectrum of the haze inasmuch as the wave-lengths in the latter are not midway between those from opposite limbs, which would result in effects of blending of different magnitude for the two limbs. It seems to the writer that many of these puzzling differences will vanish when accurate determinations of the effects of the spectrum of the haze are made. A later communication will deal with the effect in various series of observations.

SUGGESTIONS FOR FUTURE OBSERVATIONS

In the meantime it is necessary for all observers to pay special attention to the influence of the spectrum of the haze; it may be eliminated by the exact correlation of changing values of the solar

[1] See footnote 1, p. 177.

rotation with differences in value for different intensities of lines, say from two groups of lines, one greatly weakened at the limb and the other not weakened at the limb. The λ 5200 region offers the best chance for such measurement, since the strongest lines there are quite measureable, and it is possible to eliminate instrumental and other errors by using when desired either iodine or chlorine comparison spectra (as suggested by the writer[1] in 1910 and 1911 and employed by him since the installation of the limb and center prism apparatus in 1913). For these reasons I would suggest that it be considered as a common region even in preference to the λ 4250 region formerly chosen.

In measurements of line-displacements in spots, comparisons of spectra from limb and center, etc., differential effects depending on line-intensity may serve, as for rotation, in eliminating the effects of scattered light; these questions will be discussed in future communications.

CONCLUSIONS

The main conclusions from the foregoing investigation are:

1. Spectrum of haze, probably altogether terrestrial in its origin, accounts for much of the variation in the values of the solar rotation obtained by various observers at different times. Variations hitherto ascribed to the sun appear to be due to variations in scattered light.

2. Spectrum of haze, being different in character to spectrum of limb depending in general on the intensity of the line, blends with spectrum of limb in such a way as to make it appear that different spectral lines yield different values for the velocity of rotation of the sun. Such differences found in measures of the solar rotation at Mount Wilson and at Ottawa are satisfactorily explained in this manner, and it seems possible to dispense with Adams' level hypothesis.

SOLAR PHYSICS DIVISION
DOMINION OBSERVATORY, OTTAWA
April 1916

[1] De Lury, *Report of the Chief Astronomer, Ottawa*, 1910, p. 168; 1911, p. 293.

THE VARIABLE NEBULA N.G.C. 2261

By EDWIN P. HUBBLE

Recent astronomical research has been especially fruitful in the study of nebulae—a study which has now extended into the realms of dynamics. The spectroscope, with its disregard for the vast distances involved, has reaped, first—radial velocities of planetary, irregular, gaseous, and spiral nebulae; it has also shown internal motion in the great nebula of Orion, and rotation in both spiral and planetary nebulae. Patiently accumulated photographs are just beginning to be of service, as witness the proper motions of nebulae lately announced by Wolf of Heidelberg.

A striking instance of actual change in form has been found[1] in the case of the nebula N.G.C. 2261 (R.A. 6^h32^m, Dec. $+8°51'$, Epoch 1860.0, H. IV 2=h 399=G.C. 1437), one of the few real examples of cometary form in the sky and easily the finest of them. Photographically it is well defined and has almost the form of an equilateral triangle with a sharp stellar nucleus at the extreme southern point. There are faint extensions from the northern portion of it. One long streamer which projects from the northern edge extends almost due north.

Plates taken by the writer in the past winter with the 24-inch reflector of this observatory, when compared under the blink comparator with an unusually good plate taken with the same instrument by F. C. Jordan in March 1908, show changes of outline and displacements of the structural details of the nebula.

These changes, though striking, seemed improbable on account of the short interval—less than eight years—and raised a suspicion as to whether the changes could be real and were in the nebula itself or whether they might not be due to some peculiar photographic action. But repeated plates taken under different conditions of seeing, exposure time, aperture, etc., verified the changes beyond question. One of these tests was to make three successive

[1] A preliminary notice was published in the *Proceedings of the National Academy of Sciences*, 2, 230, 1916.

exposures of half, double, and the full normal time. These, when compared among themselves, did not show any change except that due to the regular building up of the image. Each of these, however, when compared with the early plates, confirmed the curious changes.

Notwithstanding the excellence of Jordan's plate, its perfectly round star-images and clear-cut nebular details, it became of first importance to find another early plate for confirmation. A reproduction was found in *Knowledge*, 24, 181, 1901, of a photograph taken at Starfield, January 27, 1900, by the late Dr. Isaac Roberts, with his 20-inch reflector of 100 inches focal length. At the request of the director of this observatory, Mme Dorothea Roberts had the great kindness to prepare and send both positive and negative copies of this invaluable piece of evidence. Long exposure had burned out some of the finer details, but happily sufficient were in evidence to fully confirm the shifts already observed. In the *Lick Observatory Bulletin No. 248*, H. D. Curtis described a photograph of this nebula which he had taken on January 31, 1913, with the 36-inch Crossley reflector with an exposure of two hours. Director Campbell has very kindly loaned us the original negative. It is of remarkably fine quality.

An attempt was made to photograph the nebula with the 40-inch refractor, but the disadvantages of a small focal ratio, 1 to 19, and a visual color-filter, were so great that an exposure of four and three-quarter hours on a brilliantly clear night registered only the nucleus and a trace of the bright band just above. Again at the request of our director, Professor Schlesinger, director of the Allegheny Observatory, very kindly had a plate taken with an hour's exposure with the 30-inch Thaw photographic refractor. The plate was taken by F. C. Jordan, and has sufficient scale to show the brighter details in their true form—coiled streamers running out from the condensed nucleus.

The nucleus of the nebula has long been known as an irregular variable star, R Monocerotis, for which a range from magnitude 9.5 to 13 has been reported. Lassell states that it is not a star, but a true nucleus, such as that of the great spiral in Andromeda, and Barnard has confirmed this opinion by visual observations

with the 40-inch refractor. The longer exposures, especially that of the Lick plate, show a very considerable nebulosity about the nucleus.

The photographs show no indication of variability of the nucleus. The writer has taken eighteen plates which cover a period of five months in the winter of 1915–1916, and three others were taken in 1900, 1908, and 1913, respectively. Small changes might easily be masked by the surrounding nebulosity and the short focus of the reflector, but there are no large differences on the dates mentioned. Such an investigation, however, belongs properly to the field of visual observation or of instruments with a long focus, and the foregoing negative results cannot be considered as conclusive. It is unfortunate that data on so interesting an object should be so scanty. Observations made at Harvard by Leon Campbell and others in the years 1904–1910 indicate a variation through two magnitudes, from 10.0 to 12.0.

Photographically, the nucleus has been about seven-tenths the way from star No. 44 to No. 73 on the Hagen chart; photovisually, about one-third the way from No. 44 to No. 62, or 10.8 on Hagen's scale and 12.0 on the Harvard scale.

For a study of details in the nebula itself, five negatives were employed (Table I). All save the last were made with reflectors. Negative copies were made of Nos. 1, 3, and 5, reduced to the scale of the 24-inch Yerkes reflector, and the entire set was compared in the blink comparator.

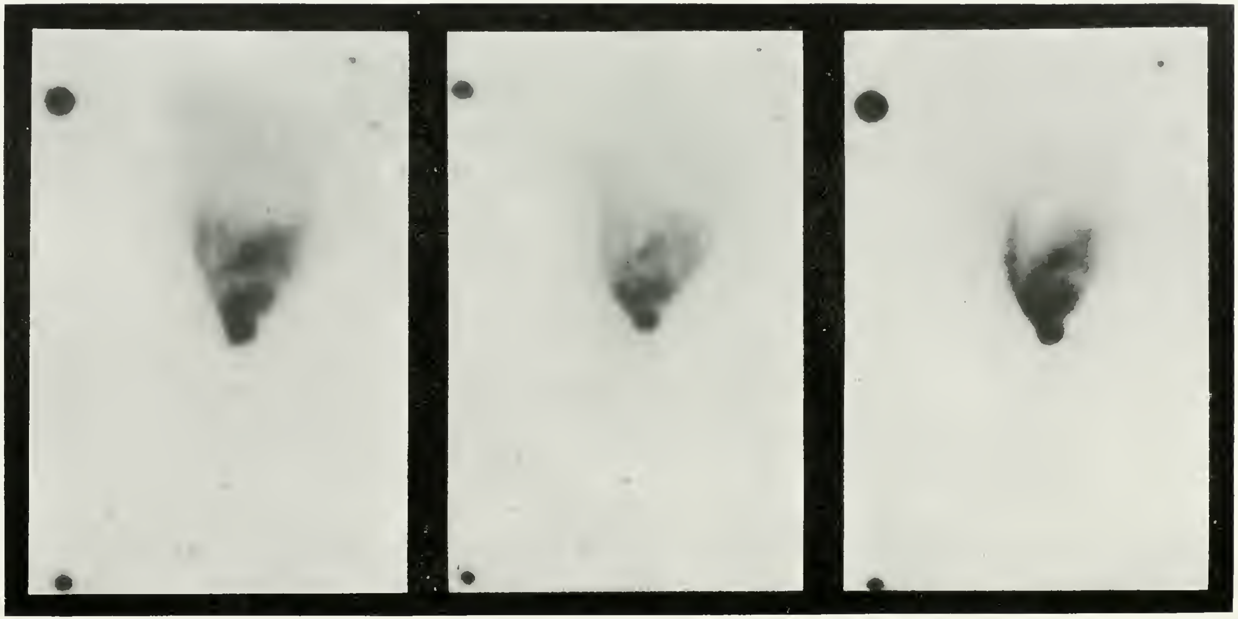
TABLE I

	Date	Aperture	Focus	Exposure	Observatory	Taken by	Notes
		in.	in.				
1	1900 Jan. 27	20	100	90^m	Starfield (England)	I. Roberts	Copies, details blurred
2	1908 Mar. 20	18	93	60	Yerkes	F. C. Jordan	Good
3	1913 Jan. 31	36	210	120	Lick	H. D. Curtis	Good
4	1916 Mar. 8	18	93	70	Yerkes	E. P. Hubble	Good
5	1916 Mar. 11	30	556	60	Allegheny	F. C. Jordan	Weak

The most striking change was what at first appeared to be a transverse shift of the bright band across the nebula just north of the nucleus, and marked *A* on the sketch. A careful examina-

PLATE V

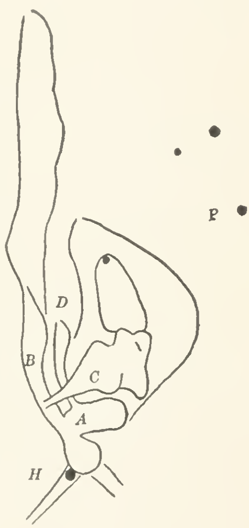
Scale: 1 mm = $3\overset{''}{.}0$



1908 March 20 Yerkes 24-inch Reflector Enlarged 24 times	1913 January 31 Lick 36-inch Crossley-Reflector Enlarged 9 times	1916 March 8 Yerkes 24-inch Reflector Enlarged 24 times

N.G.C. 2261

tion showed that the following end of *A* is coincident on plates 1 and 2, and also on plates 3, 4, and 5, but between the two sets there is a difference of about $4''.5$, in the sense that between 1908 and 1913 an extension appeared on the following edge of the band. The phenomenon is very conspicuous, and is evident in Plate V accompanying this article. Plates 3 and 5 with their larger scales show that this is due rather to the sudden appearance of a mass of nebulosity than to motion of the band itself. This new mass is apparently separated from the band, and is situated about the same distance from the nucleus, on the narrow streamer which connects the head to the body of the nebula. A new branch continues from this mass to meet the point of *C*, and another curves up and to the left, eventually mingling with the streamer which forms the following edge of the nebula, marked *B* on the sketch.



There are several other differences between the negatives which seem to be changes in the nebula itself. On the Star-field plate the north following corner of the triangle is much denser than on the others, and this apparently progresses in the sense that *B* and *D* are shifting their center of density toward the head. Considerable faint nebulosity shows around the north preceding corner and steadily drifts, as one proceeds from plate to plate, toward the center in a south following direction. The preceding end of *A* is coincident on plates 1 and 2, but from 2 to 3, and more markedly from 2 to 4, it shifts along the band toward the east.

The fainter extensions to the north of the triangle show no certain changes, nor do two extremely faint streamers running from the head to the southeast and south by southwest, respectively. In the southeast streamer, however, is a tiny mass of nebulosity, marked *H* on the sketch, so small that it appears on the short-focus plates as a star of about the sixteenth magnitude,

which exhibits a decided and irregular motion. It is very near the nucleus and seems to be covered on plate 1 by the large image of the nucleus. On plate 2 it appears clearly, just on the edge of the image of the nucleus. On plate 3 it has moved in toward the nucleus some $2''.5$ of arc, so that on plate 4 it lies within the image. Plate 5 has a larger scale and shows it clearly, but apparently coincident with the position on plate 2. That is, this curious bit of nebulosity moved in toward the nucleus just when the new masses appeared in the body; viz., between 1908 and 1913. When the nebula comes around into position this winter, some further light may be thrown on the subject by long exposures with instruments of long focus.

It would seem from the data at hand that *H* has moved not less than $0''.5$ per year between 1908 and 1913, and possibly much more. From the relation of parallax to proper motion and linear velocity, it follows that the parallax of this object is about $\frac{2.5}{V}$, where V is expressed in kilometers per second. Any velocity, therefore, up to 100 kilometers per second would suggest a sensible parallax, especially as the sharp stellar nucleus permits of accurate measurement. It is to be hoped that the nebula will find a place on the program of some of the instruments suited for such work. Careful measurements of the plates at hand fail to show any appreciable proper motion of the nucleus.

Several possibilities suggest themselves when one is seeking an explanation of the changes. The nebula may be rotating as a whole, bringing new features into view. An objection to this is that the changes are more evident at the edges, whereas a simple rotation would show its greatest effect in the middle. Further, many of the markings in various parts of the nebula show no change whatsoever, and a rotational effect should show a regularity of distribution.

Another possibility is that of local brightening and fading of stationary matter. This would satisfy most of the data, but for certain points, such as *H*, actual motion is too evident to be disregarded. Among these considerations is the current suggestion that a variable nebula might shine by light reflected from the

nucleus and the variation of the two would be directly related. In the case of this nebula, the nucleus is already believed to be irregularly variable, and the nebula might shine by reflected light, but any effects of variability of the nucleus should show a regularity that is entirely absent from the observations. One would be forced to conceive of only certain portions of the nebula being affected.

The most plausible explanation would seem to rest on actual motion of portions of nebulosity relative to the nebula as a whole. The plates indeed suggest a discharge of matter from the nucleus, northward along the following edge, where the band *A* joins to the head. However, the data at hand are too meager for conviction and the explanation must await further study of the nebula with large telescopes.

The position in the sky of N.G.C. 2261 seems highly significant. It lies in, and near the end of, a dark lane which leads up to the nebulosity around 15 Monocerotis, indicating that the nebula is nearer to us than the mass of stars blotted out by the obscuring matter in the lane. This portion of the Milky Way is rich in diffuse nebulosity, nebulous stars, and dark, obscured regions. There is another cometary nebula, N.G.C. 2245, just over two degrees north preceding, again in a dark lane, and so obviously connected with it and with the nebulous cluster, that no great stretch of the imagination is required to place the two cometary nebulae in the same category. They have so much in common that it would not be surprising to find them similar in their peculiarities, and it is to be regretted that we have no old plates of N.G.C. 2245 to compare for change.

The case is strengthened by the data from other variable nebulae. Hind's variable by T Tauri (N.G.C. 1555) is the most famous. Its remarkable career has been carefully investigated by Barnard in two papers published in *Monthly Notices*, **55**, 442, 1895, and **59**, 372, 1899. Some sixty years ago, it was a conspicuous object in a small telescope. Today it is barely discernible with the best instruments. Recent long-exposure photographs show an exceedingly faint, fan-shaped wisp of nebulosity, close to and pointing toward the variable T Tauri, which Burnham and Barnard saw as a small condensed nebula. It also is situated in a dark lane.

Schmidt, at Athens, discovered in 1861 a small nebula just beside the variable star R Coronae Australis. It is now known as N.G.C. 6729. He later announced it as a variable nebula, and Innes at the Cape in 1890 confirmed Schmidt's observations. Very recently Knox-Shaw at Helwan made a study of the nebula and removed any doubts as to its variability by photographic evidence. In this case also there is a fan-shaped wisp with a variable star at the tip. Again the variable lies in a very pronounced dark region south following the globular cluster N.G.C. 6723. Knox-Shaw read a paper on the subject before the British Astronomical Association, which is reported in the *Journal* of that association for June 1916. He affirmed that the nebula varies from week to week both in brightness and in shape. No definite period has been found for either star or nebula.

There are two other cases of variability within nebulae. Just north preceding N.G.C. 6729, and in the very heart of the dark region, is a wide double star, each component of which is the nucleus of a considerably large and bright mass of nebulosity. Innes, in a recent circular of the Union Observatory, announced that one of the nuclei is variable. The other case is that of the planetary N.G.C. 7662, the nucleus of which Barnard has observed to vary through several magnitudes. The nebula, of course, is gaseous, but the nucleus gives a strong, continuous spectrum. As a planetary, it differentiates itself from the fan-shaped nebulae.

Several plates of N.G.C. 2261 were taken here with a 15° objective-prism on the Zeiss U. V. camera. The nebula gives a strong, continuous spectrum, in which no lines were to be seen, but which, as compared with the neighboring stars of early type, weakened toward the violet, after the fashion of a spectrum of the solar type. This explains why the nebula photographs so readily with a visual color-filter: for while, with a visual color-filter and a Cramer Instantaneous plate, the usual equivalent exposure with the 24-inch reflector is about five times that for a free exposure with a Seed 30 plate, in the case of this nebula, about two and a half times the normal free exposure sufficed to give a strong image through the color-filter. N.G.C. 2245 was in the camera field and also gave a good continuous spectrum. This latter nebula is so

obviously connected with the nebulosity around the cluster, Dreyer Index Cat. 2169, that it is a fair inference to suppose that nebulosity also has a continuous spectrum. This is borne out by the speed with which it registers through the visual color-filter. The variable nebulae would seem to form a family group, characterized by the shape more or less of a fan, with a condensed variable nucleus at the tip, and having some connection with dark regions in the sky. It would be of interest to determine the nature of R Coronae Australis, whether it is not in reality a nucleus rather than a true star.

North of the nucleus of N.G.C. 2261 about 97″, and preceding it by $4\overset{s}{.}4$, is a fifteenth-magnitude star, marked *P* on the sketch, which has a proper motion of 27″ per century in a direction 164°. North following 9′ and 10′, respectively, are two variable stars whose maxima are at about 15.5 mag. There is still another variable some 17′ north preceding the nucleus, with a range of at least from magnitude 11 to 17.

Yerkes Observatory
September 14, 1916

Minor Contributions and Notes

NOTE ON A SUPPOSED VARIATION IN THE SOLAR ROTATION

In a recent number of this *Journal* is published a paper entitled "A Variation in the Solar Rotation," in which the conclusion is reached[1] "that the sun, during the summer of 1915, underwent a cyclic variation in its rotation rate with a range of 0.15 km. This variation was completed in about a month." This result appeared to me to be another case showing the effect of an overlapping spectrum of haze such as was discussed in my paper (see pp. 177–179 of this number). The observations were made with the same equipment as I have been using day by day since 1913 for the purpose of investigating any changes which might occur in the positions of lines of limb and center, so it happened that I made many observations in various regions of the spectrum during the period of the observations described. The record shows that in general high values of the rotation in the observations mentioned were obtained on the brighter days and low values on the hazier days. Selecting plates at λ 4250 on July 13 (seven double observations), and on July 20 (five double observations), on which dates the measurements mentioned above show the lowest and highest values, I measured λ 4226.9, Ca, 20, strengthened at the limb, and λ 4225.6, Fe, 3, and λ 4232.8, Fe, 2, both weakened at the limb, with results and comparisons as follows:

	July 13, Hazy	July 20, Bright
	km per sec.	km per sec.
H. H. Plaskett's values for five lines of intensities 3 to 8, λ 5900	1.846	2.026
De Lury's values:		
4225.6, Fe, 3	1.712	1.966
4226.9, Ca, 20	1.794	1.983
4232.8, Fe, 2	1.711	1.972
Difference between Ca line and Fe lines	0.082	0.014

[1] H. H. Plaskett, *Astrophysical Journal*, 43, 156, 1916.

These measurements show three results, which are all explainable by the blending of the spectrum of haze with that of the solar limb: (1) The values of the solar rotation are smaller on the hazy day than on the bright day. (2) The difference between the values for weak and strong lines is greater on the hazy day than on the bright day. (3) The values at the greater wave-length, λ 5900, are greater than those at the smaller wave-length, λ 4230, the spectrum of haze being stronger relatively to the spectrum of the limb for smaller wave-lengths than for the greater wave-lengths.

The values at λ 4230 point to a value for the equatorial velocity of about 2.03 or 2.04 km per second for a zero difference between the values for weak and for strong lines.

Measurements of the λ 5900 plates for groups of weak and strong lines will no doubt confirm the conclusion that the variation in question is due entirely to variations in the terrestrial haze.

RALPH E. DE LURY

SOLAR PHYSICS DIVISION
DOMINION OBSERVATORY, OTTAWA
August 1916

NOTICE

The scope of the Astrophysical Journal includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention is given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

Articles written in any language may be accepted for publication, but unless a wish to the contrary is expressed by the author, they usually will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right unless the author requests that the reverse procedure be followed.

Accuracy in the proof is gained by having manuscripts typewritten, provided the author carefully examines the sheets and eliminates any errors introduced by the stenographer. It is suggested that the author should retain a carbon or tissue copy of the manuscript, as it is generally necessary to keep the original manuscript at the editorial office until the article is printed.

All drawings should be carefully made with India ink on stiff paper, usually each on a separate sheet, on about double the scale of the engraving desired. Lettering of diagrams will be done in type around the margins of the cut where feasible. Otherwise printed letters should be put in lightly with pencil, to be later impressed with type at the editorial office, or should be pasted on the drawing where required.

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All papers for publication and correspondence relating to contributions should be addressed to the *Editors of the* Astrophysical Journal, *Yerkes Observatory, Williams Bay, Wisconsin, U.S.A.*

THE

ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME XLIV NOVEMBER 1916 NUMBER 4

A RELATION BETWEEN THE CONVERGENCE WAVE-LENGTHS IN SPECTRAL SERIES AND THE RADII OF THEIR RESPECTIVE ATOMS, AS COMPUTED FROM EINSTEIN'S PHOTO-ELECTRIC EQUATION AND BY OTHER METHODS[1]

By FERNANDO SANFORD

It seems to be very definitely established that electrons which are discharged from metals under the stimulus of ultra-violet light have, after escaping from the metal, velocities such that their kinetic energies are given by the Einstein equation, $\frac{1}{2}mv^2 = h\nu - p$, where h = Planck's constant, ν = the vibration frequency of the exciting light and p = the work required to remove the electron from the metal. Before leaving the metal the energy of a discharged electron must have been $\frac{1}{2}mv^2 = h\nu$.

It is hard to think of an electron as having, while associated with an electropositive sub-atom, any independent velocity except a vibrational or an orbital velocity about the electropositive sub-atom. If an electron be revolving in an elliptical or circular orbit about its positive sub-atom, it seems probable that in some circumstances its velocity may be accelerated by light vibrations

[1] Read at the meeting of the Pacific Division of the A.A.A.S., at San Diego, California, August 10, 1916.

of its own frequency until its centrifugal force exceeds the centripetal force due to the attraction of its positive nucleus, in which case it will escape as a free electron, with a rectilinear velocity only slightly less than its orbital velocity at the instant of escape.

If this represents the conditions of photoelectric discharge, and if the electrons before their escape were moving in elliptical or circular orbits with orbital velocities such that $\frac{1}{2}mv^2 = h\nu$, the Einstein equation gives us a means of computing the mean orbital radius of such electrons. Thus,

$$v^2 = \frac{2h\nu}{m} \text{ and } v = \sqrt{\frac{2h\nu}{m}}.$$

The velocity of a particle moving in a circular orbit is $v = 2\pi R\nu$. Equating these two values of v and solving for R, we have

$$R^2 = \frac{h}{2\pi^2 \nu m}$$

$$h = 6.57 \times 10^{-27}$$

$$m = 10^{-27}$$

$$2\pi^2 = 19.72$$

$$\text{Hence } R^2\nu = 0.333.$$

If we use instead of ν, λ, the wave-length of the exciting light, we have $R^2 = 1.1\lambda \times 10^{-11}$.

This equation would indicate a different electronic radius for each separate line in the spectrum, and would tell us very little about the—so-called—true atomic radius or the radius of the central positive atom about which the electrons are assumed to revolve. It is well known, however, that in a considerable number of elements there are certain series of spectral lines whose wave-lengths may be computed from more or less simple mathematical formulae. All of these series converge toward a shortest possible wave-length, which we may assume to be the shortest wave-length which could exist in the system to which the vibrating electrons belong. If the electrons which produce the spectral lines of the series are moving in orbits about a central positive nucleus, then, apparently, no electron in the system can be closer to the central nucleus than the electron which would give rise to the convergence wave-length. For pur-

poses of comparison we may accordingly assume that the orbital radius corresponding to the convergence wave-length represents the radius of the central positive mass outside of which all the electrons of the system are revolving.

Unfortunately for our comparison, only a few atomic radii have been calculated. A very few molecular radii have been computed from the kinetic gas theory, and Heydweiller has computed the relative diameters of a number of atoms by adopting the assumption of the authors of the Lorenz-Lorentz refraction formula that the ratio $\frac{n^2-1}{n^2+2}$ expresses the ratio of the true volume to the apparent volume of an atom. He has then assumed a diameter for the

TABLE I

Element	Heydweiller Radii	Cube Root of Volume	R from Einstein Equation	
			Principal Series	Subordinate Series
Hydrogen	0.85	2.18	1.5	2.0
Lithium	0.87	2.40	1.59	1.96
Sodium	0.96	2.87	1.63	2.12
Potassium	1.19	3.57	1.77	2.24
Rubidium	1.31	3.83	1.81	2.28 2.30
Caesium	1.46	4.13	1.87	2.33 2.37
			Triplet Series	
Magnesium	1.00	2.40	....	1.66
Calcium	1.21	2.94	....	1.80
Strontium	1.31	3.27	....	1.87
Zinc	1.19	2.09	....	1.59 1.60
Cadmium	1.35	2.35	....	1.62 1.64
Copper	1.19	1.91	....	1.87 1.86
Silver	1.37	2.17	....	1.90 1.87

hydrogen atom based upon the determinations of the molecular diameter calculated from the kinetic gas theory, and has made this his basis for calculating the actual diameters of the atoms. We may also assume that the cube roots of the apparent gram-atomic volumes are approximately proportional to the atomic radii, especially within a group of similar atoms, without making the

Lorenz-Lorentz correction. By comparing the atomic radii calculated by these methods with the radii given by Einstein's equation we may, perhaps, gain further evidence as to the probability of an orbital motion on the part of radiating electrons. Whether this evidence is regarded as important or not, we shall, at least, be

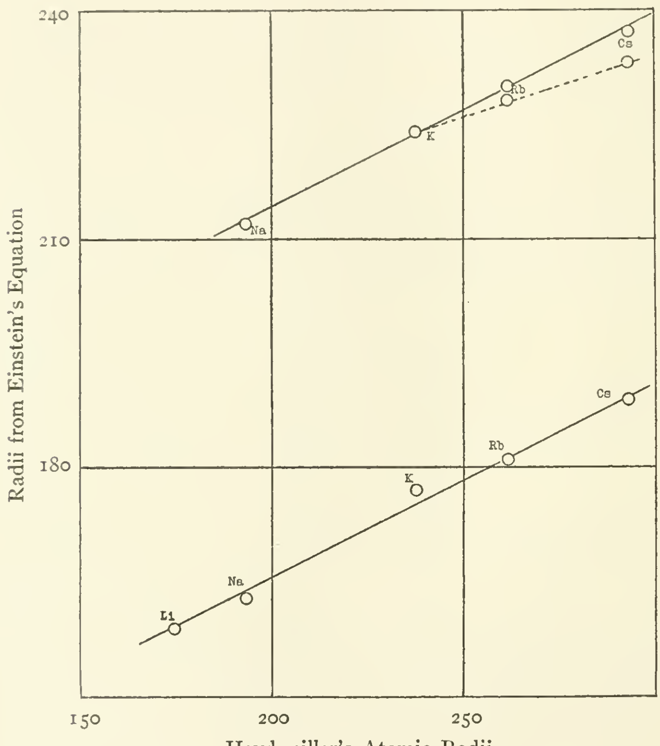


Fig. 1

able to show a relation between the convergence wave-length of a spectral series and a true physical constant of the respective atoms, viz., the radius. This is shown in Table I, in which the calculated radii are multiplied by 10^8.

It is seen from Table I that the radii calculated from the Einstein equation are approximately twice as great as those computed by Heydweiller and are not strictly proportional to them.

Within each group of similar elements, however, this proportionality is very close. This fact is shown graphically for the alkali elements in Fig. 1.

It can also be shown that the Heydweiller radii are, within similar groups, very closely proportional to the square roots of the convergence wave-lengths. In fact, the Heydweiller radii within individual groups can be calculated as accurately from the convergence wave-lengths as from Heydweiller's original data. Thus, for the first subordinate series the radii of sodium, potassium, rubidium, and caesium may be calculated from the equation $R=\frac{\sqrt{\lambda}-508}{137.5}$, as is shown in Table II.

TABLE II

Element	R from $\sqrt{\lambda}$	Heydweiller's R
Na	0.953	0.965
K	1.207	1.19
Rb	1.324	1.131
Cs	1.454	1.465

The Heydweiller radii for magnesium, calcium, and strontium may be calculated from the equation $R=\frac{\sqrt{\lambda}-304}{230}$ with the agreement shown in Table III.

TABLE III

Element	R from $\sqrt{\lambda}$	Heydweiller's R
Mg	1.	1.005
Ca	1.2	1.215
Sr	1.3	1.31

The relation of the Heydweiller radii to the square root of the convergence wave-length is shown graphically for the subordinate series of the alkali metals in Fig. 2. The relations of the atomic radii as calculated from the Einstein equation and the cube roots of the atomic volumes are shown for three spectral series in Figs. 3 and 4.

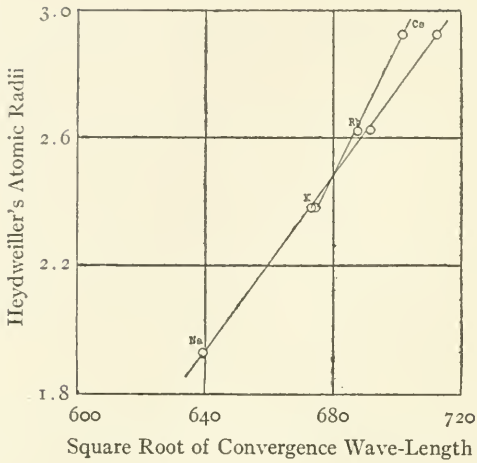

3.0
2.6
2.2
1.8
Heydweiller's Atomic Radii
600
640
680
720
Square Root of Convergence Wave-Length
Na
K
Rb
Cs

Fig. 2

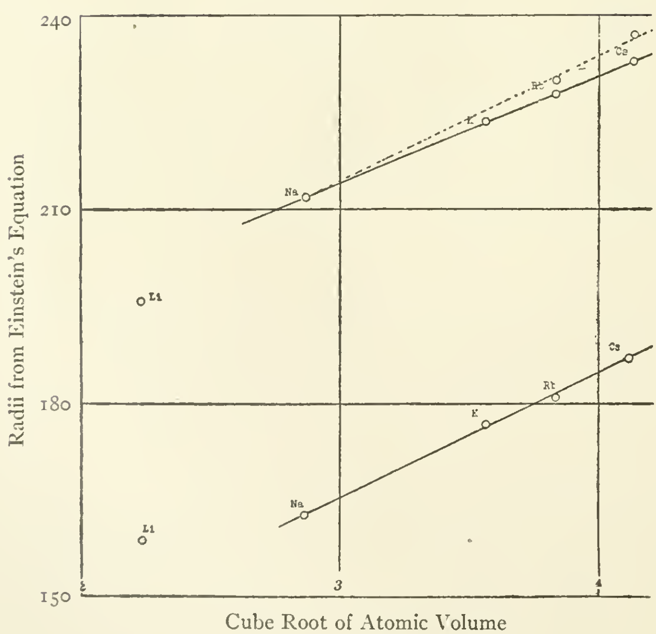

240
210
180
150
Radii from Einstein's Equation
3
4
Cube Root of Atomic Volume
Li
Na
K
Rb
Cs

Fig. 3

. The foregoing considerations seem to show that, whether the radiating electrons are revolving in circular orbits about their positive nuclei or not, the Einstein equation gives us a method of calculating an orbital radius which within several groups of similar elements is closely proportional to the respective atomic radius.

Another relation between the atomic radius and the constants used in computing wave-lengths may be found in the Ritz formula. In *Annalen der Physik,* **12,** 265, 1903, Ritz gave a presentation of his theory of serial spectra and proposed a formula which has since

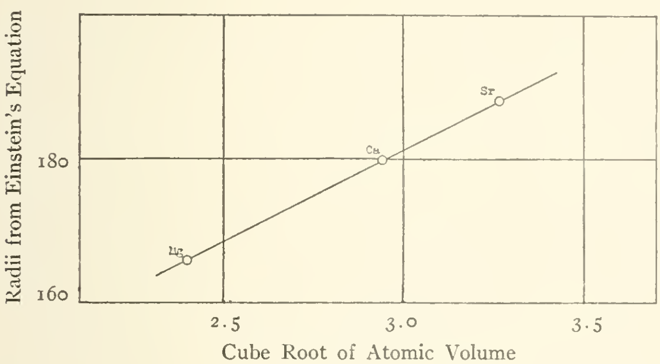


FIG. 4

been shown to give extremely close results in the alkali metals, helium and hydrogen. This formula in its simplest form is

$$\nu = A - \frac{N_0}{\left(n + a + \frac{b}{n^2}\right)^2},$$

where A is the convergence number, N_0 is the Rydberg universal constant and a and b are constants to be determined for each series.

In the article above mentioned Ritz gives his values of these constants for several series, including both the principal series and the two subordinate series of the alkali metals. In an article in *Astrophysical Journal*, **32,** 212, 1910, Birge has used the Ritz formula very successfully with the principal series of the alkalies, the Balmer series in hydrogen and a series of twelve lines in helium. In Fig. 5 Ritz's and Birge's values of a are plotted against the

atomic radius computed from the Einstein equation, and in Fig. 6 the same values of a are compared with the cube root of the respective atomic volume.

It will be seen that in both cases the agreement is remarkably close, and that there can be no reasonable doubt that the Ritz constant a is a function of the atomic radius. Already in his original paper Ritz detected a seeming relation between a and the atomic volume, but the variation was great enough to show that he had selected a wrong quantity for comparison.

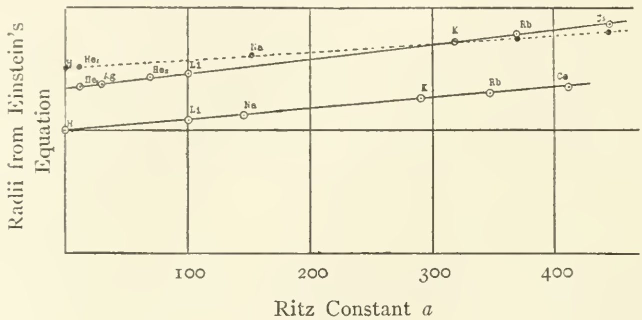


FIG. 5

The curves in Figs. 5 and 6 also give us some information about the distribution of the two subordinate series in the alkali group. They seem to show that the sodium series belong to one subordinate group and the lithium series to the other, as appears in all the curves where the radii of these series are plotted against some other constant. The Balmer series in hydrogen seems to belong to the subordinate group containing the sodium series. Two other hydrogen series having their origins at $A=48764$ and $A=48744$ (as given by Konen) seem to belong to the group of the principal alkali series. The two helium series having their origins at $A=32031$ and $A=29221$ seem to belong to the group of subordinate series containing the lithium series, while the series originating at $A=27173$ seems to belong to the sodium group. A series in silver originating at $A=31542$ also seems to belong to the lithium group. Neither Ritz nor Birge gives a value of a for copper. I have not compared the above-mentioned series to see whether they

agree in their physical properties with the foregoing classification, and I give it here only as suggested from the curves mentioned.

Fig. 6, in which the Ritz constant is plotted against the cube root of the atomic volume, enables us to estimate the atomic volumes of hydrogen and helium. From these curves the cube root of the atomic volume of hydrogen would be at 2.18 or 2.20, according to

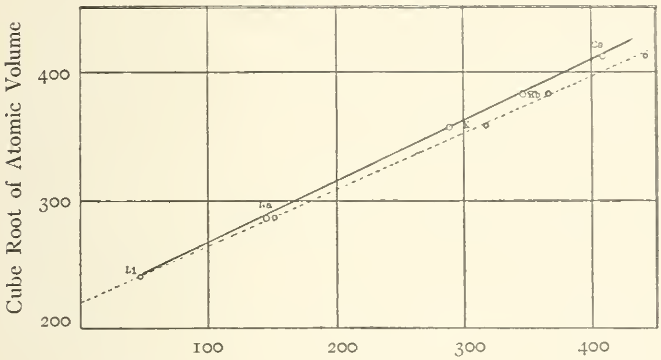


FIG. 6

which atomic system is used, and the cube root of the helium volume would be 2.24 or 2.60, according to which system is considered.

It is probably useless to raise the question at the present time as to whether the different electronic systems which give rise to different spectral series belong to the same atom, or to different atoms; but to me the indications that they actually belong to different atoms seem very strong.

LELAND STANFORD JUNIOR UNIVERSITY
August 19, 1916

PRELIMINARY EVIDENCE OF INTERNAL MOTION IN THE SPIRAL NEBULA MESSIER 101[1]

Photographs by

G. W. Ritchey, J. E. Keeler, C. D. Perrine, and H. D. Curtis

Measured and Discussed by

A. VAN MAANEN

INTRODUCTION

Although the nebulae have been under observation for many years, it is only within the last quarter-century that their motions have been detected. In 1887 Dreyer[2] stated that "so far we do not possess any clear evidence of change of form or change of place." Several of the older observations seemed to indicate such changes, but Dreyer's careful investigation of all known cases led him to the conclusion that these motions must be illusory "unless we are to believe that nebulae in the good old days moved about as they liked, but have been on their good behavior since 1861 and kept quiet."[3]

Motions of nebulae in the line of sight were first observed with the spectroscope by Keeler in 1890,[4] and since that time the radial velocities of many of them have been determined by Campbell, Slipher, and others; but we remained in ignorance as to their cross-motions for another twenty-five years. Curtis, from a comparison of plates taken with the Crossley reflector of the Lick Observatory by Keeler, Palmer, Perrine, and himself, published in 1915 the proper motions of about one hundred nebulae.[5]

In recent years evidence of internal motion has also been discovered. Here too the radial motions were detected first. In 1914, Buisson, Fabry, and Bourget demonstrated the existence of internal motions in the Orion nebula,[6] a result confirmed by Frost

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 118.

[2] *Monthly Notices*, 47, 413, 1887. [3] *Ibid.*, 47, 418, 1887.

[4] *Publications of the Lick Observatory*, 3, 217, 228.

[5] *Publications of the Astronomical Society of the Pacific*, 27, 214, 1915.

[6] *Comptes Rendus*, 158, 1017, 1914; *Astrophysical Journal*, 40, 241, 1914.

and Maney.[1] In 1914 V. M. Slipher[2] published the "Detection of Nebular Rotation," relating to the Virgo nebula, which is of the "spindle" type, and Wolf[3] detected the rotation of Messier 81, while in 1915 Campbell and Moore[4] announced the rotation of the planetary nebula N.G.C. 7009. Also Pease[5] discovered a possible internal motion in Messier 33.

In December 1915 Mr. Ritchey suggested that the two plates of Messier 101 taken by him with an interval of about five years be placed in the stereocomparator to see if they would show motion. Since no motion whatever was revealed in that way, I then proposed that the plates be placed at my disposal for measurement, to which Mr. Ritchey kindly consented. As sixteen nebulous points chosen for this purpose showed some evidence of internal motion, I at once decided to take up the work in a more extended way, measuring many more points. Feeling also the necessity of measuring at least one other pair of plates, I asked Dr. Curtis if he would be willing to place at my disposal the Crossley reflector plates of this nebula. This request was very kindly granted, and Dr. Curtis sent me three plates, one by Keeler (1899), one by Perrine (1908), and one by himself (1914). The measures on these three plates and on the two made by Mr. Ritchey are discussed in the following pages. My thanks are due to Dr. Campbell, director of the Lick Observatory, and to Dr. Curtis for the use of the Lick plates and for permission to publish my measures of them, and also to Mr. Ritchey for the use of the two plates taken by him.

MEASURES AND DISCUSSION

The measures were made with the "blink" arrangement of the stereocomparator in the manner described in "The Photographic Determination of Stellar Parallaxes with the 60-Inch Reflector."[6] The plates were measured in four pairs, combined and arranged in the stereocomparator as shown in Table I.

[1] *Popular Astronomy*, **23**, 485, 1915. [2] *Lowell Observatory Bulletin*, No. 62.

[3] *Vierteljahrsschrift der Astronomischen Gesellschaft*, **49**, 162, 1914.

[4] *Harvard College Observatory Bulletin*, No. 591.

[5] *Publications of the Astronomical Society of the Pacific*, **28**, 33, 1916.

[6] *Mt. Wilson Contr.*, No. 111, 4 ff.

The exposure times of the Lick Observatory plates were as follows: 1899, June 8, 4^h; 1908, July 28, $1^h\ 45^m$; and 1914, March 20, 2^h; the last plate being under-exposed on account of clouds. The exposure times of the Mount Wilson plates were: 1910, March 10 and 11, $7^h\ 30^m$ (aperture 54 inches); 1915, May 14, 15, and 16, $8^h\ 37^m$ (aperture 50 inches).

TABLE I

Pair	Left Plate-Carrier	Right Plate-Carrier	Observer	Remarks
R I.	1910	1915	Ritchey, Ritchey	. . . .
R II.	1915	1910	Ritchey, Ritchey	. . . .
L I.	1899	1908	Keeler, Perrine	1899 through glass
L II.	1914	1899	Curtis, Keeler	1914 through glass

As the 1899 plate of the Lick Observatory was taken with the early arrangement of the Crossley reflector, involving the use of a Newtonian flat, while the later photographs were made without this auxiliary mirror, the field of the 1899 plate is inverted with respect to that of the later photographs. In using the stereo-comparator it was thus necessary to measure through the glass of one of the plates of the Lick pairs.

The interchange of the new and old plates in the plate-carriers should in the mean eliminate any relative displacement of the images due to the measuring instrument.

Thirty-two stars, as near to the nebula as seemed safe, were used for comparison purposes. It is true, of course, that some of these objects may really belong to the nebula, but the results show that the choice of comparison stars, on the whole, was a fortunate one. To prevent the appearance of a magnitude error it was necessary to avoid a great range in brightness, and in order to increase as much as possible the weight of the plate-constants the comparison stars had to be distributed as uniformly as possible around the nebula. The co-ordinates of the comparison stars relative to the center of the nebula are given in the second and third columns of Table II, + indicating east and north for x and y, respectively.

In the choice of points in the nebula which were to be measured I tried to avoid stars. A priori no certain distinction can be made, but in general I avoided those points which through total absence of surrounding nebulosity give the impression of stars rather than of parts of the nebula. This is a hazardous procedure, and undoubtedly some of the points do not belong to the nebula. From countings on the outer parts of the plates the conclusion was reached that the area containing the 87 measured points should include about 20 measurable stars, but several of these must have been omitted in choosing the points in the nebula. Further, the points selected must be as symmetrical in appearance as possible to avoid difficulties in bisecting their images. This requirement, too, could not be rigorously fulfilled.

TABLE II

CO-ORDINATES OF THE COMPARISON STARS

	x	y		x	y
a	−10′.9	+4′.9	q	+10′.0	+2′.4
b	−10.3	−0.1	r	+10.2	+4.7
c	−6.4	+3.4	s	+8.4	+9.1
d	−6.6	−8.1	t	+4.9	+5.9
e	−6.2	−6.8	u	+3.3	+5.3
f	−3.8	−10.9	v	+9.0	+12.0
g	−3.4	−8.6	w	+6.3	+12.0
h	−0.3	−10.7	x	+2.3	+11.0
i	+1.4	−5.4	y	+1.0	+12.6
j	+1.8	−7.1	z	−0.2	+8.9
k	+2.3	−9.4	α	−0.6	+6.7
l	+7.3	−10.9	β	−3.8	+7.8
m	+7.5	−7.5	γ	−2.0	+11.5
n	+4.7	−4.7	δ	−5.9	+11.1
o	+6.5	−2.0	ϵ	−10.5	+8.4
p	+8.0	+0.1	η	−5.2	+5.9

On R I and R II, 87 nebular points were measured; on L I, 69; and on L II, 46. The co-ordinates of these points are given in the second and third columns of Table VI.

Each pair of plates was measured in four positions with east, west, north, and south, respectively, in the direction of increasing readings of the micrometer screw. In the reductions the two measures of each pair in right ascension were combined into one set; the same was done for the measures in declination.

As the measurement of a pair of plates in one position required several hours, the temperature of the room was read during every set of measures. In no case did the range during the measures in one position exceed 1° C. Disturbances from this source are therefore negligible. Moreover, several measures were duplicated to see if any shift could be detected, but no positive evidence was found.

The mean of the measured differences of every object for the two positions of the plate which differ by 180° may be called D. From the values of D the proper motions in α and δ were derived by a method analogous to that described in my paper on stellar parallaxes.[1] It was found from a preliminary reduction, however, that the quadratic terms of the co-ordinates could not here be neglected. The equations of condition therefore have the form

$$D=\mu_\alpha+a+bx+cy+dx^2+exy+fy^2$$

with a similar equation for declination.

Supposing μ_α and μ_δ of the comparison stars to be zero, we can solve the 32 equations for these stars to find the 6 plate-constants a, b, c, d, e, and f. This was done by the regular least-squares method. The constants found for each pair of plates were then substituted into the equations of condition for the stars and for the points in the nebula, thus giving μ_α and μ_δ, the proper motions in α and δ relative to the mean motion of the comparison stars.

These values of μ_α and μ_δ, still expressed in two-thousandths of a revolution of the micrometer screw, must be multiplied by 1.04, 1.04, 0.84, and 0.52 for R I, R II, L I, and L II, respectively, to give the annual proper motion expressed with 0″001 as a unit. The resulting values are entered in Table III. The first column contains the designation a, b, ... α, β, ... for the comparison stars, and the numbers 1, 2, 3, ... 87, for the nebular points.

The weight to be given to each pair of plates depends on the scale (which for the Mount Wilson plates is 1 mm = 27″20, and for the Lick plates 1 mm = 38″73), their quality, and the interval in years. As the influence of quality is difficult to determine in

[1] *Mt. Wilson Contr.*, No. 111, 7 ff.

TABLE III

TOTAL ANNUAL PROPER MOTIONS

(Unit = 0″.001)

No.	μ_α				μ_δ				Mean	
	R I	R II	L I	L II	R I	R II	L I	L II	μ_α	μ_δ
a	+20	−19	+ 8	+ 2	+24	−26	+24	+21	+ 3	+11
b	+14	− 8	−53	+ 5	−10	−17	−40	+10	−11	−14
c	−28	+ 9	−13	−16	−17	+16	− 4	+16	−12	+ 3
d	+33	+14	+46	+42	−19	+ 4	− 5	−18	+34	−10
e	−10	+18	+ 8	− 2	− 1	+ 9	− 2	+33	+ 4	+10
f	−18	−27	−23	−18	+15	− 4	+ 4	− 7	−22	+ 2
g	+ 5	+22	+18	− 2	−16	−14	−12	− 6	+11	−12
h	− 8	− 7	+ 5	−10	0	+ 8	+18	−18	− 5	+ 2
i	−35	−19	−27	−32	+41	+25	+30	+13	−28	+27
j	−12	− 4	− 3	−18	+10	+14	+10	− 5	− 9	+ 7
k	+ 1	+12	+14	+ 9	− 5	−20	+20	−14	+ 9	− 5
l	−19	−15	−44	− 9	+ 9	+17	+24	+17	−22	+17
m	+52	+31	+34	+39	+ 8	−48	−60	+ 9	+39	−23
n	+26	+ 7	+ 7	+20	−28	+27	−52	+26	+15	− 7
o	− 3	−11	−13	−29	−38	+ 3	+21	−27	−14	−10
p	+ 5	+16	− 5	+ 1	− 3	−10	−10	+ 2	+ 4	− 5
q	+10	− 9	+41	+ 1	+11	+14	+23	−15	+11	+ 8
r	−50	+12	− 6	+ 2	−32	− 5	+ 5	+ 9	−11	− 6
s	−21	−43	+16	+13	+33	+18	+ 1	+ 8	− 9	+15
t	+14	+ 5	− 4	− 2	+15	−13	+ 3	−17	+ 3	− 3
u	+24	+11	+ 2	+ 8	− 6	−18	− 1	−22	+11	−12
v	− 6	−10	−40	−36	+25	+31	+10	−12	−23	+14
w	+33	+22	+21	+20	−21	−25	−16	+14	+24	−12
x	−11	−12	−13	− 5	−14	+19	+26	− 2	−10	+ 7
y	+18	+ 7	0	+ 6	−26	−29	−18	+ 9	+ 8	−16
z	+26	+ 8	+ 6	− 6	−51	−43	− 8	+12	+ 9	−23
α	0	− 4	+ 3	+ 8	+52	−10	+17	+ 6	+ 2	+16
β	−56	− 6	−46	−11	+46	+64	+21	+14	−30	+36
γ	− 1	+ 6	+13	+12	+14	−23	−35	+13	+ 8	− 8
δ	−10	− 7	+22	+10	−16	+40	− 3	−14	+ 4	+ 2
ϵ	−10	+15	+16	−21	−12	+ 4	+18	−30	0	− 5
η	+25	− 1	+22	+ 8	+12	−14	+ 5	−32	+14	− 7
1	+43	+32	......	......	− 1	+12	......	......	+38	+ 6
2	−20	−23	−55	− 9	+ 8	+16	0	+12	−27	+ 9
3	+14	− 4	......	......	−16	+50	......	......	+ 5	+17
4	+16	− 3	+45	+ 3	+50	+50	+24	− 5	+15	+30
5	+ 7	− 4	+ 5	+ 3	+ 6	+42	− 2	+21	+ 3	+17
6	+ 7	+ 1	+68	......	+40	+24	+87	......	+25	+50
7	+26	+17	......	......	+ 4	+58	......	......	+22	+31
8	+14	+ 1	+15	−15	+18	+48	+15	+ 7	+ 4	+22
9	+35	+23	+45	+37	−18	− 7	−34	−49	+35	−27
10	+14	−10	+35	+15	+ 6	+33	+34	+ 6	+14	+20
11	− 6	+34	+ 3	......	+ 4	+56	−24	......	+10	+12
12	−22	+19	+61	......	−30	+14	− 6	......	+19	− 7
13	− 2	+ 4	+ 1	− 9	+10	+20	+ 5	+15	− 2	+13
14	+14	+ 4	+ 8	+ 3	− 2	+15	+23	+ 4	+ 7	+10
15	+38	+17	+23	+ 2	− 2	+35	+21	+28	+20	+21
16	+47	+25	+42	+12	− 1	+12	+34	+26	+32	+18

TABLE III—*Continued*

No.	μ_α RI	RII	LI	LII	μ_δ RI	RII	LI	LII	Mean μ_α	Mean μ_δ
17	+7	+25	......	......	−1	+9	......	......	+16	+4
18	+43	+33	+91	......	−10	−2	+60	......	+56	+16
19	+9	+16	−12	......	−24	+6	+19	......	+4	0
20	−21	+5	+29	......	−42	+17	+22	......	+4	−1
21	−12	−43	+18	+26	+22	+2	+58	+23	−3	+26
22	+7	−6	−8	+3	+21	+22	−8	+4	−1	+10
23	+14	−1	......	......	+4	−12	......	......	+7	−4
24	+21	+4	......	......	+18	+54	......	......	+13	+36
25	+5	+10	−9	......	+20	+9	+1	......	+2	+10
26	−19	−36	......	......	−29	−8	......	......	−28	−19
27	+11	−15	−26	−16	−40	−2	+6	+20	−12	−4
28	+19	+15	+15	......	−75	−1	+30	......	+16	−15
29	+52	−2	−55	......	+4	−31	−81	......	−2	−36
30	−6	−47	......	......	−10	−31	......	......	−27	−21
31	+45	−8	+23	......	+23	−27	−15	......	+20	−6
32	+4	−49	+29	−27	−24	+16	+28	+28	−11	+12
33	−20	−28	−22	......	+16	+2	+9	......	−23	+9
34	−11	−44	+15	−13	−22	+12	+66	+16	−13	+18
35	+27	−21	−32	−32	−1	−10	+28	+16	−15	+8
36	+9	−30	−12	−21	+5	+24	−8	−9	−14	+3
37	−23	−12	......	......	−9	+27	......	......	−18	+9
38	+23	−23	......	......	+19	+4	......	......	0	+12
39	−25	−36	+13	−7	+5	+6	−8	+27	−14	+8
40	−44	−16	......	......	−21	−18	......	......	−30	−20
41	+24	+16	......	......	−12	−19	......	......	+20	−16
42	−48	−44	−19	......	−16	−27	−29	......	−37	−24
43	+58	−35	......	......	−22	−45	......	......	+12	−34
44	−3	+11	−15	......	−44	−60	+19	......	−2	−28
45	+14	−28	+64	......	+8	+1	−36	......	+17	−9
46	−11	−29	−36	−20	+29	−16	−12	−38	−24	−9
47	−38	−9	−15	−7	+20	−26	−30	−28	−17	−16
48	+6	−29	−18	−14	−15	−25	+30	−24	−14	−9
49	−9	−57	+30	−3	−10	−42	−3	−37	−10	−23
50	+5	−43	+37	−16	+5	−46	−37	−20	−4	−25
51	+30	−40	+46	......	−11	−49	−34	......	+12	−31
52	−69	−83	+5	......	−6	−38	−60	......	−49	−35
53	+50	−17	+4	−1	−14	−28	−39	−12	+9	−23
54	−15	−34	−32	−45	−32	−73	−18	−21	−32	−36
55	−49	−77	−19	−28	−36	−11	+21	+21	−43	−1
56	+47	+1	+51	−1	−45	−66	−19	−27	+25	−39
57	+15	−11	+18	−4	−49	−57	+11	−3	+5	−25
58	−5	+10	+59	......	−51	−52	−44	......	+21	−49
59	+33	−6	+24	−9	−73	−60	−89	−38	+11	−65
60	+25	+9	−14	......	−15	−54	−34	......	+7	−34
61	−14	−35	−10	+3	−46	−41	−66	−2	−14	−39
62	+7	+1	+44	−20	−52	−51	−19	−16	+8	−35
63	+21	−20	−7	−9	−74	−60	−46	−21	−4	−50
64	+15	−57	+76	−28	−77	−11	+41	−5	+2	−13
65	+28	−29	+19	−7	−58	−38	−23	−31	+3	−38
66	+60	−28	+23	−27	−30	−22	−44	−41	+7	−34
67	+58	−4	+8	−6	−48	−31	−31	−48	+14	−40

TABLE III—*Concluded*

No.	μ_α				μ_δ				Mean	
	R I	R II	L I	L II	R I	R II	L I	L II	μ_α	μ_α
68........	+ 1	−10	+69	......	−59	−69	−66	......	+20	−65
69........	+ 7	−28	......	......	−46	−29	......	......	−11	−38
70........	+15	+24	+24	+10	−70	−55	−25	−27	+18	−44
71........	−11	−21	−23	+11	+12	−28	−39	+10	−11	−11
72........	+21	−15	+18	− 3	−42	−46	−13	−12	+ 5	−28
73........	+38	− 1	+33	......	−59	−72	−14	......	+23	−48
74........	+11	− 4	......	......	−107	−55	......	......	+ 3	−81
75........	+12	+23	+ 5	......	−35	−18	−33	......	+13	−29
76........	+41	+27	......	......	−38	−34	......	......	+34	−36
77........	+29	+37	+115	......	−17	−57	−10	......	+60	−28
78........	+37	+33	+13	+13	+ 1	−33	−33	−36	+24	−25
79........	+41	+ 9	+24	− 3	−28	−68	−71	−38	+18	−51
80........	+14	+18	+37	− 3	−21	−41	− 6	−35	+17	−26
81........	+ 2	−19	+34	− 5	+ 5	−61	+24	−29	+ 3	−15
82........	+ 4	+46	+42	− 5	−34	−75	−14	−24	+22	−37
83........	+31	+ 5	+20	+ 6	−80	−63	−31	+ 4	+16	−43
84........	+81	+28	+56	......	+ 2	+11	+71	......	+55	+28
85........	+16	+34	......	......	−17	+ 3	......	......	+25	− 7
86........	−17	+20	+47	+16	−11	+36	− 7	+18	+17	+ 9
87........	+43	+31	......	......	−33	+31	......	......	+37	− 1

advance, all pairs were provisionally given the same weight and combined to form mean motions for the 32 comparison stars and the 46 nebular points which were measured on all four pairs.

The average deviation from these means for each pair of plates was as shown in Table IV (unit=0."001). Since the mean deviations

TABLE IV

Pair	R I	R II	L I	L II
Comparison stars..........	12	11	11	13
Nebular points............	16	16	17	14

are sensibly the same for the different pairs, it seems permissible to use the same weights for all pairs, the quality very nearly counterbalancing the influence due to scale and time interval. The adopted mean μ_α and μ_δ are accordingly those given in the last two columns of Table III. There is of course quite a range in the agreement for the different points, but the mean probable error of a mean μ_α or μ_δ is only 0."008.

The results found may be due partly to a translation of the nebula as a whole, and partly to internal motions. To derive the motion of translation of the nebula three methods were employed.

1. The mean of the proper motions μ_α and μ_δ for all the nebular points was found.

This gives $\mu_\alpha = +0''.005$, $\mu_\delta = -0''.012$. The Mount Wilson plates treated separately give: $\mu_\alpha = +0''.002$, $\mu_\delta = -0''.016$, while the Lick plates alone give $\mu_\alpha = +0''.007$, $\mu_\delta = -0''.008$.

2. Whatever the character of the internal motions of a nebula like Messier 101, we may expect them to be nearly symmetrical with respect to the center of the nebula. If, therefore, we divide the points into four groups corresponding to the four quadrants, the two pairs of opposite quadrants should give a good agreement. Using all the 87 points for this purpose, the results are as summarized in Diagram I, which gives for each quadrant the mean values of μ_α and μ_δ, and the number of points measured.

DIAGRAM I

		N		
	I		II	
	$+0''.002$		$-0''.010$	
	$+0''.008$		$-0''.023$	
	31		18	
E				W
	$+0''.018$		$+0''.013$	
	$-0''.001$		$-0''.040$	
	16		22	
	IV		III	
		S		

Mean of I and III $\begin{cases} \mu_\alpha = +0''.007^5 \\ \mu_\delta = -0''.016 \end{cases}$

Mean of II and IV $\begin{cases} \mu_\alpha = +0''.004 \\ \mu_\delta = -0''.012 \end{cases}$

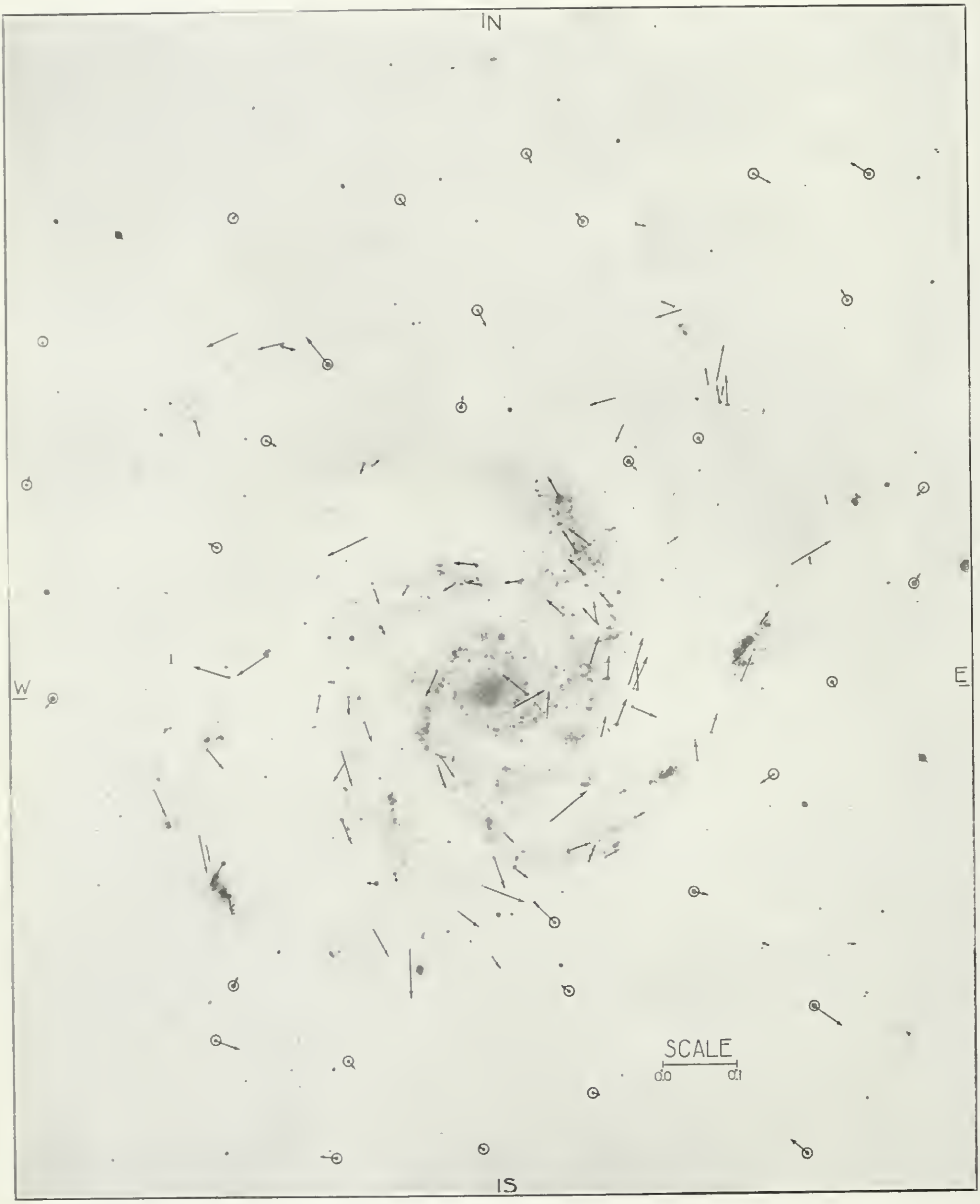
3. As, however, the distribution of the points measured is far from uniform in the four quadrants, it seemed worth while also to derive analogous values for the central parts of the nebula, within a radius of $5'.0$, where the distribution of the points is more symmetrical. The results may be seen in Diagram II.

The results of the three methods are given in Table V. The agreement throughout is as good as could be expected. For the final motion of translation the values $\mu_\alpha = +0''.005$, $\mu_\delta = -0''.013$

INTERNAL MOTIONS IN MESSIER 101

The lines indicate the direction and magnitude of the annual motions of the nebulous points as derived from the different pairs of plates. Their scale (0.″1) is indicated on the illustration. The scale of the nebula is 1 mm = 9″1.

PLATE VII



Internal Motions in Messier 101

The arrows indicate the direction and magnitude of the mean annual motions. Their scale (0″1) is indicated on the illustration. The scale of the nebula is 1 mm = 10″5. The comparison stars are inclosed in circles.

have been used. No great weight, however, should be given to this motion of translation, as the hour angles and the parallax factors of the different plates may have vitiated the results. With

DIAGRAM II

	N	
I		II
$-0''.004$		$-0''.009$
$+0''.015$		$-0''.024$
14		9
E		W
$+0''.020$		$+0''.014$
$-0''.003$		$-0''.037$
14		11
IV		III
	S	

Mean of I and III $\begin{cases} \mu_\alpha = +0''.005 \\ \mu_\delta = -0''.011 \end{cases}$

Mean of II and IV $\begin{cases} \mu_\alpha = +0''.005^5 \\ \mu_\delta = -0''.013^5 \end{cases}$

the material at hand, however, it is the best we can do. Subtracting the adopted μ_α and μ_δ from the total motions in Table III, the results are what may be called the internal motions. They are given in the fourth and fifth columns of Table VI.

TABLE V

Method	μ_α	μ_δ
1................	$+0''.005$	$-0''.012$
2................	$+0.006$	-0.014
3................	$+0.005$	-0.012

The accompanying illustrations show these motions, freed from the motion of translation of the whole nebula. The density of the center of the prints used for the engravings was reduced in order to show the motions more clearly. Plate VI indicates the motions given by each pair of plates; they show in general a good agreement, although there are several large discrepancies. Plate VII shows the mean annual proper motion for each of the 87 points, and also for the comparison stars, which are surrounded by circles. The scale for the annual motions is indicated on the plates.

If the results as illustrated in Plate VII could be taken at their face value, they would certainly seem to indicate a motion of

rotation or possibly motion along the arms of the spiral. Without expressing a final opinion as to the character of the motion, which must be determined by future work, it may be of interest to examine the evidence afforded by the existing material.

To discuss the internal motions from the standpoint of rotation, they were analyzed into components along and perpendicular to the radius; the latter for convenience will be spoken of as the rotational component. The results are given in the last four columns of Table VI. In view of the uncertainty of the measures it seems well to treat the available material in two ways: first, by taking all points measured, and, secondly, by excluding those points which, because of internal disagreement of the measures on the four sets, seem to be uncertain in their final motion. As the probable error in μ_α or μ_δ derived from a single pair of plates is a little over $0''.013$, I have thought it well to exclude in the latter case points whose deviation in μ_α or μ_δ for one or more of the pairs of plates is $>0''.040$. In this way the following 25 points were excluded: Nos. 6, 11, 12, 18, 20, 28, 29, 34, 35, 43, 44, 45, 49, 50, 51, 52, 53, 64, 66, 67, 68, 77, 81, 83, and 84. This procedure seems the more reasonable as 18 of these 25 points are involved asymmetrically in the nebulosity; the measures in such cases must be very uncertain if the plates compared are not of exactly the same density.

The results are as follows, numbers in parentheses relating to the restricted group: 78 (55) points have a left-handed motion,[1] only 9 (7) moving right-handedly; 58 (44) points appear to be moving outward, while 28 (17) show motion inward. The rotational motion is the larger in the majority of cases, viz., for 63 (45) points. The mean rotational motion is $0''.022$ ($0''.021$) left-handed; the mean radial motion $0''.007$ ($0''.009$) outward.

The probable reality of the result is indicated by the satisfactory agreement of the pairs of plates as shown in Table VII, where the + sign indicates left-handed and outward motions, respectively. For this comparison only the 46 objects common to all plates were used.

[1] The rotation directions refer to the illustrations. See Plate VII, noting the orientation.

TABLE VI

Co-ordinates and Internal Motions

(Unit=0".001)

No.	x	y	μ_α	μ_δ	$\mu_{rad.}$		$\mu_{rot.}$	
					Out	In	L.H.	R.H.
1.....	+0'.7	−0'.3	+33	+19	21	....	32	....
2.....	+0.9	−0.1	−32	+22	....	34	20	....
3.....	+1.3	−0.6	0	+30	....	13	27	....
4.....	+2.3	+0.6	+10	+43	20	....	39	....
5.....	+2.8	+0.3	− 2	+30	1	....	30	....
6.....	+3.2	+0.1	+20	+63	22	....	62	....
7.....	+3.4	0.0	+17	+44	17	....	44	....
8.....	+3.5	0.0	− 1	+35	....	1	35	....
9.....	+3.3	−0.4	+30	−14	31	....	....	11
10.....	+3.0	−0.8	+ 9	+33	0	0	34	....
11.....	+2.6	−1.1	+ 5	+25	....	5	25	....
12.....	+3.4	−3.0	+14	+ 6	7	....	14	....
13.....	+4.8	−1.7	− 7	+26	....	15	22	....
14.....	+5.2	−1.0	+ 2	+23	....	2	23	....
15.....	+5.9	+0.2	+15	+34	17	....	33	....
16.....	+5.7	+0.6	+27	+31	30	....	28	....
17.....	+6.3	+1.5	+11	+17	15	....	14	....
18.....	+7.1	+2.9	+51	+29	58	....	8	....
19.....	+7.6	+2.9	− 1	+13	4	....	12	....
20.....	+7.9	+4.3	− 1	+12	5	....	11	....
21.....	+5.7	+6.7	− 8	+39	25	....	31	....
22.....	+5.4	+6.7	− 6	+23	14	....	19	....
23.....	+5.5	+6.9	+ 2	+ 9	8	....	4	....
24.....	+5.4	+7.2	+ 8	+49	44	....	24	....
25.....	+5.2	+7.1	− 3	+23	17	....	16	....
26.....	+4.6	+8.9	−33	− 6	....	20	17	....
27.....	+4.4	+9.0	−17	+ 9	1	....	19	....
28.....	+3.5	+10.9	+11	− 2	2	....	....	11
29.....	+3.1	+6.2	− 7	−23	....	24	....	4
30.....	+3.0	+6.8	−32	− 8	....	20	26	....
31.....	+4.2	+3.4	+15	+ 7	16	....	....	4
32.....	+1.6	+4.6	−16	+25	19	....	24	....
33.....	+2.5	+3.4	−28	+22	1	....	36	....
34.....	+2.0	+3.2	−18	+31	17	....	32	....
35.....	+2.2	+2.8	−20	+21	3	....	29	....
36.....	+2.8	+2.0	−19	+16	....	7	24	....
37.....	+2.5	+1.5	−23	+22	....	8	31	....
38.....	+2.5	+1.6	− 5	+25	9	....	23	....
39.....	+1.7	+1.8	−19	+21	2	....	28	....
40.....	−4.8	+8.2	−35	− 7	12	....	34	....
41.....	−4.9	+8.1	+15	− 3	....	10	....	11
42.....	−5.9	+8.4	−42	−11	15	....	40	....
43.....	−6.9	+6.5	+ 7	−21	....	19	11	....
44.....	−2.9	+5.4	− 7	−15	....	10	14	....
45.....	−2.7	+5.3	+12	+ 4	....	2	....	13
46.....	−0.3	+3.0	−29	+ 4	7	....	28	....
47.....	+0.7	+2.6	−22	− 3	....	9	20	....
48.....	−0.2	+2.5	−19	+ 4	5	....	18	....

TABLE VI—*Continued*

No.	x	y	μ_α	μ_δ	$\mu_{rad.}$ Out	$\mu_{rad.}$ In	$\mu_{rot.}$ L.H.	$\mu_{rot.}$ R.H.
49	−0′.9	+2′.5	−15	−10	....	5	17	....
50	−1.9	+2.5	− 9	−12	....	4	15	....
51	−2.8	+2.4	+ 7	−18	....	17	9	....
52	−2.9	+3.6	−54	−22	16	....	56	....
53	−2.6	+1.6	+ 4	−10	....	9	6	....
54	−5.2	+0.9	−37	−23	33	....	29	....
55	−6.2	+0.4	−48	+12	48	....	....	8
56	−6.7	−1.3	+20	−26	....	15	29	....
57	−7.5	+0.9	0	−12	....	2	12	....
58	−8.0	−2.2	+16	−36	....	6	39	....
59	−6.9	−3.3	+ 6	−52	16	....	49	....
60	−6.8	−3.6	+ 2	−21	8	....	19	....
61	−6.4	−4.0	−19	−26	30	....	12	....
62	−6.2	−4.7	+ 3	−22	11	....	19	....
63	−1.3	+0.5	− 9	−37	....	5	38	....
64	−3.7	+0.3	− 3	0	3	....	0	0
65	−4.1	−0.1	− 2	−25	3	....	25	....
66	−3.4	−0.1	+ 2	−21	....	1	21	....
67	−3.0	−0.7	+ 9	−27	....	3	28	....
68	−3.5	−1.3	+15	−52	4	....	54	....
69	−3.4	−1.6	−15	−25	24	....	16	....
70	−3.5	−3.0	+13	−31	11	....	32	....
71	−2.7	−4.5	−16	+ 2	7	....	....	15
72	−2.3	−4.3	0	−15	13	....	7	....
73	−2.8	−5.5	+18	−35	23	....	32	....
74	−2.0	−6.0	− 2	−68	65	....	19	....
75	−0.1	−6.2	+ 8	−16	16	....	8	....
76	−0.9	−5.1	+29	−23	17	....	33	....
77	−0.2	−4.6	+55	−15	12	....	56	....
78	+0.5	−4.1	+19	−12	14	....	18	....
79	0.0	−3.9	+13	−38	38	....	13	....
80	+0.3	−3.3	+12	−13	14	....	11	....
81	−0.9	−1.5	− 2	− 2	3	....	....	1
82	−1.2	−1.5	+17	−24	8	....	28	....
83	−1.3	−1.7	+11	−30	17	....	27	....
84	+1.4	−3.1	+50	+41	....	17	63	....
85	+2.6	−3.9	+20	+ 6	6	....	20	....
86	+2.2	−4.0	+12	+22	....	13	21	....
87	+1.8	−3.8	+32	+12	2	....	34	....

TABLE VII

	$\mu_{rot.}$	$\mu_{rad.}$
R I	+0″.021	+0″.004
R II	+0.032	+0.012
L I	+0.017	+0.006
L II	+0.012	+0.007

Of the comparison stars, 13 have left-handed and 19 right-handed motion; 15 move outward and 15 inward; the mean rotational motion, as well as the mean radial motion, is 0".000; a glance at Plate VII shows that very few of the stars can share in the motions of the nebula.

The measures indicate a small and hardly trustworthy decrease of rotational motion with increasing distance from the center, as shown by Table VIII.

TABLE VIII

Distance	Mean Distance	Rotational Component	Number of Points
<3'.1	2'.2	0".024 (0".026)	19 (14)
3'.1 to 5'.0.....	3.9	0.028 (0.024)	29 (16)
5.1 to 7.0.....	5.9	0.014 (0.015)	18 (15)
>7.0	8.9	0.019 (0.021)	21 (17)

The change in the rotational component with increasing distance from the center is of interest in connection with the assumption that the nebular points are moving about the nucleus in elliptical orbits. For different points the observed orbital motion should then be inversely proportional to the square root of the mean distance from the center of the nebula. It does not seem safe to attempt final conclusions on the basis of the material now available.

The annual rotational component of 0".022 at the mean distance from the center of 5' corresponds to a rotation period of about 85,000 years. If we knew the parallax of the nebula, and if we could assume that the motions and the distances of the points from the center are mean values for elliptical orbits, the central mass could be calculated. The parallax is unknown and the assumption concerning distances and motions is probably far from the truth. Nevertheless, such an assumption, combined with more or less probable values for the parallax, may give a notion as to whether we are dealing with masses of the order of those of the stars or with much greater aggregations of material. It goes without saying that the universality of the Gaussian gravitation constant must also be assumed.

The relation between the parallax and a central mass which would cause a particle to move in an elliptical orbit at a mean distance of 5′ with a period of 85,000 years can be derived from Kepler's third law; it is $M = 0.0037\,\pi^{-3}$. We thus find the corresponding values of parallax and mass given in Table IX.

TABLE IX

π	Mass (Sun = 1)	Mean Distance (Astron. Units)	Rotational Compt. km/sec.
0″0001....	3,700,000,000	3,000,000	1,038
0.0004....	57,812,500	750,000	260
0.0016....	903,320	187,500	65
0.0064....	14,114	46,875	16
0.0256....	220	11,719	4
0.1024....	3.4	2,930	1

The order of the parallax of Messier 101 is suggested by the following considerations: Curtis gives 0″033 as the average annual motion of 66 large spiral nebulae.[1] Could we assume that these objects are comparable with the stars in the matter of proper motion and distance, we should have, supposing them to be of the twelfth magnitude, 0″005 for their mean parallax. Again, we may compare the cross-motion of 0″033 with the observed radial velocities. Only a few of the latter have been determined as yet, but those found indicate high speeds. Assuming them to be distributed at random in space, Curtis finds the distance to be of the order of 10,000 light-years,[2] corresponding to a parallax of 0″0003. Various objections to the acceptance of these results, even as rough guesses, immediately suggest themselves, but they are the best we have. At the moment, therefore, this method of discussion suggests large values for the masses involved.

An inspection of Plate VII conveys the impression that the general drift of the motion is outward along the branches. The direction of the branches in the vicinity of 52 of the points can be specified with fair accuracy, and a comparison with the observed motions shows a mean divergence for the latter of $7^\circ \pm 4^\circ$ toward the *concave* side of the spiral.

[1] *Publications of the Astronomical Society of the Pacific*, 27, 217, 1915.

[2] *Ibid.*, 218, 1915.

It would be of interest to consider the further development of a nebular mass in which, as apparently is the case with Messier 101, the direction of motion is so closely coincident with the branches of the spirals, and to examine the evidence afforded by other nebulae as to the probability of the continued existence of such motion. For example, continued outward motion should result in forms in which the distribution of the nebular material is widely different from that in Messier 101. Interesting cases such as Messier 77, Messier 81, Messier 101, Messier 100, and possibly N.G.C. 7293 immediately suggest themselves in this connection; in these we see a gradual transition from a nebula with the bulk of the material strongly concentrated in the center, while the spirals contain relatively little material, to the other extreme, where the spirals, containing most of the material, seem to be connected hardly at all with the central body.

All such questions as those touched on above, which naturally are highly speculative, are here considered because of the part played by elliptical motion in the Chamberlin-Moulton hypothesis as to the origin of spiral nebulae. The explanation which they have given, at least for the typical case, requires that the motion of a particle belonging to one of the branches should be inclined at a considerable angle toward the convex side of that branch. The departure of Messier 101 from the typical case imagined by Chamberlin and Moulton is sufficient to excite comment. Professor Chamberlin, however, kindly allows me to quote parts of his manuscript of a book, *The Origin of the Earth*, bearing on this question. He states:

> . . . . that the paths pursued by the projectiles are not identical with the spiral chain of projectiles into which they are forced to arrange themselves. The divergence between the paths and the chain of nebulous matter may vary widely. A much closer approach to coincidence between the paths of the projectiles and the chain of projectiles is assignable in certain cases of closer approach and more violent projection.

And also:

> . . . . the relative amount of this forward or tangential pull is a critical factor; its value is obviously dependent on the relative distance to which the bolt was projected.

The results can therefore perhaps be reconciled with the Chamberlin-Moulton hypothesis, by assigning sufficiently high values to the "disruptive" forces as compared with the central attractions; but all such questions can well await the accumulation of further observational data. In the meantime my measures of two photographs of Messier 81 (made by Mr. Ritchey in 1910 and 1916) give preliminary evidence of internal motion similar to that revealed in the case of Messier 101.

For the precision of such results the time factor is of course the most important, but when the photographs are to be used for the study of internal motions, various precautions will also contribute in no small degree:

1. Several photographs of each object should be taken; 4 or 5 old plates compared with 4 or 5 new ones should enable us to derive the motions satisfactorily.

2. Care should be taken that both hour angles and parallax-factors are similarly distributed in both the old and the new series of plates. A neglect of this precaution undoubtedly will introduce errors difficult or impossible of elimination; some of the discrepancies in the measures on Messier 101 are probably due to this cause.

3. As it is impossible to measure points near the center, if the exposures are long enough to show the fainter parts of a nebula, two sets of plates of the same object are desirable, one with long, the other with short, exposures.

4. The plates should be taken with a telescope of the greatest possible focal length, in order to increase the scale.

5. It is advisable to insert a star-trail on at least one plate for purposes of orientation. This of course is not necessary in deriving possible internal motions, but for the motion of translation it will afford more accuracy.

6. For a determination of the parallaxes of spiral nebulae, the distribution of the plates, if feasible, should be as outlined in my paper on stellar parallaxes.[1] The short exposures used for the proper motions might be utilized in part for this work.

[1] *Mt. Wilson Contr.*, No. 111.

7. On account of asymmetry in the nebulous points, the plates should be measured in a purely differential manner, so that the settings for any point can be made on both plates in quick succession. The chance of choosing different parts of the image for the settings on the two plates is thereby greatly decreased. The stereocomparator is an admirable apparatus for such measures, but if not available, two miscroscopes may be used on an ordinary measuring machine, adjusted in such a way that they point to the same part of the field on the two plates, which are mounted side by side on the plate-carriage.

To test the possibility that the results found might be a consequence of systematic personal or instrumental errors, it was important that the plates should be measured with another machine and, if possible, by another measurer. I am glad to acknowledge the great service rendered by Mr. Seth B. Nicholson, who willingly spent a large amount of time in remeasuring parts of the Mount Wilson plates with two different machines. We first mounted a second microscope alongside the first, on one of our regular instruments used for measuring spectra. The two plates were mounted on the plate-carriage moved by the micrometer screw in such a way that the microscopes were directed toward nearly identical points on the two plates. In this way we were able to bisect corresponding points one after the other. As the instrument allowed only a part of a plate to be measured without readjustment on the carriage, 8 or 9 points were selected on each side of the center of the nebula, and their relative shift in declination was derived with respect to a dozen comparison stars. Mr. Nicholson found an annual difference of motion in declination of 0".083 (the east side of the nebula moving northward as compared with the west side), while my measures with the stereocomparator had given for the same points a relative shift in the same direction of 0".063. The agreement is satisfactory.

After having thus proved that the motion found was not due to any defect of the stereocomparator, Mr. Nicholson kindly undertook with this instrument a set of measures of 53 points of the nebula, using all the 32 comparison stars for reference. The plates were measured in all four positions and reduced in exactly the same way

as the original measures. Of the 53 points measured, 11 are points mentioned as difficult on p. 220. The results for all 53 points, and also for the 42 remaining after excluding these 11 difficult points, are summarized in Table X.

TABLE X

Annual Motions	53 Points	42 Points	Agreement of Sign with Van Maanen	
			53 Points	42 Points
μ_α of translation	$+0''.003$	$+0''.002$	72%	76%
μ_δ of translation	-0.013	-0.011	75	83
$\mu_{rad.}$	-0.003	-0.002	68	69
$\mu_{rot.}$	$+0.009$	$+0.010$	75	76

The agreement of the motion of translation with that given on p. 218 is very satisfactory. Although Mr. Nicholson's value of the rotational component is smaller than that found above, a comparison with the data on p. 222 shows that the difference is within the uncertainty of the determination.

I wish to express my thanks to Mr. Hale and Mr. Seares for many suggestions they have made during the work on the plates discussed here, and to Miss Helen Davis for much assistance in the computations.

Mount Wilson Solar Observatory
March 19, 1916

ON THE RELATION BETWEEN LINES OF THE SAME SPECTRAL SERIES

By W. M. HICKS

The very accurate determination of the wave-lengths of the first 57 members of the principal series of sodium published by Messrs. Wood and Fortrat in a recent number of this Journal (**43**, 73, 1916) affords data which help to throw some light on the question as to the nature of the formulae giving series lines, or as to whether in fact such formulae actually exist. For the measures a degree of accuracy involving errors not exceeding a few thousandths of an angstrom is claimed, even up to within the last few members—a claim which is probably well justified. There can be no doubt but that the frequencies of any of the recognized series can be approximately given by a formula of the type $n=A-\frac{N}{[f(m)]^2}$ and I have shown that, with the exception of a few cases only, the form $f(m)=m+\mu+\frac{d}{m}$, suggested by Mogendorff[1] and myself[2] as a modification of Rydberg's, gives the frequencies to within a few hundredths, except in many cases for $m=1$, and that it is in fact better than the Ritz formula used by Wood and Fortrat. In general the constants are determined from the lines for $m=2, 3, 4$, for which of course the calculated and observed values agree, but even so their values will be subject to uncertainty, due to the observational errors of the three lines. When we come to lines beyond $m=4$, the observational errors are usually so considerable that no conclusion is possible beyond the fact that any given formula can or cannot represent the observations approximately. The limit A of the series is determined with considerable accuracy, in nearly all cases from the lines for $m=2, 3, 4$. This in itself is a remarkable result. The actual calculated value of A, however, not only suffers from the errors of observation of the three lines,

[1] *Proc. Roy. Acad. Amsterdam*, **9**, 434, 1906. [2] *Phil. Trans.*, A, **210**, 57, 1909.

but also depends to some extent on the form of $f(m)$ employed. For instance, the value of the wave-numbers may differ by a few units—or say 1 in 10,000—according as the formula α/m, or Ritz's β/m^2 is used. It might be thought that in the case where many members of a series are known the value might be best obtainable by considering those of the highest order. But this is not the case, because not only are the higher orders faint and subject to large observational errors in wave-length, but being in general in the ultra-violet, these errors are multiplied by large factors in the wave-numbers. For instance, in the case of Na, the limit is close to 2400 A.U., or a wave-number about 41450. An error of 0.1 A.U. is multiplied by 17.2, whilst errors in λ considerably larger than 0.1 may be expected. The great importance of Wood and Fortrat's measurements is that even up to $m=54$ the observations are probably correct to within a few thousandths of an A.U. Therefore we may use the last half-dozen preceding this to calculate asymptotically the value of A, and thus obtain the true numerical value of $f(m)$ for any line, independent of any supposition as to its general form, and only affected with the single observational error of the line itself. For the lines of high order an ordinary Rydberg formula is accurate to the degree of approximation attainable.

In the numerical work which follows I have used Wood and Fortrat's wave-lengths given in international units and reduced them to vacuo by means of the table given by Kayser in his *Handbuch*, **2**, 514, the numbers in the tables being increased by $\frac{1}{60}$ to render them applicable to wave-lengths in air at 15° C. instead of 20° C. For Rydberg's constant I have used $N=109679.2$, the value on the international scale deduced from the careful measures by W. E. Curtis[1] of the spectrum of H. For the determination of the limit the formula $f(m)=m+1.148$ is sufficient. Using this and any value of the limit obtained in the ordinary way, the more accurate value 41449.00 was found. The frequencies calculated from $41449.00+\xi-\dfrac{N}{(m+1.148)^2}$ for $m=49\ldots\ldots54$ exceed the observed by respectively -0.01, $+.00$, $-.03$, $-.03$, $+.01$, $+.00$,

[1] *Proc. Roy. Soc.*, A, **90**, 605, 1914. This agrees very closely with Rydberg's value on the Rowland scale, reduced to the international, viz., 109678.6.

or an average of $\xi-0.01$. An error of 0.01 may arise as much from residual errors of calculation in the last significant figures as from any other cause and will make no change in λ up to 0.001 A.U. and only a few units in the sixth decimal place of $f(1)$. The formulae for P_1 and P_2, using this limit and the first and third lines (the second is affected with a possible observational error of $\lambda=0.030$), are then

$$P_1 \qquad n=41449.00-\frac{N}{\left\{m+1.148088-\frac{.031215}{m}\right\}^2}$$

$$P_2 \qquad n=41449.00-\frac{N}{\left\{m+1.147256-\frac{.031126}{m}\right\}^2}.$$

The values of $d\lambda$ = calculated − observed wave-lengths are given under H in the table on p. 232. The corresponding values deduced from Wood and Fortrat's paper are given under W. F. The wave-numbers used are given in the second column.

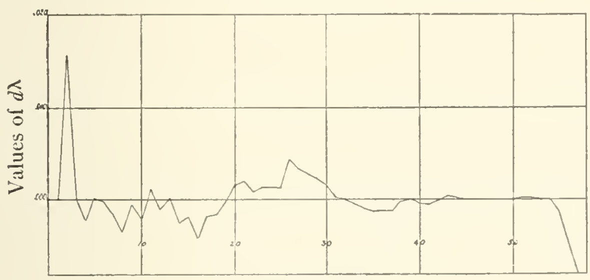
The deviations shown by the three last lines from their calculated values are so large that they can scarcely belong to the typical P series. The lines from $m=44$ to 54 agree exactly with their calculated values. Of the three in question the first and third agree with Ritz combinations, $s(1)-d(55)$ and $s(1)-d(56)$, where s, d denote respectively the sharp and diffuse sequences, $s(1)$ being the limit $P(\infty)$. Their corresponding differences from the observed lines are respectively -0.001 and $+.002$. It is not clear what relation the second has to known sequences. It differs from the calculated combination $s(1)-s(55)$ by -0.013, so that this allocation seems doubtful. They suggest that in the ionised vapor the molecular or atomic configurations which would give diffuse or sharp series up to $m=54$ do not exist, but have all transformed to p configurations. Above $m=54$, however, they have in this case not been so further transformed, and the corresponding p configurations are in consequence absent; above $m=56$ not even these are formed and no further systems capable of absorbing light of p, s, or d frequencies exist. It would be interesting to know whether a different treatment as to heat or the presence of other ionising

sources in the vapor would modify the last few lines observable in all cases, whatever the number of lines showing.

m	n	H.	W.F.	m	n	H.	W.F.
1.....	{16956.16	0.000	0.014	26.....	41300.92	0.043	0.042
	{16973.34	.000	.000	27.....	41311.13	.033	.032
2.....	{30267.38	.172	.141	28.....	41320.39	.028	.027
	{30272.87	.157	.306	29.....	41328.71	.022	.021
3.....	{35040.18	.000	−.076	30.....	41336.20	.015	.012
	{35042.66	.000	.000	31.....	41342.90	.002	−.001
4.....	{37296.23	−.035	.032	32.....	41349.18	.000	−.004
	{37297.73	−.021	−.027	33.....	41354.87	−.004	−.007
5.....	{38540.10	−.040	−.005	34.....	41360.07	−.009	−.012
	{38541.58	.002	−.018	35.....	41364.85	−.012	−.015
6.....	{39298.47	−.022	−.002	36.....	41369.31	−.012	−.014
	{39299.26	−.002	−.014	37.....	41373.53	−.012	−.010
7.....	{39793.66	−.075	−.010	38.....	41377.39	−.002	−.005
	{39794.96	−.014	−.061	39.....	41380.95	.000	−.002
8.....	40136.75	−.034	−.022	40.....	41384.15	−.004	−.006
9.....	40383.17	−.005	.003	41.....	41387.18	−.005	−.006
10.....	40565.65	−.020	−.015	42.....	41390.05	−.002	−.002
11.....	40705.74	.011	.022	43.....	41392.78	.003	.002
12.....	40814.13	−.010	−.008	44.....	41395.21	.001	.000
13.....	40900.92	.002	.004	45.....	41397.50	.000	.000
14.....	40970.46	−.025	−.022	46.....	41399.66	.000	−.002
15.....	41027.95	−.019	−.026	47.....	41401.69	.000	.000
16.....	41075.21	−.042	−.046	48.....	41403.57	.001	−.003
17.....	41115.59	−.019	−.021	49.....	41405.40	.000	.000
18.....	41149.53	−.016	−.018	50.....	41407.08	.000	.000
19.....	41178.72	−.003	−.005	51.....	41408.70	.002	.002
20.....	41203.97	.015	.014	52.....	41410.20	.002	.003
21.....	41225.71	.019	.018	53.....	41411.58	.000	−.001
22.....	41244.42	.008	.014	54.....	41412.94	.000	.000
23.....	41261.13	.013	.014	55.....	41413.99	−.013	−.014
24.....	41275.75	.012	.014	56.....	41414.63	−.046	−.047
25.....	41288.77	.012	.009	57.....	41415.24	−.079	−.079

As to the other lines, the table shows clearly that if the observational errors do not amount to more than a few thousandths of an angstrom, neither formula is sufficient to the degree of accuracy here in question, although that in a/m gives slightly better results than that in β/m^2. To produce differences of the magnitude shown, for instance, between $m=14$ and 36, the values of the denominators $f(m)$ must differ very largely from those given by the formulae, and the formulae give the results so closely as they do because m is so large and only the first few digits in $f(m)$ are required. Messrs. Wood and Fortrat point out that the deviations appear to show signs of periodicity. This can best be exhibited graphically. In the accompanying diagram the abscissae are values of m and the

ordinates of $d\lambda$. Two phenomena are clear at a glance. If regard be had to the fact that there will be some disturbance in the curve for low values of m, because the deviations for $m = 1$ and 3 are forced to be zero, there would appear to be a mean set of deviations, going through a period of about 16 in m. Superposed on these is another variation alternately positive and negative which is almost certainly real. Such cannot be represented by any simple algebraic function, although the latter effect might be represented by a term $a(-)^m$ in $f(m)$. It would seem that if $f(m)$ is a function of m alone, circular or transcendental functions are required.



But it is possible that $f(m)$ may not be a function of m. If the various frequencies in a series all arise as different modes of vibration of the same configuration, we should expect such a function to exist. If, however, each line comes from a special rearrangement of the components of an atom, this might no longer happen. For instance, with the same m (say m electrons absent) a number of rearrangements might exist, each capable of giving different spectral lines. That something of this nature may be a reality is indicated by the existence of what I have called lateral displacement.[1] These depend on multiples of the oun.[2] It would seem that the atom could not adjust itself to the exact conditions given by a function $f(m)$, but as if it could only get as near as possible by a number of discrete changes. Just as the shape of a brick wall can be adjusted

[1] "A Critical Study of Spectral Lines," Part III, *Phil. Trans.*, A, **213**, 336, 1913.

[2] The word "oun" is used by the author to designate $90.47\,w^2$, where $w = \dfrac{\text{atomic weight}}{100}$.—Eds.

to a curve $y=f(m)$ only roughly by abrupt steppings of the individual bricks, or a multiple of bricks. Here the bricks are the ouns. In any case it is clear that in a diffuse series of strong lines and satellites, the two sets cannot both be determined by functional forms of $f(m)$. In a large number of elements, the corresponding denominators $f(m)$ of the two sets (strong and satellite) differ by one number of ouns for $m=1$, and by a number different from this but the same for other values of m.[1] Moreover, it is possible in diffuse series in all cases to express the denominators so that the differences of their successive decimal parts differ by an integral number of ouns. It is interesting to see whether a similar rule is applicable to the principal series of Na. In sodium the oun is given by $4\delta_1=\delta=19.17$. Δ, which gives the doublet separation, is $743=155\delta_1$. The quantities are so small that their values can only be determined to three significant figures, and therefore it is not possible to draw any convincing conclusions. If, however, the fourth significant figure in Δ is 7, so that $\Delta=743.7$, $\delta=19.19$, the observed differences do appear to come very close to such multiples. Taking the observational errors in λ to be p in the third decimal place (p probably <2), these denominators are given in the following list from $m=1$ to 7. The numbers in parentheses denote maximum possible errors, the third column gives their differences, whilst the oun multiples are given on the right.

m.			
1.	2.116873(0)		
		15810	$21\Delta+10\delta=15809.6$
2.	3.132683(38)		
		5000	$6\Delta+28\delta=4999.5$
3.	4.137683(3.8p)		
		2317	$3\Delta+5\delta=2327.0$
4.	5.140000(7.6p)		
		1977	$2\Delta+25\delta=1967.1$
5.	6.141977(16p)		
		833	$\Delta+5\delta=839.6$
6.	7.142810(25p)		
		277	$15\delta=287.8$
7.	8.143087(39p)		

The differences 2317, 1977 differ by 10 or $2\delta_1$, in opposite directions from the multiples given. It may be due to an error of 0.0013 A.U. in the observation for $m=4$. This discussion would have been wholly fanciful were it not for the very accurate measurements of wave-length. But, as is seen, even now, the possible observational errors themselves are comparable with δ, even for

[1] *Phil. Trans.*, A, 210, 72, 1910; 213, 347, 1913; e.g., in Cd.

$m=3$, so that no certain conclusion is possible. The cases of K, Rb, Cs are, however, different, the values of δ for these being respectively 55.45, 263.77, 638.22. One main object of the present note is to draw attention to the particular importance of using the same instrument in Weiss' laboratory to extend observations to these elements. Bevan has shown that Wood's method is applicable to all the alkalies as well as to Na.

Even with Na, however, it is possible to draw an important conclusion with certainty. It refers to a curious irregularity in the differences of the denominators for corresponding lines in the P_1 and P_2 series. As these depend on close lines any observational errors are probably of about the same amount and produce little effect on their differences. I have shown[1] that whilst in many elements these differences for $m \gtreqless 2$ may be the same and considerably less than for $m=1$, in the alkalies the values for $m \gtreqless 2$ are larger. The present measures enable us to test this question for six intervals with very considerable exactitude. The result is as follows (taking as above $\Delta=743.7$).

m			
1.	743	$\Delta=155\delta_1$	
2.	769	$\Delta+5\delta_1=40\delta=$	767.7
3.	802	$\Delta+3\delta=$	801.3
4.	928	$\Delta+38\delta_1=$	926.1
5.	1563	$2\Delta+4\delta=$	1564.1
6.	1313	$2\Delta-9\delta=$	1314.6

The agreement is close, especially when it is remembered that errors of 1 or 2 units in the last place may be introduced as residuary errors due to calculations with seven-place logarithms of numbers of 7 significant figures. Again, however, on account of the smallness of δ, we can feel no absolute certainty as to the multiples of the oun. But the changes, the large change for $m=5$ and return to a smaller for $m=6$, must certainly be real, unless the list contains misprints. Both series can scarcely be represented by any simple algebraic function of m.

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[1] *Phil. Trans.*, A, 212, 33, 1912.

AN EXPERIMENTAL STUDY OF A THEORY OF THE COMPLEX ZEEMAN EFFECT

By A. E. BECKER

INTRODUCTION

Various attempts have been made to account for the complex Zeeman effect. Preston's[1] suggestion that it may be due to complexity in the parent line has since been restated by Drude[2] and Bohr.[3] Wali-Mohammed[4] also suggests that the thallium line λ 5351 is really double, as it gives six Zeeman components which can be symmetrically arranged in two triplets. Apparently he does not extend the idea to other complex types. On the other hand, Paschen[5] has shown that narrow series doublets (or triplets), whose components give complicated magnetic effects for low fields, may combine into a single normal Zeeman triplet at high fields. The writer, however, has been led by his investigations on series spectra to an explanation identical with the suggestion of Preston. Even in the face of Paschen's work it seems worth while to present the results obtained by such a study of the complex Zeeman effect.

The fundamental hypothesis is that the complex Zeeman effect is due to complexity in the structure of the parent line, which is supposed to consist of a number of components of equal or nearly equal frequency. Each of these gives rise to a Zeeman triplet of normal or nearly normal type. It is assumed that the number of components in the parent line equals the number of p-components (i.e., Zeeman components polarized in a plane perpendicular to the lines of force). Each of the latter is accompanied by two s-components (i.e., Zeeman components circularly polarized about the lines of force), exactly as in the simple effect. Such a triplet will be called a reduced triplet. It is not maintained that the p-components are necessarily at the positions of the parent-line

[1] *Dublin Society Transactions*, **6**, 385, 1898; *Philosophical Magazine*, **45**, 331 1898; *Nature*, **59**, 224, 1899.

[2] *Theory of Optics*, p. 447.

[3] *Philosophical Magazine*, **27**, 518, 1914.

[4] *Annalen der Physik*, **39**, 248, 1912.

[5] *Ibid.*, **39**, 897, 1912.

components, for in that case change of field-strength would not affect their positions, which is contrary to experimental fact. The inference is that the number of p-components is the same as the number of components in the parent line, and vice versa. Thus the application of the hypothesis leads to an investigation of the two propositions:

1. In any complex Zeeman effect the number of s-components is always twice the number of p-components.

2. These Zeeman components may be arranged in symmetrical reduced triplets, one for each p-component, of normal or nearly normal interval.[1]

APPLICATION OF THE HYPOTHESIS

Let us first study the established complex types as illustrated by Voigt,[2] a reproduction being given in Fig. 1. The continuous vertical line in the middle of this diagram indicates the positions of the parent lines. Two parallel lines are drawn at a normal distance, a, one on either side of the central one. The letters p and s denote the p- and s-components respectively. A', A, A''; B', B, B'', etc., indicate respectively the components of the reduced triplets. In any given type these triplets are all of the same interval. This is given in terms of a at the right of the types in Fig. 1.

It is now evident that type I reduces to three triplets, as it has three p-components, A, B, and C, together with the necessary six s-components, A', A''; B', B''; and C', C'', all properly placed. Type III is a triplet. It is obvious that type IV reduces to three triplets; type V, to three; type VIII, to two; type IX, to three; type X, to three; and type XII, to five.

In type VI one encounters difficulties. But Runge and Paschen,[3] who established these various types, seem to have been uncertain of the number of components in this one. It must therefore be regarded as an exception needing further experimental investigation.

Since type II has two p- and four s-components, it satisfies proposition 1. But the reduced triplets are unsymmetrical, the

[1] In this connection the reader will find the article by Ritz (*Annalen der Physik*, 25, 676, 1908) of importance.

[2] *Magneto- und Electro-optik*, p. 87.

[3] *Astrophysical Journal*, 15, 235, 1902.

interior s-components, A'' and B', and the exterior ones, A' and B'', being respectively at intervals $2a$ and $3a/2$ from their p-components, A and B. Thus type II is an exception to proposition 2. But

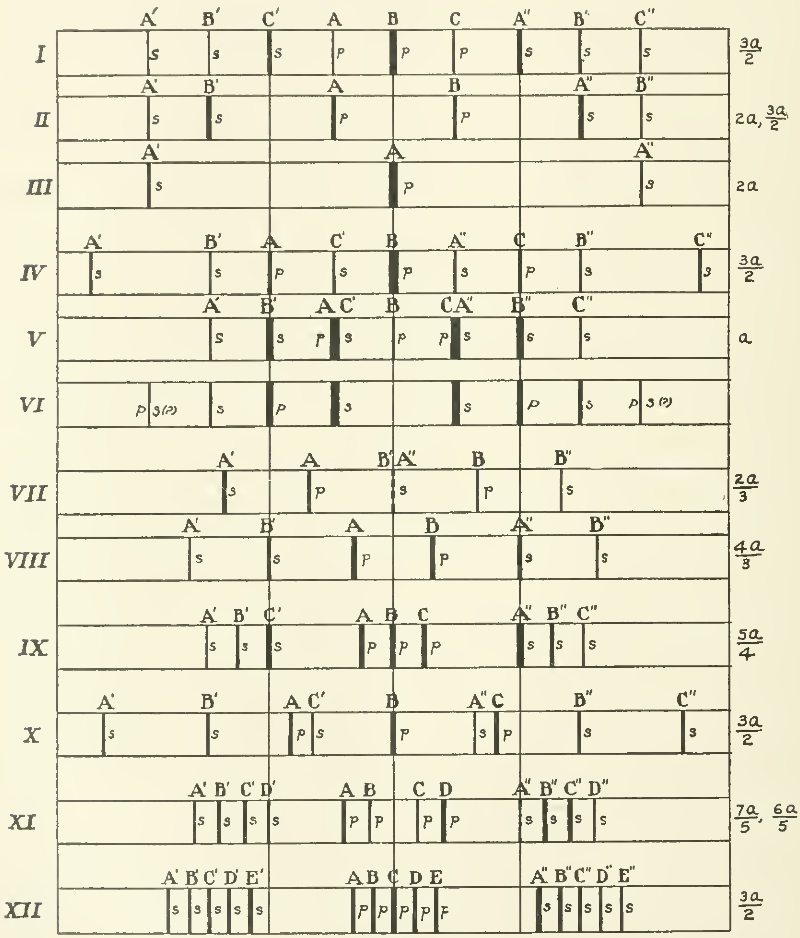


FIG. 1

it is significant that these unsymmetrical displacements are those of the reduced triplets of types III and I respectively; for, as Voigt[1] points out, any three parent lines which give rise to types I, II, and III, respectively, are closely related from the standpoint of series

[1] *Loc. cit.*

spectra. And Paschen[1] shows that series combination terms of the form (np_2-mS) give Zeeman triplets of interval $3a/2$, which is also significant, since np_2 is taken from the formula for lines giving type II. Similarly, type XI may be regarded as four unsymmetrical reduced triplets, the displacements being $7a/5$ and $6a/5$.

Type VII satisfies neither proposition. But by assuming two s-components, as yet unobserved, at the position of the parent line, it reduces to two symmetrical triplets of interval $2a/3$, which is just half that of those in type VIII. Thus the assumption has merit, since series spectra again show that parent lines giving rise to types VII and VIII are very closely allied. One would thus expect each of these parent lines to be close doublets, which is the conclusion of Michelson[2] and others for the D_1 and D_2 lines.

The assumption of the superposed s-components in type VII affords opportunity to appeal to experiment. Along the lines of force only the s-components are visible. By analogy with the simple effect, the primed s-components of the reduced triplets should be circularly polarized in one direction and the double-primed ones in the other. The interesting case is that in which a circularly right-handed polarized component falls on a circularly left-handed polarized component, as in type VII at the assumed central s-component. Such a double component should be unpolarized when viewed along the lines of force.

We have thus found that of the twelve types of Fig. 1 eight satisfy both propositions 1 and 2; two others satisfy proposition 1; another requires further experimental investigation; and the remaining one points out the method for a beautiful experimental test of the truth of the two propositions.

If the assumed s-component of type VII really exists, it is probably of weak intensity and thus rather unsuitable for examination along the lines of force. However, a study of King's[3] results for iron and titanium shows that several complex types afford opportunity for carrying out the experiment. A convenient type

[1] *Loc. cit.*

[2] *Philosophical Magazine*, 34, 290, 1892.

[3] King, "The Influence of a Magnetic Field upon the Spark Spectra of Iron and Titanium," *Carnegie Institution of Washington, Publication No. 153*, pp. 22–42.

is that illustrated in Fig. 2, where *a* represents the parent line; *b*, the *p*-components; *c*, the *s*-components at right angles to the lines of force; *d*, the *s*-components which one expects to be circularly polarized in one direction; and *e*, those in the other. That is, one expects the central *s*-component to be unpolarized along the field.

This expectation is verified experimentally, as shown by the writer's photographs, Plate VIII, Figs. 3 and 4. Another type which occurs several times in both spectra is that of Plate VIII, Fig. 5, which again has the central unpolarized *s*-component. It thus

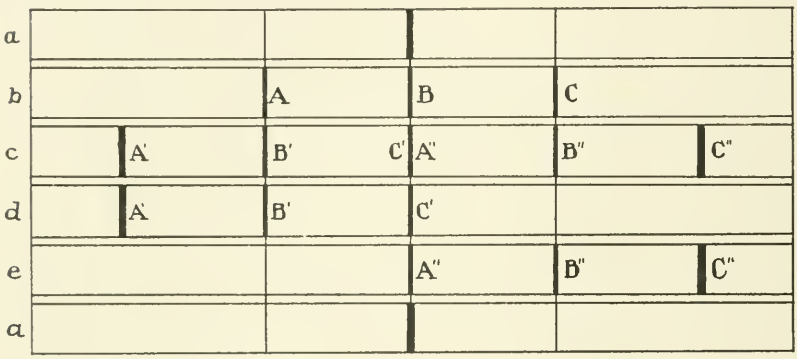
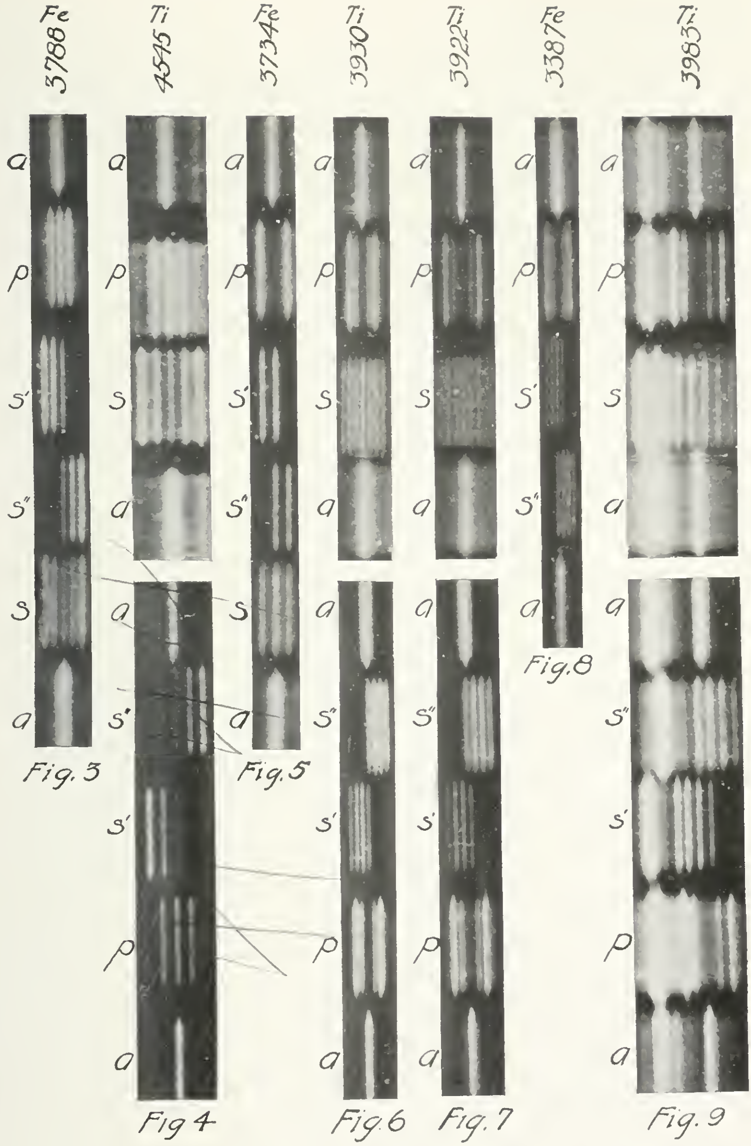


Fig. 2

has four *s*- and two *p*-components, as called for by proposition 1. Fig. 6, on the same plate, presents another type, which is seen to consist of twelve components instead of eleven, as it appears when viewed at right angles to the field. The same is true in Plate VIII, Fig. 7, except that the central *s*-components are not exactly superposed. A further verification of proposition 1 is shown by Plate VIII, Figs. 8 and 9, in each of which there are two of these unpolarized *s*-components, thus giving eight *s*- and four *p*-components. Becquerel and Deslandres[1] had already discovered the unpolarized *s*-component in Plate VIII, Figs. 3 and 5, but the writer was able to predict that there would be two of these unpolarized *s*-components in cases such as shown in Plate VIII, Figs. 8 and 9. There are several other types, especially in titanium, which in the same

[1] *Comptes rendus*, **127**, 18, 1898.

PLATE VIII



a = No magnetic field. p = p-components. s = s-components. s' = s-components circularly polarized in one direction. s" = s-components circularly polarized in other direction. Field strength = 23,800. Scale of about 1 Angstrom per cm.

way verify the hypothesis that there are twice as many *s*- as *p*-components in every complex Zeeman effect.

To test the hypothesis in another way, the writer repeated the experiments of King.[1] A field of 23,800 gausses was used, the investigation extending from λ 5050 to λ 3230 for iron and titanium. Some of the complex types found do not agree with King's results. In titanium the region common to both investigations is from λ 5050 to λ 3659, for which King records 165 Zeeman triplets, 23 lines showing complex effects, 1 unaffected line, and 133 lines whose Zeeman effect was doubtful. For the same 322 lines the writer finds 157 Zeeman triplets, 45 lines showing complex effects, 1 unaffected line, and 119 lines of a doubtful number of Zeeman components. Thus the writer finds a larger number of lines of complex Zeeman effect, in which the components are photographically well-defined and sharp, than does King.

Between λ 5050 and λ 3230, the writer finds in titanium 67 lines showing complex Zeeman effects of unquestionable character. Of these lines 62 have twice as many *s*- as *p*-components and therefore satisfy proposition 1. Of the remaining 5, 3 are apparently quartets, 1 an octuplet, and the other a quintuplet having but one *p*-component. This latter may be regarded as two reduced triplets in which the *p*-components for some reason do not separate when the field is applied. There is also the line λ 4296 which is apparently unaffected by a magnetic field. Careful measurements show that of the 62 lines of complex Zeeman effect which satisfy proposition 1, 70 per cent can be reduced to symmetrical triplets and thus satisfy proposition 2. Similar results are obtained for the iron spectrum.

APPARATUS

To carry out the experimental work a large concave grating of 21-foot focus, 20,000 lines per inch, and a 6-inch ruled surface was used. The form of mounting is that of Paschen at Tübingen, the grating and slit being fixed. The spectrum can be examined from λ 2000 in the first order to λ 7000 in the second; thus photographs may be taken simultaneously in both orders. The grating remains

[1] *Loc. cit.*

in perfect adjustment for long periods of time, it now having been used for more than a year without readjustment. The temperature of the grating-room remains so constant that exposures of several hours are successfully made.

The source of light was an electric spark. A condenser of variable capacity from 0.01 to 0.06 m.f. was used in the oscillatory circuit, the spark length being about half a centimeter. The use of self-induction was avoided.

The earlier experiments were made with an electromagnet giving a field of 14,000 gausses with a gap of 8 mm. A Weiss electromagnet is now in use which gives a field of 23,800 gausses for a gap of 12 mm. This is especially convenient, since it can be rotated so that the spectrum may be examined along the lines of force or at right angles to them.

At right angles to the field a nicol prism is used to separate the *p*- and *s*-components. Along the field a Fresnel rhomb and a nicol prism are used to separate the two kinds of circularly polarized components. All of these components, as well as comparison spectra, may be obtained on the same negative by means of a slit diaphragm placed on a separate holder immediately in front of the plate-holder.

SUMMARY

Some work on series spectra, which the writer hopes to publish soon, led him to suspect that certain spectral lines are in reality close doublets, and that therefore they may be expected to show a complex Zeeman effect consisting of two *p*- and four *s*-components. This is indeed found to be the case. Conversely, the writer conceived the idea, independently of other investigators, that complex Zeeman effects occur because spectral lines which give rise to more than three Zeeman components are themselves complex in structure, and that the number of components in such a parent line is exactly equal to the number of *p*-components which it shows when placed in a magnetic field.

Investigation has shown that, in the titanium spectrum between λ 5050 and λ 3230, 62 out of 67 lines of complex Zeeman effect have twice as many *s*- as *p*-components; and that at least 70 per

cent of these may be reduced to symmetrical Zeeman triplets, one for each *p*-component. In each complex type the reduced triplets are all of the same interval, which varies, however, from type to type in the same manner as for ordinary Zeeman triplets. Further study led to the prediction that in certain cases there are present one or more *s*-components which should appear unpolarized when viewed parallel to the field. This has been verified experimentally.

Several important questions still remain to be solved. One is the reason for the unsymmetrical distribution of the intensities which frequently occurs in the components of the reduced triplets. Another is that of the separation of the *p*-components when the magnetic field is applied. A third is that of the abnormal Zeeman intervals. But the same fundamental cause which gives rise to ordinary abnormal Zeeman triplets is probably responsible for the abnormal intervals of the Zeeman triplets to which the complex types may be reduced.

In conclusion, the writer wishes to recognize the valuable help of Professor Theodore Lyman, of Harvard University, under whose supervision this work has been done. His many suggestions, particularly as to the best method of presenting the subject, have been invaluable.

Jefferson Physical Laboratory
Cambridge Mass.
October 1916

THE NATURE OF THE CONSTANT-ERROR TERM K

SECOND PAPER

By C. D. PERRINE

In my first paper on this subject[1] the question was raised whether the term K, found in the solution for solar motion from radial velocities, was in fact constant. Some later evidence makes it desirable to consider this matter again. Unfortunately this is a question which cannot be answered directly by obtaining positive evidence as to whether there is or is not a pressure-effect in the atmospheres of any of the stars of a nature to cause displacements of the spectral lines, but must be attacked more or less indirectly, at least for the present. The solar motion, general systematic motion of the stars, and constant error are so linked together that it appears impossible completely to separate any one. It may be possible later to obtain a consistent apex for the solar motion from proper motions. The velocity of the system, which is really the important factor, would still be in doubt, however.

It may be pointed out in further explanation that in the solutions of the radial velocities for apex and velocity of solar motion there is usually an excess which cannot be represented by the rigid spherical relations existing between the coefficients of the three unknowns in the equations of condition. From the nature of the observed velocities a perfect solution is scarcely to be expected. In the spectral classes B, K, and M these excesses are large and have the same algebraic sign. The introduction into the equations of a constant term for each star or similar one independent of the spherical relations referred to will absorb the excess found. The evidence for such a constant-error term appears to rest wholly upon the excesses mentioned above in solutions for solar motion.

In my first paper the results were derived by freeing the stars from the solar motion on the basis of common apex and velocity, in that case approximately those derived from the same system

[1] *Astrophysical Journal*, 43, 286, 1916.

of stars. While this is probably justifiable in fact as evidence, in this particular instance it does not seem as logical or strong as to make separate solutions in each spectral class for apex and velocity of solar motion and K term simultaneously. The desirability of doing this was accentuated by the finding of differences in the position of the apex in the different spectral classes. Such results are, then, the best representation which can be obtained from the data, and the values of the constant-error term are directly comparable with values derived in a similar way from all of the stars.

The finding of systematic differences in the proper motion of the stars of Class B in the northern and southern sky, as well as of apparent peculiarities in the radial velocities in these two regions, led to a more careful investigation of these regions, in connection with the question of the constant error.

In a paper[1] giving the results of separate determinations from northern and southern stars and according to proper motions, the values of K were uniformly small when obtained from these two regions separately. This again is not entirely independent evidence. If, however, it can be shown that certain groups of stars or those in certain regions have produced the excesses giving rise to the K term, in general solutions from all parts of the sky, such evidence is independent and of great weight, as such values have resulted in exactly the same way as the original (large) values of K.

It was found that there were large excesses of positive velocities and of stars in the southern sky in the classes B, K, and M (those yielding considerable values for K), whereas there was practical equality in the classes A, F, and G.

Further investigation by correcting the different spectral classes by *regions* for a common solar motion of 19.5 km toward $\alpha = 270°$, $\delta = +30°$, showed that there are large systematic velocities in regions of considerable size, that the same general preferences of these excesses are found in all of the spectral classes for the northern sky and generally for the southern sky also in the region from right ascension 18^h to 4^h, but that in the southern sky from approximately 4^h to 18^h there is a radical difference. In this region large negative velocities are all but universal in the types A, F, and G,

[1] *Astrophysical Journal*, 44, 103, 1916.

whereas in the types B, K, and M, negative velocities are either almost entirely wanting or they are restricted to smaller regions. The residual velocities of these type areas are of considerable size and uniformity. The positive residuals appear to increase from the B stars to the K and M stars. They appear to lie in general on great circles, but not parallel to the Milky Way. These peculiarities will be dealt with in a separate paper. For our present purpose it seems only necessary to say that their size and the general correctness of the assumption as to elimination of solar motion make it seem unlikely that they are fictitious. This conclusion is further strengthened by the fact that the very high velocities of all spectral classes (which were not used in these solutions) show somewhat similar distributions. It was found that in these regions the large positive velocities usually fell above a fairly well-defined limit and could be approximately isolated by that means. The criterion was adopted, therefore, with the lower included limits as follows:

B ....+ 6 km
K ....+ 11
M....+ 15

Only two or three exceptions to these limits occurred, and these were either accidental or, in one case, due to the fact that a region was somewhat isolated. The regions rejected are given in Table I. v is the residual velocity after correction for the solar motion toward $\alpha=270°$, $\delta=+30°$. The B stars were corrected for a solar velocity of 20 km, and the K and M stars for 19.5 km. These regions (averaging about a third of the total) were then rejected and solutions made from the remaining ones for the apex and velocity of solar motion and constant error. The results are given in Table II.

The effect of omitting these large positive regions in these three classes is very marked. The constant-error term has practically disappeared and the values of the solar motion have been brought into much better agreement with the values from Classes A, F, and G. There has been an improvement also in the agreement of the declinations of the apex.

It may be argued that the use of a given set of elements for the solar motion and the rejection of those most divergent will of necessity tend to a closer agreement with the adopted elements. While

TABLE I

$3^M.0$ AND FAINTER

Spectral Class	α	δ	No. of Stars	Obs. V.	v
				km	km
B	$4^h 16^m$	$+64°$	2	+ 6	+ 8
	16 47	+43	4	−12	+ 7
	18 12	+23	4	−10	+10
	22 31	+14	3	+ 6	+14
	5 54	− 9	14	+25	+ 7
	6 0	−68	2	+22	+ 6
	7 3	−25	4	+33	+14
	9 11	−63	4	+23	+ 9
	10 0	− 7	2	+19	+ 9
	13 6	−47	15	+12	+ 9
	14 46	−45	9	+ 6	+ 6
K	1 15	+15	6	+20	+17
	4 30	+16	6	+23	+11
	8 58	+16	7	+23	+14
	8 58	+74	2	+ 4	+10
	9 0	+40	3	+17	+14
	10 43	+ 6	2	+16	+12
	19 9	+68	3	+21	+36
	20 57	+26	4	0	+15
	3 17	−70	4	+27	+14
	5 9	−69	5	+29	+14
	6 50	−69	5	+31	+16
	7 4	−44	8	+34	+16
	8 36	−68	4	+26	+12
	9 3	−18	7	+30	+16
M	4 52	+40	3	+23	+17
	10 32	+16	2	+20	+17
	22 26	+53	2	+ 9	+21
	3 28	−17	2	+45	+30
	7 2	−46	2	+38	+20
	8 15	−27	2	+32	+15
	11 28	−31	1	+22	+15
	13 8	− 8	2	+18	+22
	19 56	−28	1	+14	+22
	20 50	−19	3	+ 8	+16

this is undoubtedly true, a more important consideration would seem to be the proportion of such rejections and the nature of the observations omitted. In the present case, the regions (and individual stars) rejected constitute a third or less of the total in

each of the three classes. The residuals in these rejected regions after correction for solar motion, in each class, show considerable uniformity in size, an increase from class to class, and, perhaps most important of all, average from two to four times the size of the constant error found from the solutions. A conclusion which rests upon only one-third of the data can scarcely be considered as established. No one would think, for example, of proposing as characteristics an increase of velocity with increasing age, or a magnitude-velocity equation, or almost any other, if it were

TABLE II*

	A	D	$V_\odot$	K	No. of Stars	No. of Equations
			km	km		
B {3.M0 and fainter—all..	276°0	+29°6	−20.3	+4.0	193	31
B {Rejecting 11 regions.	274.1	+28.5	−19.4	+1.2	130	20
K {3.M0 and fainter—all..	274.2	+25.6	−20.7	+3.6	391	63
K {Rejecting 14 regions.	268.1	+30.5	−17.7	+0.1	325	49
M {3.M0 and fainter—all..	269.7	+31.7	−22.9	+4.0	65	31
M {Rejecting 10 regions.	265.4	+31.9	−18.8	−3.2	45	21
2.M9 and brighter—All spectral classes ...	258.0	+41.5	−18.9	+1.0	110	33

*The results in this table depend upon Campbell's catalogues of radial velocities.

Velocities of 50 km and over in class K, and 40 km and over in class M, were omitted.

Each equation of condition is the mean of all the stars in a region 2^h in right ascension by 30° in declination.

known that two-thirds of the data failed to show it. The constant error in question seems to be no exception. I see no reason why this conclusion as to the sufficiency of evidence should not apply to the question of a constant error, whether there are any peculiarities of distribution or not—that even without any other evidence we should be fully justified in concluding that the excesses observed cannot be ascribed to a constant error. In any case the burden of proof is certainly upon the constant-error explanation. But there are other points bearing upon its reality as a constant error:

1. There is undoubted system in the distribution of these large residual velocities.

2. The assumption of a constant error does not reconcile the values of the solar velocity derived from the different spectral classes.

3. The brighter stars by themselves show only a very small excess although more than half belong to the spectral classes B, K, and M.

4. There is strong evidence that the general motions of the stars of different spectral and other classifications are different. It therefore seems not improbable that the observed residuals are nothing more than motion and may be related to these differences of general motion.

5. It seems more natural and reasonable to ascribe the residuals observed to motion, which we know exists, than to a physical cause of which there is no direct evidence whatever, even if there were considerable uniformity in those excesses.

The system of 1300 stars upon which these investigations are based yields an apex differing 5° from that used in the elimination of the solar motion for the purpose of isolating the peculiar regions. The application of the correction for this difference of apex and the constant error of $+1.9$ km from all of these stars would not obliterate the large values which have been rejected—those peculiarities would stand out almost as strikingly as they do without such corrections. The very fact that the values of the solar velocity and of the K term from all of the stars are a compromise of the values found from the spectral classes separately is further evidence that constancy is not necessarily implied but must be proven quite independently of the solutions.

The foregoing results appear to be sufficient, without discussing other evidence, to justify the conclusion that the observed residuals in the solutions for solar motion from radial velocities are not sufficiently uniform to be treated as a constant error, but are much better explained by motion.

Observatorio Nacional Argentino
Córdoba
August 2, 1916

ON THE ORBITS OF THE SPECTROSCOPIC BINARIES α ORIONIS AND α SCORPII

By JOSEPH LUNT

These two stars α Orionis and α Scorpii have the peculiarity of showing the smallest range of variation of radial velocity of any of the spectroscopic binaries for which orbits have been derived, the semi-amplitude of the velocity-curve being only 2.45 km per sec. in the case of α Orionis according to Bottlinger's elements,[1] and 2.12 km according to Halm's elements[2] in the case of α Scorpii. As these quantities are comparable with the accidental errors of the observations, it is of interest and importance to examine the evidence on which the orbits are based and to supplement it by measures of a large number of plates taken at the Cape, of which duplicate measures have, in most cases, been made by Halm and myself. It is desirable to call attention to the fact that in spite of the large number of determinations of velocity of these two stars which have been made, it is still necessary to continue the accumulation of material, as the orbits—particularly in the case of α Scorpii—are obviously in need of revision.

Ludendorff called Bottlinger's attention to the similarity of the orbits derived for these two stars, and the latter states that they are the only spectroscopic binaries of Class III*a* of which the elements are known.

The case of α Orionis is of further interest from the fact that Bottlinger has attempted to connect the variations of radial velocity with the variations of brightness of the star and suggests a relationship similar to that found to exist in the case of stars of the δ Cephei type, noting that the period and type of spectrum are very different.[3] Charles P. Olivier,[4] from a discussion of 293 observations of brightness extending over a period of nearly eleven

[1] *Astronomische Nachrichten*, 187, 33, 1911.

[2] *Annals of the Cape Observatory*, 10, Part III, 56*c*.

[3] *Astronomische Nachrichten*, 187, 33, 1911. [4] *Ibid.*, 194, 81, 1913.

years, from 1901 to 1912, concluded that "it was quite impossible to find any regularities in the light-changes." On plotting his observations together with the velocity-determinations and theoretical velocity-curve, no connection appears to exist, and the observations negative the idea of a relationship similar to that found in the case of δ Cephei and other analogous stars.

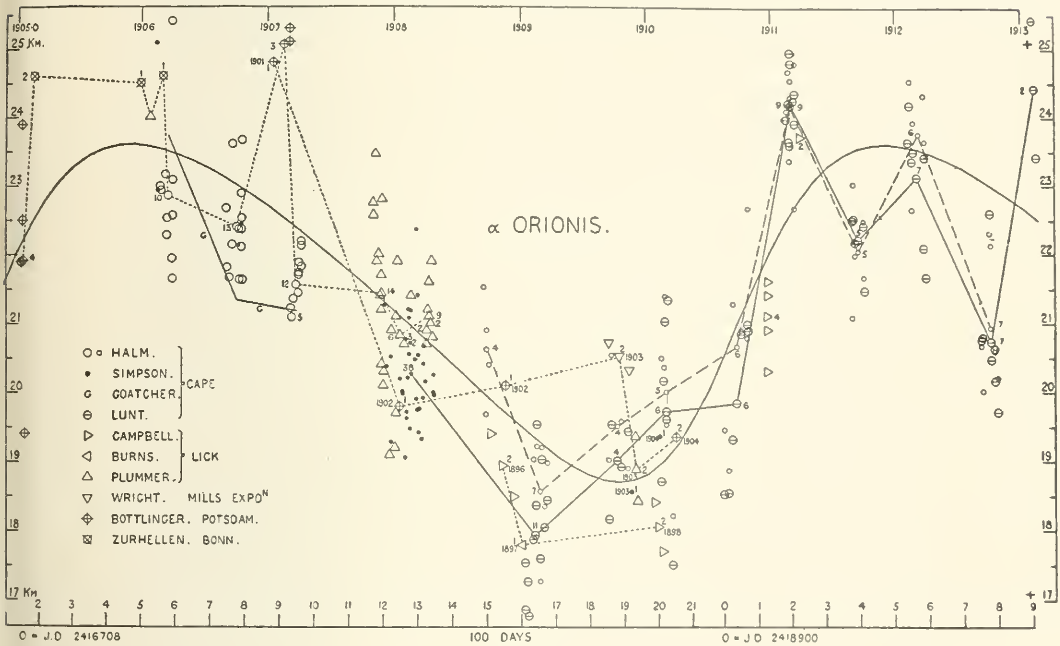


FIG. 1

In order to exhibit the facts as regards the determination of the orbits in a graphic manner, two charts have been prepared which clearly show the present position of the problem and indicate what further work is necessary in order to arrive at a satisfactory solution. The original charts were made on millimeter-squared paper on a large scale, viz., 1 cm = 0.2 km, 1 cm = 40 days, and were reduced photographically to a convenient size for the pages of the *Journal*.

α ORIONIS

In Fig. 1 the sinuous line represents the theoretical velocity-curve derived from the elements by Bottlinger. The individual

observations on which the orbit is based are shown, and the means of each group are connected by the fine broken line, the number of plates in each group being denoted by numerals at each of the connected points. The observations of different observers are differentiated in the diagram and they are divided into three periods, viz., 1896–1898, 1901–1904, and 1905–1913. The observations in the first two periods are denoted by dates at the mean points in the diagram. To the rest of the observations the dates at the head of the diagram and the 100-day scale at the foot of the diagram apply. Included in the additional observations here given are 38 measures by Simpson[1]—published after the appearance of Bottlinger's paper—from plates taken at about the same time as Plummer's series. Halm's measures, mainly duplicating those of the same plates measured by me but using a different standard plate, are indicated by the coarse broken line from 1908 to 1912, my measures being indicated by a continuous line. The continuous line 1906–1907 represents the means of Goatcher's measures,[2] on the Zeiss comparator, of the same plates measured by Halm on the Hartmann comparator.

It is apparent from the diagram that no point on the velocity-curve can be satisfactorily fixed by the measures of only one or two plates, and that Bottlinger's determinations of the important maximum and minimum points of the curve are weak, inasmuch as they depend on scanty and non-homogeneous material. The later observations, depending on a larger number of plates, indicate that the minimum is earlier and lower than Bottlinger's orbit demands and that the maximum is earlier and higher. Whether the pronounced departures from simple orbital motion which are so strikingly shown in the duplicate measures of the same plates by Halm and myself are real, and analogous to those regarded as established by Campbell in the case of ζ Geminorum, or whether they are instrumental in origin, is a point which can be settled only by further observations, preferably at different observatories with different instruments.

It is evident from Bottlinger's measures of the Potsdam plates, from Plummer's measures of the Lick plates, and from measures of

[1] *Annals of the Cape Observatory*, 10, Part III, 91*c*. [2] *Ibid.*, 10, Part I, 54.

the Cape plates by various observers that isolated observations are of little value in determining orbits of binaries showing such small amplitude, but groups of ten or twelve plates a few months apart, particularly at maximum and minimum, are much needed. Plummer's series of measures of Lick plates and Simpson's of Cape plates at about the same epoch afford a good opportunity of comparing results and indicate a difference Lick − Cape of 0.64 km.

In the period from December 17, 1907, to April 23, 1908:

Plummer 17 plates	= 20.88 km	Mean phase 1291^d
Simpson 36 "	= 20.24 "	" " 1286^d

In determining orbits of stars such as these, with such a small variation in velocity, there is some danger in combining heterogeneous material from different observatories when corrections have to be applied to bring them into agreement.

Campbell records that fifteen observations by Belopolsky at Pulkowo in 1898 and 1899 on the average are 6 km above his values for ζ Geminorum. Bottlinger applied a correction of 2 km to the Potsdam and Bonn plates, but Küstner and Zurhellen, from observations of the starlike mountain peaks near the lunar terminator, arrive at a correction of only 1 km. Zurhellen's values would therefore be 1 km higher than is shown in the diagram. It is not considered advisable to attempt to revise Bottlinger's elements until further observations are secured.

The observations are given in Table I.

The measures of forty-four plates by Simpson are published in the *Annals of the Cape Observatory*, **10**, Part I, 54. The six measures of three Lick plates, 1910–1911, are published in *Lick Observatory Bulletin*, **6**, 144. One plate taken on December 21, 1910, was measured independently by Miss Hobe and Messrs. Burns, Wilson, and Young, and two plates taken on March 27 and 28, 1911, were measured by Miss Hobe. In the diagram these are marked as Campbell plates.

The Cape measures were made on a Hartmann spectrocomparator. Halm used plate 3453 of α Orionis as standard plate, the shift of which has been taken provisionally as +45.81 km, the mean of three comparisons with the plate 2145 of α Tauri used as

TABLE I

α ORIONIS

Plate No.	Date	Phase*	Radial Velocity		L.—H.
			Lunt	Halm	
			km	km	km
1901	1908 Sept. 14	1491[d]	....	+21.52	
1909	21	98	....	19.68	
1913	22	99	....	20.90	
1922	30	1507	....	20.40	
	Mean, 4 plates....	1499	....	20.62	
2117	1909 Jan. 13	1612	+17.55	....	
2123	14	13	16.87	....	
2146	21	20	17.28	....	
2159	23	22	16.77	....	
2207	Feb. 6	36	17.88	19.03	
2226	15	45	18.37	17.92	
2229	16	46	19.53	19.22	
2244	25	55	17.60	17.28	
2258	Mar. 2	60	19.03	19.20	
2271	10	68	18.05	18.35	
2282	19	77	18.44	18.98	
	Mean, 11 plates....	1641	17.94	....	
	Mean, 7 plates....	1655	....	18.57	−0.63
2501	1909 Sept. 15	1857	18.17	19.02	
2515	22	64	19.53	20.54	
2542	Oct. 20	92	18.92	19.58	
2566	Nov. 9	1912	19.43	18.90	
	Mean, 4 plates....	1881	19.01	19.51	−0.50
2675	1910 Feb. 15	2010	18.71	20.49	
2683	22	17	20.16	20.36	
2689	25	20	21.02	21.39	
2694	Mar. 1	24	19.60	19.53	
2705	5	28	21.33	....	
2707	19	42	17.51	18.21	
	Mean, 5 plates.....	2023	....	20.00	
	Mean, 6 plates.....	2024	19.72	....	−0.28
2796	1910 Aug. 19	3	18.52	19.45	
2806	Sept. 2	17	18.54	18.87	
2818	13	28	19.31	21.28	
2840	Oct. 6	51	20.84	20.89	
2852	25	70	20.98	20.79	
2855	28	73	20.89	22.67	
	Mean, 6 plates.....	40[d]	+19.85	+20.66	−0.81

*Zero phase is taken as J. D. 2416708 = 1904 August 15, and the period as 2192 days.

TABLE I—*Continued*

Plate No.	Date	Phase	Radial Velocity		L.—H.
			Lunt	Halm	
			km	km	km
3008	1911 Feb. 16	184^d	+23.97	+24.09	
3021	22	190	24.20	24.67	
3024	24	192	23.64	24.03	
3027	25	193	23.59	23.36	
3029	27	195	24.95	24.54	
3032	Mar. 1	197	24.79	24.29	
3042	8	204	24.24	24.94	
3048	11	207	23.90	22.67	
3049	13	209	24.34	24.78	
	Mean, 9 plates	197	24.18	24.15	+0.03
3260	1911 Aug. 28	377	22.49	21.08	
3265	31	380	22.51	23.02	
3268	Sept. 5	385	22.17	21.98	
3283	29	409	22.41	22.47	
3290	Oct. 2	412	21.47	21.66	
	Mean, 5 plates	393	22.21	22.04	+0.17
3453	1912 Feb. 6	539	23.63	Standard	
3470	12	545	24.17	24.53	
3480	17	550	23.35	22.65	
3489	21	554	23.49	23.92	
3499	23	556†	26.94†	27.36†	
3547	Mar. 23	585	22.09	24.32	
3553	25	587	23.41	23.65	
3567	29	591	21.67	23.45	
	Mean, 7 plates	564	23.12	....	
	Mean, 6 plates	569	....	23.75	−0.63
3796	1912 Sept. 6	752	20.77	20.67	
3803	10	756	....	20.02	
3825	29	775	20.80	22.31	
3830	30	776	22.60	....	
3835	Oct. 4	780	20.48	22.14	
3847	14	790	20.64	20.60	
3849	15	791	20.17	20.66	
3864	23	799	19.72	20.21	
	Mean, 7 plates	780	20.74	....	
	Mean, 7 plates	778	....	+20.94	−0.20
3969	1913 Feb. 3	902	25.43	....	
3973	14	913	23.42	....	
	Mean, 2 plates	907^d	+24.43	....	

† Plate rejected.

standard plate by Lunt. The plate of α Tauri has a shift of +77.38 km, as determined by comparison with twenty solar plates. Halm's measures may require a further correction when the shift of his standard plate is better determined.

α SCORPII

The diagram of velocities, Fig. 2, is self-explanatory and the same general remarks apply as in the case of α Orionis. Halm's

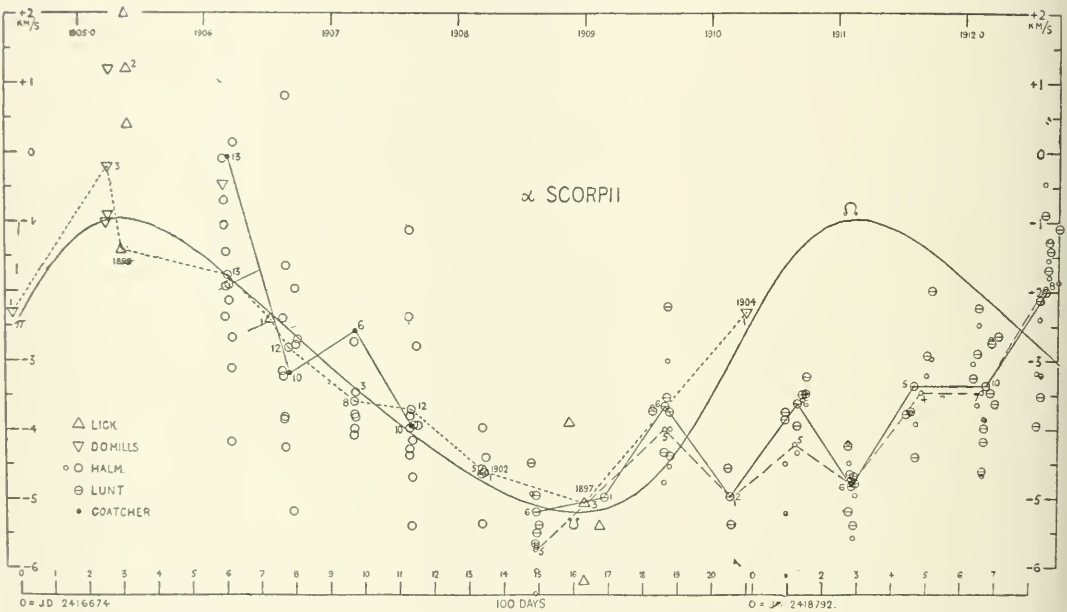


FIG. 2

published measures are confined to a part of the downward branch of the velocity-curve covering a period of a little over two years and comprising fifty plates. The individual measures of groups of twelve or thirteen plates show fluctuations which in range exceed the total variation of velocity required by the orbit derived, and show that measures of isolated plates are of little or no value in fixing a point on the velocity-curve. The remaining and major part of the velocity-curve depends on only nine plates, five secured at Lick and four by the Southern Mills Expedition at Santiago. The apex of the curve depends on approximate measures of three

TABLE II

α Scorpii

Plate No.	Date	Phase*	Radial Velocity		L.–H.
			Lunt	Halm	
			km	km	
1786	1908 July 30	1479ᵈ	–4.48	–4.93	
1807	Aug. 10	1490	5.65	6.38	
1813	11	91	5.18	5.70	
1816	12	92	4.95	5.65	
1822	13	93	5.50	6.05	
1846	21	1501	5.38	Standard	
	Mean, 5 plates	1489	....	5.74	
	Mean, 6 plates	1491	5.19	....	+0.55
2247	1909 Feb. 25	1689	5.38†	....	
2414	July 15	1829	3.73	3.67	
2456	Aug. 17	62	4.33	4.76	
2464	26	71	3.53	....	
2468	27	72	2.22	3.00	
2475	Sept. 2	78	3.74	4.53	
2481	3	79	4.38	3.99	
	Mean, 5 plates	1864	....	3.99	
	Mean, 6 plates	1865	3.66	....	+0.33
2682	1910 Feb. 20	2049	4.56	....	
2698	Mar. 1	2058	5.38	4.97	0.00
	Mean, 2 plates	2054	4.97	....	
2774	1910 Aug. 2	94	3.85	4.49	
2777	3	95	3.74	5.22	
2809	Sept. 6	129	3.94	4.33	
2825	21	144	3.49	3.56	
2831	29	152	3.48	3.47	
2835	Oct. 4	157	3.23	....	
	Mean, 5 plates	123	....	4.21	
	Mean, 6 plates	129	3.62	....	+0.59
2969	1911 Jan. 31	276	4.24	4.19	
2975	Feb. 2	278	5.19	4.49	
2990	6	282	4.65	4.72	
3003	15	291	5.39	5.57	
3010	16	292	4.68	4.74	
3014	20	296	4.78	4.95	
	Mean, 6 plates	286ᵈ	–4.82	–4.78	–0.04

* Zero phase taken as J.D. 2416674 = 1904 July 12, and the period as 2118 days.

† Radial velocity 4.98 in diagram in error.

TABLE II—*Continued*

Plate No.	Date	Phase	Radial Velocity		L.–H.
			Lunt	Halm	
			km	km	km
3231	1911 July 18	444^d	−3.77	....	
3240	31	457	3.73	−3.74	
3245	Aug. 14	471	4.40	3.91	
3278	Sept. 15	503	2.92	3.22	
3280	29	517	1.99	2.97	
	Mean, 5 plates	478‡	3.36	....	
	Mean, 4 plates	487	....	3.46	+0.10
3443	1912 Jan. 29	639	3.25	3.04	
3462	Feb. 8	649	2.89	3.64	
3473	12	653	2.23	2.48	
3495	22	663	4.60	4.67	
3505	27	668	4.17	3.84	
3508	28	669	3.97	3.85	
3542	Mar. 17	687	3.46	....	
3545	21	691	2.74	2.68	
3568	31	701	3.62	....	
3570	April 8	709	2.64	....	
	Mean, 7 plates	662	....	3.46	
	Mean, 10 plates	673	3.36	....	+0.10
3739	1912 July 29	821	3.94	3.18	
3756	Aug. 7	830	2.12	2.40	
3763	10	833	3.51	3.21	
3768	19	842	0.90	0.45	
3782	30	853	1.69	1.55	
3789	Sept. 2	856	1.29	1.80	
3792	6	860	1.42	1.25	
3827	30	884	1.09	1.87	
	Mean, 8 plates	847^d	−2.00	−1.96	−0.04

‡ Phase shown as 465 days in error.

Santiago plates—mean, −0.2 km per sec.—and one Lick plate belonging to a previous period. Two other Lick plates which place the observed maximum at +1.2 km have been left out of account, but if these are included and the revised[1] values of the three Santiago plates are used, the mean of the five 1905 plates is 1.09 km above the theoretical maximum of the curve, and the two Lick plates place it 2.16 km above the curve, leaving the maximum very uncertain. The minimum of the curve depends on three Lick plates taken in 1897, and the period depends mainly on the

[1] *Publications of the Lick Observatory*, **9**, 257.

coincidence in velocity between one Lick plate at 1902.4 and five Cape plates at 1908.2, giving a period of 5.8 years. On the ascending branch of the curve there is only a single Lick plate.

The velocity-curve derived from measures of fifty plates from July, 1908 to October, 1912—subsequent to the first series measured by Halm—does not follow the course prescribed for it by the orbit, but indicates that the maximum of the curve had not then been reached. It indicates that the period must be lengthened very considerably and that a satisfactory orbit cannot be derived until the observations are extended to give a determination of the apex of the curve. In view of the small range of variability it is desirable to secure observations in groups of plates, say ten to twelve, at intervals of four to six months, and that the observations should be duplicated at different observatories.

The observations are given in Table II.

Halm used Plate 1846 of α Scorpii as standard plate.

Lunt used Plate 2145 of α Tauri as standard plate.

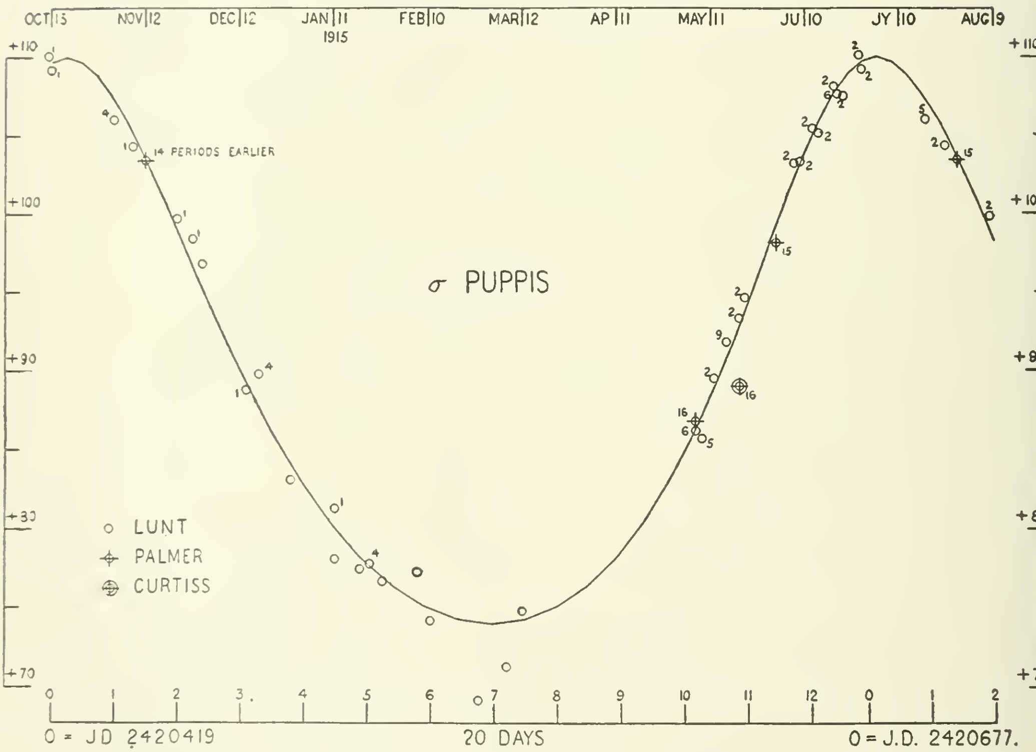
Shift of 1846 taken as +23.43 km, the mean of four comparisons with plate 2145. Shift of 2145 taken as +77.38 km, the mean of comparisons with twenty solar plates. Halm's measures may require a further correction when the shift of his standard plate is better determined.

Royal Observatory
Cape of Good Hope
April 1916

ON THE ORBIT OF THE SPECTROSCOPIC BINARY σ PUPPIS

By JOSEPH LUNT

The variable radial velocity of this star (for which $\alpha = 7^{h}26^{m}.1$, $\delta = -43^{\circ}6'$ [1900]; magnitude, 2.99; type, K5M) was announced in *Lick Observatory Bulletin*, No. 75 (3, 111) as detected by Dr. Palmer



in measures of 4 plates taken by the D. O. Mills Expedition, Santiago, in 1904.

Thirty-three plates of the spectrum of this star were obtained at the Cape between January 4, 1909, and March 11, 1915. The measures of these confirm the variability, and suffice for a prelim-

inary determination of the orbit, which appears not to have been published previously.

Table I gives the observations, in which the four Santiago plates[1] are included. The diagram shows the observations compared with the theoretical velocity-curve computed from the following elements derived by graphical methods:

TABLE I

Plate No.	Date	Radial Velocity	Plate No.	Date	Radial Velocity
		km			km
	1904 Jan. 15*	+ 86.8	4207......	1914 Jan. 22	+107.5
	29*	89.0	4215......	27	110.1
	Oct. 25*	98.1†	4216......	28	109.2
	Dec. 22*	103.4	4236......	Feb. 23	104.3
2101......	1909 Jan. 4	91.8	4250......	Mar. 9	99.7
2991......	1911 Feb. 7	86.2	4257......	14	98.4
3082......	Mar. 25	107.6	4273......	31	88.8
3323......	Oct. 25	85.7	4296......	Apr. 28	78.1
3406......	1912 Jan. 5	106.0	4455......	Nov. 30	96.5
3485......	Feb. 20	89.8	4468......	Dec. 28	83.1
3556......	Mar. 26	77.8	4471......	1915 Jan. 11	81.3
4172......	1913 Dec. 11	89.5	4476......	19	77.6
4180......	19	93.3	4482......	26	76.7
4182......	21	94.6	4489......	Feb. 4	77.0
4190......	1914 Jan. 6	103.2	4494......	10	74.2
4193......	8	103.3	4506......	25	69.2
4199......	12	105.4	4513......	Mar. 6	71.3
4200......	14	105.1	4519......	11	74.8
4206......	19	108.1			

* Santiago plates.

† Correction +1.1 km applied.

$$
\begin{aligned}
P &= 258 \text{ days} \\
T &= \text{J.D. } 2420419 \\
\omega &= 350^\circ \\
e &= 0.2 \\
K &= 18.0 \text{ km} \\
a \sin i &= 62{,}570{,}000 \text{ km} \\
V_o &= +88.5 \text{ km}
\end{aligned}
$$

[1] In *Publications of the Lick Observatory*, **9**, 148, the following revised values of these velocities are given:

+85.37 +89.00 +96.95 wt ½ +102.79 km

The figures placed beside the observations represent the number of periods which must be added to the date of observation to bring it up to the date given at the head of the diagram.

The determination of the minimum of the velocity-curve is weak and includes two doubtful observations, but as no opportunity occurs to revise it until March-May 1917, it seems desirable to publish the data already secured.

In Table II the observed velocities, with the values derived from the theoretical curve, and their differences, are given in order of phase.

TABLE II

OBSERVATIONS OF σ PUPPIS ARRANGED IN ORDER OF PHASE

Phase	O.	C.	O.–C.	Phase	O.	C.	O.–C.
	km	km	km		km	km	km
0d....	+109.2	+109.7	−0.5	149d....	+ 74.8	+ 74.2	+0.6
20.....	106.0	107.3	−1.3	204.....	86.8*	86.2	+0.6
26.....	104.3	105.3	−1.0	204.....	86.2	86.2	0.0
30.....	103.4*	103.5	−0.1	206.....	85.7	87.0	−1.3
40.....	99.7	99.2	+0.5	210.....	89.5	88.9	+0.6
45.....	98.4	96.8	+1.6	214.....	91.8	90.6	+1.2
48.....	96.5	95.4	+1.1	218.....	89.0*	92.6	−3.6
62.....	88.8	89.2	−0.4	218.....	93.3	92.6	+0.7
66.....	89.8	87.7	+2.1	220.....	94.6	93.6	+1.0
76.....	83.1	84.1	−1.0	230.....	98.1*	99.3	−1.2†
90.....	78.1	80.0	−1.9	236.....	103.2	102.2	+1.0
90.....	81.3	80.0	+1.3	238.....	103.3	103.2	+0.1
98.....	77.6	78.2	−0.6	242.....	105.4	105.0	+0.4
101.....	77.8	77.6	+0.2	244.....	105.1	105.9	−0.8
105.....	76.7	76.9	−0.2	249.....	108.1	107.7	+0.4
114.....	77.0	75.6	+1.4	250.....	107.6	108.0	−0.4
120.....	74.2	75.0	−0.8	252.....	107.5	108.5	−1.0
135.....	69.2	74.0	−4.8	257.....	110.1	109.5	+0.6
144.....	71.3	74.0	−2.7	258 or 0d	109.2	109.7	−0.5

* Santiago observations.

† Correction +1.1 km applied.

The measures were made on the Hartmann spectrocomparator, using Plate 2145, of α Tauri, as standard.

ROYAL OBSERVATORY
CAPE OF GOOD HOPE
April 1916

ERRATA

Vol. 42, December 1915, "The Elements of the Eclipsing Systems TV, TW, TX Cassiopeiae and T Leonis Minoris," by R. J. McDiarmid:

Page 422, last line, *for* ak^2 *read* $a_2^2k^2$

" 431, sixth line, *for* c *read* C

" 432, Table VII, the following numerical values should be substituted:

	Uniform	Darkened
a_b	0.285	0.287
b_b	0.271	0.277
a_f	0.301	0.325
b_f	0.285	0.314
ρ_b	0.137	0.127
ρ_f	0.055	0.044

Distance in light-years 1030

This change in dimensions of stars means that the larger star is the fainter in both cases, so that in Fig. 2 the stars are interchanged.

Vol. 43, June 1916, "Researches on Solar Vortices," by Carl Störmer:

Page 348, second equation, *for* ϕ *read* ϕ_0.

" 352, *delete* fourth and fifth lines from bottom.

" 363, *interchange* legends of two parts of Fig. 9.

" 401, second formula, the quantities between the inequality signs are two parts of a single expression. The comma should be replaced by multiplication sign.

Delete last sentence of the second paragraph of Section XIV.

Vol. 44, September 1916, "Some Determinations of the Apex and Velocity of Solar Motion from the Radial Velocities of the Brighter Stars, Including an Apparent Relation to Proper Motion," by C. D. Perrine:

Page 107, Table I, fifth column, $V_\odot$ (North), next to last line, *for* $+2.3$ *read* $+0.7$.

Page 114, Table VII, fifth column, K, next to last line, *for* $+4.3$ *read* $+3.6$.

Vol. 44, September 1916, "The Minimum Radiation Visually Perceptible," by Herbert E. Ives:

Page 127, fifth line, *for* $0.38\times 10^{-8}\ \frac{\text{ergs}}{\text{sec. sq. cm.}}$ *read* 0.38×10^{-8} ergs per second.

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All papers for publication and correspondence relating to contributions should be addressed to the *Editors of the* ASTROPHYSICAL JOURNAL, *Yerkes Observatory, Williams Bay, Wisconsin, U.S.A.*

THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XLIV DECEMBER 1916 NUMBER 5

MUTUAL REPULSION OF SPECTRAL LINES AND OTHER SOLAR EFFECTS CONCERNED WITH ANOMALOUS DISPERSION[1]

By SIR JOSEPH LARMOR

In *Philosophical Transactions of the Royal Society of London*, **189**, 240, 1897, which I believe was the earliest attempt at detailed discussion of such matters, the refractive index of a gas in the neighborhood of an *ideally simple* line is given by

$$\frac{\mu^2-1}{\mu^2+2}=\frac{ng_1}{p_1^2-p^2}$$

where

$$p_1=\frac{2\pi c}{\lambda_1}\,,\quad p=\frac{2\pi c}{\lambda}\,;$$

n is the number of molecules per unit volume, g_1 a molecular constant for the line (p_1).

The absorption line is black *absolutely* from

$$p^2=p_1^2-ng_1 \text{ to } p^2=p_1^2+2\,ng_1;$$

thus its breath $\delta p\left(=-\frac{2\pi c}{\lambda_1^2}\,\delta\lambda\right)$ is given by

$$\frac{\delta\lambda}{\lambda}\left(=-\frac{\delta p}{p}\right)=\frac{3\,ng_1}{2\,p_1^2}\,.$$

[1] From letter to G. E. Hale, of date April 1916, prompted by S. Albrecht's paper; revised August 26.

It is asymmetric with respect to the emission line by one-sixth of its breath toward the violet end.

How far from this absorption line is the extra refractive power of the medium, due to the material that produces the line, recognizable? The pressure-effect in a gas is substantially, or largely, an effect of density operating through the increased value of the dielectric coefficient K or μ^2. The train of ideas in *Astrophysical Journal*, **26**, 120, 1907, which of necessity is most rough, only indicating the order of magnitude to be expected, shows that change of μ from 1 to 1.003 in the surrounding gas may be expected to alter the wave-length of a vibration of a molecule immersed in that gas by an amount of the order given by

$$\frac{\delta\lambda}{\lambda}\left(=-\frac{\delta p}{p}\right)=10^{-6}.$$

This uses the data of Humphreys' original observations. *A change in* λ *of* 0.001 *A means* $\frac{\delta p}{p}=\frac{1}{5}\cdot 10^{-6}$, *say, and to produce it would thus require the change in* μ *to be of the order* $\frac{1}{2}\cdot 10^{-3}$. Using this value $\frac{1}{2}\cdot 10^{-3}$ for $\delta\mu$, put then for the gaseous atmosphere

$$\mu^2=1+\varepsilon+10^{-3},$$

where $1+\varepsilon$ is ordinary, owing to bands at a distance, and 10^{-3} is local, selective, or anomalous (the value of the constant of astronomical refraction makes ε for air at the bottom of the atmosphere 3.10^{-4}) and the first formula gives, as μ^2+2 is practically 3,

$$\frac{10^{-3}}{3}=\frac{ng_1}{p_1^2-p^2};$$

while the breadth of the absolutely black part of the line is

$$(\delta\lambda)_0=\frac{3\lambda_1}{2p_1^2}ng_1.$$

This gives

$$p_1^2-p^2=3\cdot 10^3\, ng_1=2\cdot 10^3\,\frac{p_1^2}{\lambda_1}\,(\delta\lambda)_0$$

or

$$1-\frac{\lambda_1^2}{\lambda^2}=2\cdot 10^3\,\frac{(\delta\lambda)_0}{\lambda_1},$$

giving

$$\frac{\lambda\delta\lambda}{\lambda^2} = 10^3 \frac{(\delta\lambda)_0}{\lambda_1},$$

or say

$$\delta\lambda = 10^{-3}(\delta\lambda)_0.$$

Thus μ is altered in the gas by the assumed amount $\frac{1}{2}\cdot 10^{-3}$ for radiation at a distance $\delta\lambda$ from the line λ_1 causing the alteration, which is given by $\delta\lambda = 10^3$ $(\delta\lambda)_0$; viz., if $(\delta\lambda)_0$ is the breadth of the part of an absorption line of *simplest type that is absolutely black, the wave-length of an adjacent independent line will be affected to the order 0.001 A if that line is at a distance from it of 1000* $(\delta\lambda)_0$. The molecules of the gas are supposed to be stationary in this deduction: thus really $(\delta\lambda)_0$ is the very small residue that remains when the Doppler-Fizeau effect of molecular translatory motion is subtracted.

This is very rough and incomplete, and we have had to introduce the unknown $(\delta\lambda)_0$. But Becquerel made observations on the sodium D lines long ago, from which the anomalous change $\delta\mu$ in their neighborhood can be measured directly without doubt: in *Comptes Rendus*, **127**, 899, 1898; **128**, 146, 1899, quoted by Kelvin in *Philosophical Magazine* (5), **46**, 494, 1898, or *Baltimore Lectures*, p. 176, where, however, the investigation is imperfect (except for a gas) and should be replaced by the above. R. W. Wood has, I think, measured the deviation more precisely since. It is of course easy to measure it with adequate apparatus. Using such direct data, the only remaining question is whether the rough hypothesis quoted above from the *Astrophysical Journal* gives the right order of magnitude; it errs, I think, as an underestimate.

I *now see* that King's flexured spectra give pertinent information. The flexure is due to the light passing across a horizontal trough of vapor, something like a prism, which involves deviations of the order $\mu - 1$ as the angle of the prism is about half a right angle or more. If this angular deviation is about $\frac{1}{2}\cdot 10^{-3}$ (which would give $\delta\lambda$ about 0.001 of the black breadth of the absorption line), it must give rise to a transverse displacement in the spectrum of $\frac{1}{8}$ inch, as it appears on the photograph, if the plate was at a distance of 1000/4 inches, say 20 feet, from the vapor prism, lenses

in the path being allowed for. Even in that experiment it is thus likely that the density of the vapor was below the limit that could be expected to show a dynamical influence on the wave-length of an adjacent line (separated by 1000 $(\delta\lambda)_0$ from it).

I have now looked up R. W. Wood's paper (*Philosophical Magazine*, 8, 295, 1906) and that of Julius (*Astrophysical Journal*, 25, 95, 1907). The latter gives on p. 99 a table of densities of saturated sodium vapor; at temperature 420°C. the density is only $\left(\frac{0.0013}{0.000007}\right)^{-1}$ or 1/200 of that of atmospheric air. Such figures seem to give no chance of affecting sensibly the wave-length of an adjacent line.

Julius speaks of $\mu-1$ being so very great as 0.36 at 0.4 of an angstrom from a D line, quoting Wood. This would imply deviation of that radiation (0.4 from D) of the order $\mu-1$ radians in getting refracted obliquely out of such a region of sodium vapor. He says nothing about the breadth of Wood's D line, probably far removed from the ideal simple type with which we have been dealing; if it came anywhere near to 0.4 A, such adjacent light just outside the margin of the line cannot travel far without absorption or scattering in the sodium vapor. In any case its elimination would only produce a blurring of the D line, what, I think, Julius calls a "dispersion band." This does not exist sensibly in the solar spectrum, though it does in Wood's vapor—thus indicating that the density of vapor in the sun is much less than Wood's, which makes it to me unexpectedly, but of course not surprisingly, small and obliterates anomalous influences—extending even to the case of the arc when its lines are sharp, for which the Pasadena laboratory has now a negative result, even in an electric furnace. If one had thought of all the above, the experiment on mutual repulsion of furnace lines might have been unnecessary.

As to the suggested distortion of the form of solar prominences by anomalous refraction: Consider a volume of glowing vapor as represented in Fig. 1. Light emitted from a point A in it is refracted to the observer as if it came from A'. This crude representation exaggerates the effect. Even if the anomalous part of $\mu-1$ were 0.36, according to Wood's measurement, which is

absurd in the present circumstances, the angle ACA' would be only about 20°, so that the abnormal displacement (of A to A') would be less than one-third of the depth AC of the highly refracting vapors, and that in the most favorable case; but under such conditions the path of the light from A to C could not be more than a few miles before extinction arrives, so that the distortion of position (A to A') could not possibly be so much as one-third of this. On the other hand, if the vapor-density is much smaller, so is the angle ACA', to an extent compensating the longer possible path AC.

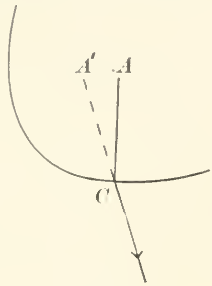

FIG. 1

The stratification of the vapor must be on the whole parallel to the sun's surface. If then distortions existed, they would be more prevalent and conspicuous and the prominence lines would appear more ragged nearer the limb, where the light issues obliquely, than around the center of the disk.

The abnormal flare of the Sherman F line of 1872, figured in Abbot's *The Sun*, p. 163, is surely broadening owing to the condensation or other affection of the gas which is revealed by the concurrent continuous spectrum.

If anomalously refracted light from A thus appears to come from A', x miles distant, while the other parts of the light appear to come from the true source A, the spectroscope will receive the former at an inclination x/D to the true line of sight, where D is the sun's distance. If AA' were as much as 10 miles, the inclination would be 1/42 of a second of arc. This would show itself as an apparent change in the angular dispersion for that constituent of the light, and therefore as an apparent proportionate change of λ, of the same order when a high-dispersion grating is used; it would then amount to about $\frac{1}{2}\cdot 10^{-3}$ of an angstrom in the visible spectrum.[1] On the other hand, the first estimate made above refers to a true change of wave-length (or period) of the source, arising from the influence of neighboring sources of adjacent wave-length. The considerations advanced by Professor W. H. Julius (*Astrophysical*

[1] See footnote *infra*.

Journal, 43, 43, 1916) refer to deviation of rays in the solar atmosphere by anomalous refraction without change of period; these apparently are subject to the extreme estimate just now made, and thus seem to be beyond present instrumental means.[1]

The considerations *supra* as to change of period hold good also for influence of an adjacent line of the *same* substance, provided this line arises from a separate independent vibration. A very searching test would be afforded by the behavior of the satellites of a line, when the vapor-density of the source is increased; but a negative result can be taken as an indication that the satellite is an essential part of the vibration of the main line, and not due to a different source which can be varied independently of it; cf. Evershed's observations, recently reported in *The Observatory*, **39**, 59, 1916, and the suggestion on these lines that followed in connection therewith.

The case of rays part of whose path is tangential to the gaseous strata of density needs further consideration. By James Thomson's principle, the curvature of a ray in a heterogeneous medium is $d \log \mu/dn$, where dn is measured along its normal drawn inward. For a gas this is practically $d\mu/dn$; and if the ray travels for a considerable distance nearly tangential to a stratum across which the gradient of density is very steep, its total deviation will be large, giving rise to possibly hundreds of times the $\frac{1}{2} \cdot 10^{-3}$ of an angstrom mentioned above. But would it be shown by the spectroscope?

Suppose the curves of the diagram (Fig. 2) to represent the stratification of density in the mass of gas. The light from the different point-sources, in or behind the mass, which is caught up by the spectroscope, is a narrow ray from each point, all parallel. Where the ray is tangential to a surface of stratification it is drawn around toward it, as regards that constituent of frequency susceptible to anomalous refraction by the gas, while the rest passes

[1] If I am not under a misapprehension in this intricate subject, it seems to follow that true change of period, such as arises from line-of-sight motion, implies angular displacement of the lines increasing in proportion to the dispersive power of the spectroscope that is employed, while mere anomalous deviation of the rays, without change of period, would give the same angular deviation for all dispersive powers.

on; but elsewhere the paths are practically straight. If in its path the ray is anywhere tangential to a stratum,[1] its anomalous constituent is thus thrown off the slit of the spectroscope; but its place is taken there by the same constituent from a source more to the right in the diagram, by a few hundreds of miles at the very most (cf. *supra*), owing to the shallowness of the solar atmosphere. The light thus caught by the telescope from this source, at one side of the source at which it is pointed, is not quite parallel to the main path of the light; and this difference of direction would come out as a displacement of the spectral line, which might amount to 10^{-2} of an angstrom, thus simulating motion of the original source in the line of sight. But the spectroscope could hardly show it.

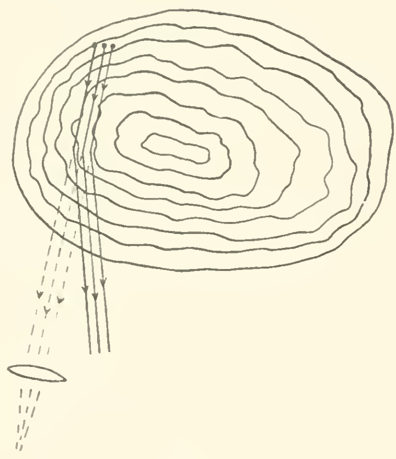

FIG. 2

The smallest available breadth of the slit represents hundreds of miles in the region of the sun whose image is thrown on it. Thus, for these phenomena to be observable without entire loss of sharpness, the parallel rays from a band of sources hundreds of miles wide must be deflected to nearly the same extent where each is tangential to a stratum. This necessary regularity requires that the gradient of density should remain about the same, and of substantial amount along hundreds of miles across the gas; and, in order to account for the observed shifts of lines, bearing in mind the small atmospheric pressure notwithstanding high gravity (cf. the remark on scale, *infra*), this would seem to pile up the density of the gas of the solar atmosphere to quite impossible values.

Actually in the higher levels the densities are for various reasons probably almost infinitesimally small.

[1] It cannot be anywhere tangential to the strata if its source is in the front half of the mass of gas.

A general grasp of these matters, in their relation to laboratory experience, is perhaps facilitated by use of a principle which follows from the formula for curvature of the rays, viz., that the deviations of rays in the solar atmosphere are the same as they would be in a model atmosphere, on a scale reduced uniformly in all three dimensions of space, but with the same densities of the gaseous constituents at all corresponding points.

Cambridge, England
August 26, 1916

THE VARIATIONS IN SPECTRAL TYPE OF TWENTY CEPHEID VARIABLES[1]

By HARLOW SHAPLEY

A considerable amount of material relative to the spectral types of variable stars of the Cepheid class has been accumulated at the Mount Wilson Observatory during the past year, chiefly with the aid of the 10-inch photographic telescope. Earlier work on some of these stars at the Lick, Harvard College,[2] and Pulkova Observatories had shown or suggested variations in certain characteristics of the spectra. The data collected in the present paper, however, indicate that distinct changes in spectral type, accompanying regularly the periodic variations in light and apparent velocity, constitute one of the general and fundamental properties of Cepheid variables. A statement of the Cepheid problem at the time it was taken up at Mount Wilson is outlined in *Contribution* No. 92,[3] together with a summarized account of the most relevant previous work. The significance of spectral variations in the interpretation of variability in light and velocity is noted in the same article and in subsequent papers on the light-curves and spectra of Cepheid variables.[4] The following pages will be devoted to the observations of the spectra of individual stars.

A list of all the Cepheids for which definite variations of spectral type have been observed is given in Table I. The positions in the second and third columns are from *Harvard Annals*, **56**, No. 6, Table VIII. The data for the light-variations are taken from the same source except that the magnitudes for RT Aurigae are by

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 124.

[2] See, for instance, the remark by Miss Cannon on the spectrum of δ Cephei in *Harvard Annals*, **56**, 110, 1912.

[3] *Astrophysical Journal*, **40**, 448, 1914.

[4] *Mt. Wilson Contr.*, Nos. 104, 112; *Astrophysical Journal*, **42**, 148, 1915; **43**, 217, 1916; *Mt. Wilson Communications*, Nos. 14, 21, 22, 27; *Proceedings of the National Academy of Sciences*, **1**, 452, 1915; **2**, 132, 136, 208, 1916.

Kiess,[1] those for RR Lyrae by Shapley,[2] and those for RS Boötis and XZ Cygni from Hartwig's *Ephemeris*.[3] Precise information as to the maximum magnitude and range of variation of these stars, even of the much observed brighter ones, is unfortunately not available.[4] Widely differing values are given in the catalogues by different observers. The uncertainty arises partly from probable

TABLE I

List of Spectral Variables

Star	R.A. 1900	Decl. 1900	Visual Light		Period	Spectral Variation	No. of Photographs
			Maximum	Range			
TU Cassiopeiae..	0h 20m.9	+50° 44′	7M.2	1M.4	2d.139	F1–F8	5
SU Cassiopeiae..	2 43.0	+68 28	5.9	0.4	1.950	A8–F5	24
SZ Tauri......	4 31.4	+18 20	7.2	0.5	3.149	A9–F7	13
T Monocerotis.	6 19.8	+ 7 8	5.7	1.1	27.012	F5–G2	9
RT Aurigae.....	6 22.1	+30 34	5.1	0.9	3.728	A7–G1	30
W Geminorum.	6 29.2	+15 24	6.7	0.8	7.916	F2–G1	9
RS Boötis......	14 29.2	+32 11	9.2	1.0	0.377	B8–F0	13
X Sagittarii...	17 41.3	−27 48	4.4	0.6	7.012	F1–G5	24
Y Ophiuchi....	17 47.3	− 6 7	6.1	0.4	17.113	F5–G3	8
W Sagittarii...	17 58.6	−29 35	4.3	0.8	7.595	A8–G2	25
Y Sagittarii...	18 15.5	−18 54	5.4	0.8	5.773	F4–G4	10
RR Lyrae......	19 22.3	+42 36	6.8	0.9	0.567	B9–F2	17
U Aquilae.....	19 24.0	− 7 15	6.2	0.7	7.024	F6–G2	4
XZ Cygni......	19 30.4	+56 10	8.7	0.6	0.467	A0–A6	2
U Vulpeculae..	19 32.3	+20 7	6.5	1.1	7.990	F7–G5	8
SU Cygni......	19 40.8	+29 1	6.2	0.8	3.846	A6–F7	17
η Aquilae.....	19 47.4	+ 0 45	3.7	0.8	7.176	A8–G5	29
S Sagittae....	19 51.5	+16 22	5.5	0.6	8.382	F4–G3	18
T Vulpeculae..	20 47.2	+27 52	5.5	0.6	4.436	A9–G1	17
δ Cephei.....	22 25.4	+57 54	3.5	0.8	5.366	F0–G2	46

irregularities in the variations, but chiefly because most of the light-observations have been naked-eye comparisons with stars for which various estimates of magnitudes have been adopted. The values in the fourth and fifth columns, therefore, serve in most cases only to indicate approximately the visual brightness and range of light-change. The point is of some importance in that it shows the

[1] *Laws Observatory Bulletin*, No. 23, 1915.

[2] *Mt. Wilson Contr.*, No. 112; *Astrophysical Journal*, **43**, 217, 1916.

[3] *Vierteljahrsschrift der Astronomischen Gesellschaft*, **49**, 287, 301, 1914.

[4] Exceptions are δ Cephei, RR Lyrae, and perhaps a few others. See note to Table VIII below.

futility of attempting at present to study the relation of range in light to range in spectral variation. The data are sufficient, however, to show the relation of spectral type to period (which had already been noted for a larger number of stars on the basis of the Harvard classification),[1] although in a number of cases the extreme range of spectrum is not recorded when observations near maximum or minimum light are wanting.

Of the 328 photographs of spectra used in deriving the results for the stars in Table I, 311 were made by the writer with the 10-inch portrait lens and objective prism. The remainder were made with the 60-inch reflector—13 by Mr. Pease and 4 by Mr. Adams. For nearly all the work with the 10-inch, a 15° prism is employed. The refracting edge is set perpendicular to the hour circle, and the spectrum, drifting in right ascension, is widened as desired by adjusting the driving clock. For some of the brighter variables, a small 30° prism was used on a few nights in conjunction with the 15° prism. Such instances are designated by the note "45°" in the columns of remarks in Tables III–XIX. Except in the cases noted, all photographs are on Seed "27" plates.

With the single prism the dispersion is $H\beta - H\varepsilon = 5.2$ mm; for the two prisms $H\beta - H\varepsilon = 12$ mm. The uncertainty of a determination of spectral type with the lower dispersion is one- or two-tenths of a spectral interval, but there may be a small systematic difference between my classification and that made at Harvard. So far as possible, however, the choice of criteria for type has been based upon the classifications of the *Draper Catalogue*.

In preparing the material for tabular presentation, all the spectra were classified before phases were computed. This procedure lessens the possibility of errors of prejudice where small changes are concerned. Only two or three discordant results remained after some errors of identification were eliminated, and such discordances may be due to irregularities in the variations or to erroneous light-elements.

As remarked in earlier papers, the changes observed in the spectra of these stars are between known, normal types. No peculiarities appear other than those to be expected in the spectra

[1] *Mt. Wilson Contr.*, No. 92, p. 16; *Astrophysical Journal*, 40, 463, 1914.

of stars of very high luminosity. Possible exceptions are the spectrum of W Geminorum, also noted at Harvard,[1] and the occasional unequal sharpness of the hydrogen lines in spectra made during increasing light. With high dispersion other peculiarities may appear, as, for instance, those indicated in the special study of δ Cephei.[2]

Continuous and periodic changes of spectral type are shown distinctly for fifteen of the stars over several epochs of maximum, and are definitely indicated for the remainder of the list. No Cepheid variable observed has failed to show variability of spectrum. Upon the basis of this result we may believe that for the two or three thousand variable stars which belong to the same class similar disturbances of the radiating surfaces underlie similar periodic oscillations in light and spectral type.

The variation in spectrum of a Cepheid is undoubtedly as important a part of the phenomenon as the fluctuation in light; moreover, it should be as definite a method of detecting a star's peculiar variability as the measures of magnitude. By collecting the classifications of spectrum into normals (using the period that defines the variation in light) and plotting the mean spectrum against phase, we may obtain a curve closely similar to the curves of variation in magnitude and velocity. Such a curve is shown for δ Cephei in Fig. 1, based upon the data in Table II. It differs in form from the light-curve on a later page in certain respects—the spectrum falls away from its maximum (bluest) value more rapidly than the light falls from its maximum, and the change from red to blue spectrum at the end of minimum light is frequently abrupt. To some extent this latter result probably reflects the oscillation in the time of the rise to maximum light, which has been shown to be conspicuous for several Cepheids.

The changes in the spectrum of RR Lyrae are discussed in *Contribution* No. 112. Data for RS Boötis are given by Pease in *Publications of the Astronomical Society of the Pacific*, **26**, 256,

[1] *Harvard Annals*, 55, 38, 1907.

[2] *Mt. Wilson Communications*, No. 22; *Proceedings of the National Academy of Sciences*, **2**, 136, 1916.

1914.[1] The observations of XZ Cygni by Adams will be published elsewhere. For the other seventeen stars listed in Table I the variations are detailed in the accompanying series of tables and figures.

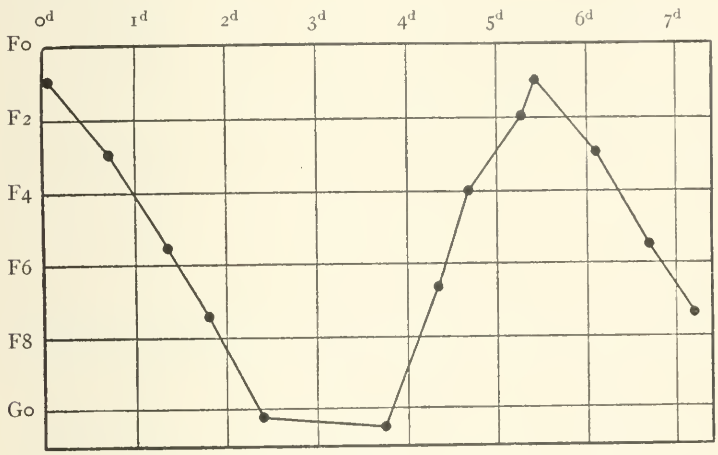

FIG. 1.—The curve of spectral variation of δ Cephei

TABLE II

SPECTRAL VARIATION OF δ CEPHEI

Mean Phase	Mean Spectrum	Number Spectra	Weight
+0ᵈ.04	F1.0	7	12
0.74	F3.0	6	11
1.36	F5.6	5	8
1.82	F7.5	4	8
2.40	G0.2	4	8
3.76	G0.5	4	8
4.37	F6.7	4	7
4.72	F4.0	5	10
5.29	F1.9	7	14

[1] A later curve of the spectral variation is shown in *Contribution* No. 112 and *Communication* No. 21.

TABLE III

TU CASSIOPEIAE

Max.=J.D. 2419302.12+2ᵈ139·E*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
	1916					
71......	Feb. 2	$9^h 21^m$	13^m	$1^d.048$	F6	Faint
85......	5	9 46	18	1.926	Fo:	Faint
100......	6	8 44	25	0.744	F1	
		9 04	16	0.758	F2	
108......	7	7 46	37	1.704	F8	

* *Vierteljahrsschrift der Astronomischen Gesellschaft*, **49**, 334, 1914. The visual range given by Hartwig is $1^M.1$.

TABLE IV

SU CASSIOPEIAE

Max.=J.D. 2417287.30+1ᵈ9498·E*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
	1915					
48......	Dec. 9	$12^h 04^m$	8^m	$0^d.051$	F1	
	1916					
61......	Jan. 31	8 23	10	0.253	Fo:	Out of focus
		36	10	0.262	Fo:	
		46	10	0.269	Fo:	
62......	31	10 36	11	0.345	F3	Poor plate, wind
		46	11	0.352	F4	
		56	6	0.359	F2:	
70......	Feb. 2	8 55	6	0.325	F5:	
		9 05	12	0.332	F5	
81......	5	6 56	15	1.292	A9	
		7 09	10	1.301	A8	
87*b*.....	5	11 42	11	1.491	F2	
		50	5	1.497	Fo:	Bad seeing
		59	10	1.503	Fo	
101......	6	9 34	12	0.453	F2	
		46	8	0.461	F1	
		53	4	0.466	F1	
109......	7	8 27	12	1.406	A9	
		39	10	1.415	Fo	
		52	15	1.424	F2	
110......	7	9 09	14	1.435	F1	Clouds
		24	14	1.446	Fo	
		42	21	1.458	Fo	

* The phases are computed with the elements derived by Müller and Kempf (*Astronomische Nachrichten*, **173**, 307, 1907). The agreement with spectral type is not good, but the difficulty probably is attributable to inaccurate light-elements rather than to an anomalous relation between spectrum and light-variations. Using the later elements by Parkhurst (*Astrophysical Journal*, **28**, 279, 1908):

Max.=J.D. 2417287.30+1ᵈ9490·E,

the agreement is equally bad; but with an intermediate period, 1ᵈ94935, the representation is all that could be expected when the small range and the quality of the observations are considered. Direct observations of the brightness of the spectral images on the plates in Table IV verify the supposition that the light-elements are erroneous.

TABLE V

SZ TAURI

Max. = J.D. 2410000.60 + $3^d.1487 \cdot E$*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
	1915					
26......	Nov. 14	9^h46^m	180^m	$0^d.356$	F2	Overexposed
59......	Dec. 12	14 59	60	0.235	F0	
		15 39	7	0.262	A9	
	1916					
63......	Jan. 31	11 08	5	2.844	F3:	High wind
		18	10	2.851	F4	
64......	31	13 11	8	2.929	F0:	Field low
		22	10	2.937	F2	
88......	Feb. 5	12 20	17	1.597	G	Faint
97......	6	6 36	3	2.358	F6	
		42	7	2.362	F7	
102......	6	10 02	3.5	2.501	F5	
		21	30	2.514	F6	
113......	7	12 16	33	0.446	F0	

* Leavitt, *Harvard Circulars*, No. 186, p. 2, 1914.

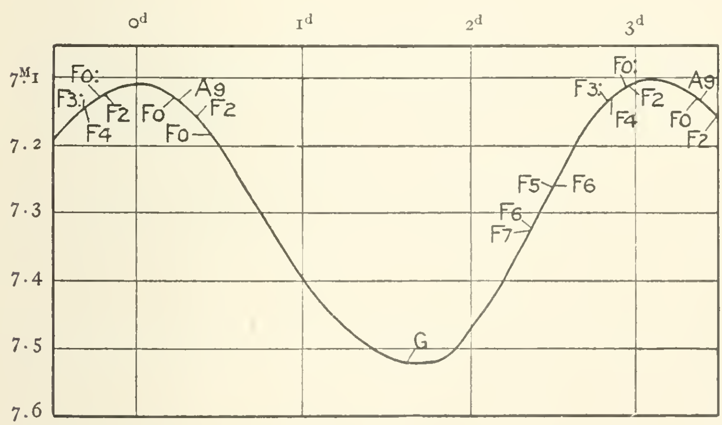

FIG. 2.—SZ Tauri. Mean light-curve by Miss Leavitt (*op. cit.*). An increase of five seconds in the length of the period (a change well within the uncertainty of the light-elements) would place the plotted spectral observations a fourth of a day earlier and harmonize better with the light-curve.

TABLE VI

T MONOCEROTIS

Max. = J.D. 2409633.63 + 27ᵈ0122·E*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
	1916					
78......	Feb. 2	13^h34^m	10^m	26ᵈ193	F8	
		45	10	26.201	F8	
89......	5	13 06	20	2.162	F6	Wind
111......	7	11 02	25	4.076	F6	Clouds
114......	7	12 54	33	4.153	F7	Clear
121......	Apr. 7	8 41	30	9.953	G2	
160......	25	9 26	5	0.972	F5	Wind
169......	27	8 54	16	2.950	F6	
174......	28	8 58	29	3.953	F8	Clouds

* Yendell, *Astronomical Journal*, **16**, 149, 1896.

TABLE VII

RT AURIGAE

Max. = J.D. 2417173.459 + 3ᵈ72806·E*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
	1916					
65......	Jan. 31	13^h38^m	4^m	0ᵈ839	F2	
		46	10	0.844	F2	
67......	Feb. 1	6 31	10	1.542	F7	Plate fogged
		41	10	1.549	F5	
77......	2	12 56	10	2.810	F7	Haze
		13 03	4	2.815	G0	
		11	10	2.820	F9	
90......	5	13 47	11	2.117	G0	
		57	10	2.124	G1	
104......	6	11 38	12	3.027	G0	
112......	7	11 28	13	0.292	A8	
		38	5	0.299	A8	
119......	Mar. 30	11 14	9	0.090	A7	
		22	8	0.095	A8	
120*a*......	Apr. 7	7 35	7	0.481	F0	
120*b*......	7	7 48:	4:	0.490:	A9	Time uncertain
		55	8	0.495	A9	
122......	7	9 10	17	0.547	F1	
145......	9	10 04	7	2.585	F8	
		11	6	2.590	F8	
146......	9	10 21	10	2.597	F7	Faint
159......	25	9 01	8	3.629	F0	
		08	5	3.634	F1	
		11	2	3.636	A9	
167......	26	7 30	3	0.838	F4	Strong twilight
168......	27	7 24	10	1.834	F8:	Faint
170......	27	9 15	12	1.911	F9	
		24	5	1.917	F8	
175......	28	9 21	12	2.916	F7	
		38	21	2.927	F6	

* Kiess, *Laws Observatory Bulletin*, No. 23, 1915.

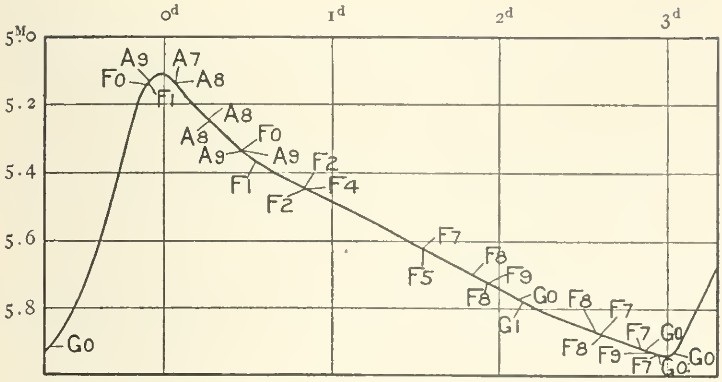

FIG. 3.—RT Aurigae. Mean visual light-curve by Kiess (*op. cit.*), neglecting supposed oscillations in the descending branch.

TABLE VIII

W GEMINORUM

Max. = J.D. 2413266.34 + 7ᵈ.91603·E*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
	1916					
78......	Feb. 2	13^h34^m	10^m	$7^d.422$	F2	Haze
		45	10	7.429	F2	
89......	5	13 06	20	2.486	F7:	Wind; faint
103......	6	11 16	20	3.410	G	Faint
111......	7	11 02	25	4.400	F6:	Clouds
114......	7	12 54	33	4.478	G0	Clear
121......	Apr. 7	8 41	30	0.974	F4	
169......	27	8 54	16	5.151	G1	
174......	28	8 58	29	6.153	F3	Haze

* *Vierteljahrsschrift der Astronomischen Gesellschaft*, 49, 334, 1914. The visual range in Table I is $0^M.8$; according to Hartwig it is $1^M.3$, and from unpublished observations made at Utrecht and generously placed at my disposal by Dr. Van der Bilt it appears to be less than $0^M.4$.

TABLE IX

X SAGITTARII

Max. = J.D. 2403169.93 + $7^{d}.01190 \cdot E$*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
	1916					
69......	Feb. 1	$17^{h}34^{m}$	3^{m}	$0^{d}.052$	F2	Dawn
		38	5	0.055	F3	
96......	5	17 36	7	4.053	G	Dawn
130......	Apr. 7	15 49	10	2.872	F8	
		57	5	2.878	F8	
		16 03	6	2.882	F6:	
140......	8	13 22	7	3.770	F8:	
156......	9	15 39	14	4.865	F8	Wratten "M" plate
158......	9	16 24	10	4.897	F8	Wratten "M" plate
164......	25	15 43	10	6.844	F1	
		50	5	6.849	F1	
183......	28	14 14	4	2.770	F7	
		20	9	2.774	F6	
187......	July 9	11 36	13	4.542	G1	
		48	8	4.550	G1	
198......	28	9 51	10	2.433	F4	45°
205......	29	10 00:	10	3.439:	F8:	45°, faint
219a.....	30	10 10	23	4.446	G5	45°, faint
220......	30	10 52	20	4.475	G4	45°, faint
236......	31	11 47	38	5.513	F5	45°
244......	Aug. 1	8 59	26	6.397	F1	45°, Hγ sharp
256......	2	11 42	34	0.498	F	45°, faint
261......	3	8 03	8	1.346	F3	
		16	18	1.355	F2	

* Hellerich, *Dissertation*, p. 9 (Bonn, 1913).

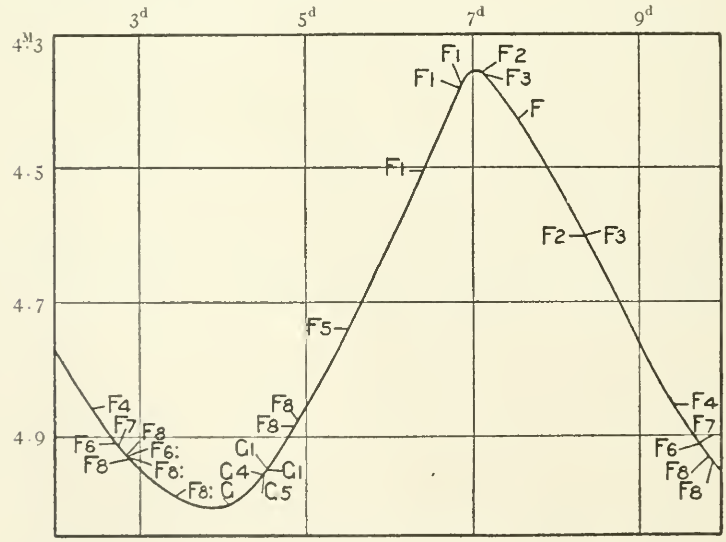

FIG. 4.—X Sagittarii. Mean light-curve by E. C. Pickering (*Harvard Annals*, 46, 155, 1903).

TABLE X

Y OPHIUCHI

Max. = J.D. $2408697.25+17^{d}.1130 \cdot E$*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
	1916					
68......	Feb. 1	17^h14^m	7^m	$14^d.35$	G0	Field low
		23	10	14.35	F8:	Weak
94......	5	16 22	21	1.20	F5	Field low
		40	15	1.21	F7	
131......	Apr. 7	16 16	5	11.85	G2	Dawn
		22	10	11.86	G3	
139......	8	13 00	9	12.72	G	Field low
181*a*.....	28	13 26	10	15.62	F6	

* Hellerich, *op. cit.*, p. 11.

TABLE XI

W SAGITTARII

Max. = J.D. $2403198.88+7^{d}.594386 \cdot E$*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
	1916					
96......	Feb. 5	17^h36^m	7^m	$6^d.267$	F5:	Dawn
130......	Apr. 7	15 49	10	7.438	F0	
		57	5	7.444	F0	
		16 03	6	7.448	A9	
140......	8	13 22	7	0.742	F2	
		31	2	0.748	F1	Narrow
		40	5	0.754	F2	
156......	9	15 39	14	1.837	F4	Wratten "M" plate
158......	9	16 24	10	1.868	F4	Wratten "M" plate
164......	25	15 43	10	2.651	F5	
		50	5	2.656	F6	
183......	28	14 14	4	5.589	G0	
		20	9	5.593	G2	
187......	July 9	11 36	13	1.535	F6	
		48	8	1.544	F6	
198......	28	9 51	10	5.274	G1	45°
205......	29	10 00:	10	6.280	F8	45°, time uncertain
219*a*.....	30	9 58:	1:	7.278:	A8	45°, time uncertain
		10 10	23	7.287	F0	
220......	30	10 52	20	7.316	A9	45°
236......	31	11 47	38	0.760	F1	45°
244......	Aug. 1	8 59	26	1.643	F3	45°
256......	2	11 42	34	2.756	F6	45°, faint
261......	3	8 03	8	3.604	F9	
		16	18	3.613	F9	

* Hellerich, *op. cit.*, p. 9.

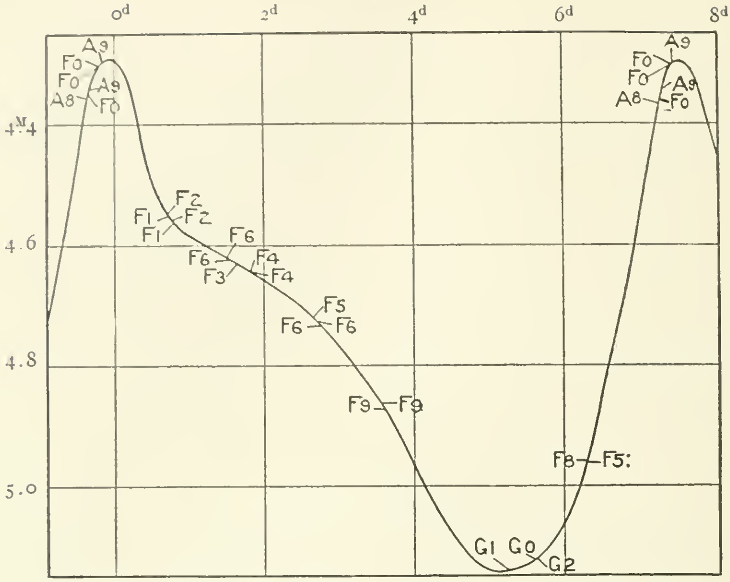

FIG. 5.—W Sagittarii. Mean light-curve by E. C. Pickering (*Harvard Annals*, 46, 155, 1903).

TABLE XII

Y SAGITTARII

Max.$=$J.D. $2410175.08+5^{d}.773268\cdot E$*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
	1916					
130......	Apr. 7	15^h49^m	10^m	$2^d.448$	G0	
		57	5	2.453	F8:	Diffuse
		16 03	6	2.458	F8	
140......	8	13 22	7	3.346	G2	
164......	25	15 43	10	3.124	G:	Wind; diffuse
183......	28	14 14	4	0.289	F4	
		20	9	0.293	F4	
187......	July 9	11 36	13	2.900	F8	Moon near field
		48	8	2.908	G	
261......	Aug. 3	8 16	18	4.668	G4	

* Hellerich, *op. cit.*, p. 10.

TABLE XIII

U AQUILAE

Max. = J.D. 2410170.325+7^d.02387·E*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
	1916					
142......	Apr. 8	15^h33^m	15^m	3^d.992	G2	
181*b*.....	28	13 48	12	2.847	F6	Diffuse
191......	July 10	10 06	17	5.455	F8	
230......	31	8 14	17	5.305	F6	

* *Vierteljahrsschrift der Astronomischen Gesellschaft*, 49, 335, 1914.

TABLE XIV

U VULPECULAE

Max. = J.D. 2414200.253+7^d.98950·E*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
	1916					
128......	Apr. 7	14^h38^m	21^m	2^d.573	F7	
		54	8	2.584	F7:	Narrow
141......	8	14 30	27	3.567	G4	Diffuse
152......	9	13 46	20	4.537	G4:	Faint
153......	9	14 14	15	4.556	G0	
161......	25	13 31	12	4.547	G5	
178......	28	12 25	20	7.501	F8p	Hβ strong
211......	July 29	14 50	22	3.728	G5	Difficult

* *Vierteljahrsschrift der Astronomischen Gesellschaft*, 49, 335, 1914.

TABLE XV

SU CYGNI

Max. = J.D. 2414202.820+3^d.845612·E*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
	1916					
128......	Apr. 7	14^h38^m	21^m	2^d.383	F6	
		54	8	2.394	F7	Drift in declination
141......	8	14 30	27	3.377	A6	
152......	9	13 46	20	0.501	F1	
		58	5	0.510	F0:	Narrow
161......	25	13 31	12	1.108	F3	
178......	28	12 25	20	0.217	A8	
184*b*.....	July 9	9 42	15	2.883	F5	
		56	11	2.892	F5:	
188......	10	8 21	10	3.826	A6	
		38	24	3.838	A7	
211......	29	14 33	12	0.011	A8	
		50	22	0.023	A9	
216......	30	8 34	17	0.637	F4	
231......	31	8 38	21	1.640	F6	
258......	Aug. 2	13 26	21	0.119	A8	
262......	3	8 40	20	0.920	F2	

* Luizet, *Astronomische Nachrichten*, 173, 196, 1906.

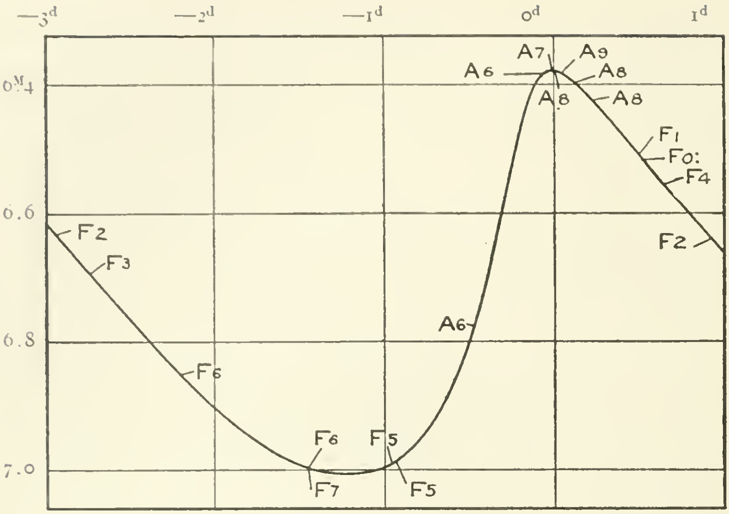

FIG. 6.—SU Cygni. Mean light-curve from observations made at Utrecht; data furnished in manuscript by Dr. Van der Bilt.

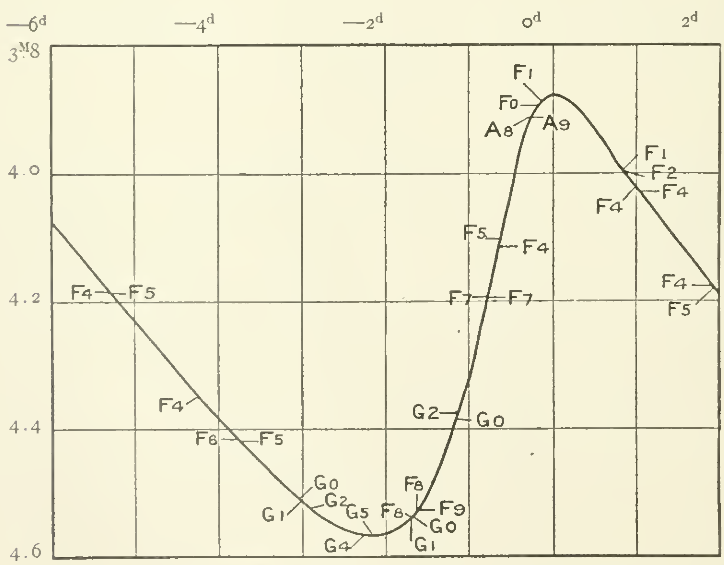

FIG. 7.— η Aquilae. Mean light-curve from observations made at Utrecht; data furnished in manuscript by Dr. Van der Bilt.

TABLE XVI

η AQUILAE

Max. = J.D. 2396168.732 + $7^d.176382 \cdot E$*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
	1916					
142......	Apr. 8	$15^h 33^m$	15^m	$7^d.053$	F0	
		44	5	7.061	F1	
162......	25	13 40	3	3.418	F5	
		43	2	3.420	F6	
172*b*.....	27	14 40	3	5.460	G0	
		42	2	5.461	G1	
181*b*.....	28	13 40	4	6.418	F7:	Diffuse
		48	12	6.424	F7	
184*a*.....	July 9	9 16	7	5.471	F8	
		21	4	5.474	F8	
		26	5	5.478	F9	
190......	10	9 45	15	6.491	F4	Hγ very sharp
191......	10	10 06	17	6.506	F5	
196......	28	8 49	10	2.923	F4	45°
210......	29	13 44	9	4.128	G0	45°
		54	11	4.135	G1	
213......	29	15 43	10	4.210	G2	
215......	30	8 12	5	4.897	G4	
		17	4	4.901	G5	
230......	31	8 02	5	5.890	G0	
		14	17	5.899	G2	
243......	Aug. 1	8 20	2	6.903	A8	45°
		31	12	6.910	A9	
254......	2	9 47	10	0.787	F1	45°
		10 01	18	0.797	F2	Hγ very sharp
259*b*.....	2	14 04	7	0.966	F4	
		12	7	0.971	F4	
269......	3	13 00	5	1.921	F5	45°
		08	11	1.926	F4	

* Hellerich (*op. cit.*, p. 6) finds that the elements derived by Luizet (*Astronomische Nachrichten*, 163, 361, 1903) are satisfactory without the use of the harmonic term adopted by the latter.

TABLE XVII

S SAGITTAE

Max. = J.D. 2409863.324 + $8^{d}.381613 \cdot E$*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
	1916					
144......	Apr. 8	$16^{h} 18^{m}$	11^{m}	$2^{d}.433$	F5	Dome
		26	6	2.438	F6	Diffuse
153......	9	14 14	15	3.347	F9	
172*a*.....	27	14 28	7	4.593	G1	
		34	5	4.597	G2	
179......	28	12 42	9	5.519	G3	
		50	5	5.525	G2	
184*c*.....	July 9	10 12	10	1.981	F7	
		20	6	1.987	F6	
		24	3	1.990	F6:	Narrow
189......	10	9 07	26	2.936	F8	
		24	6	2.948	F7	
212......	29	15 11	12	5.426	G2	
227......	30	15 16	27	6.429	G	45°, faint
233*b*.....	31	9 38	10	7.194	F6	Wind
		44	1	7.199	F5	
237......	31	12 41	51	7.322	F6	45°
259*a*.....	Aug. 2	13 50	15	0.488	F4	Very thick

* *Vierteljahrsschrift der Astronomischen Gesellschaft*, 49, 335, 1914.

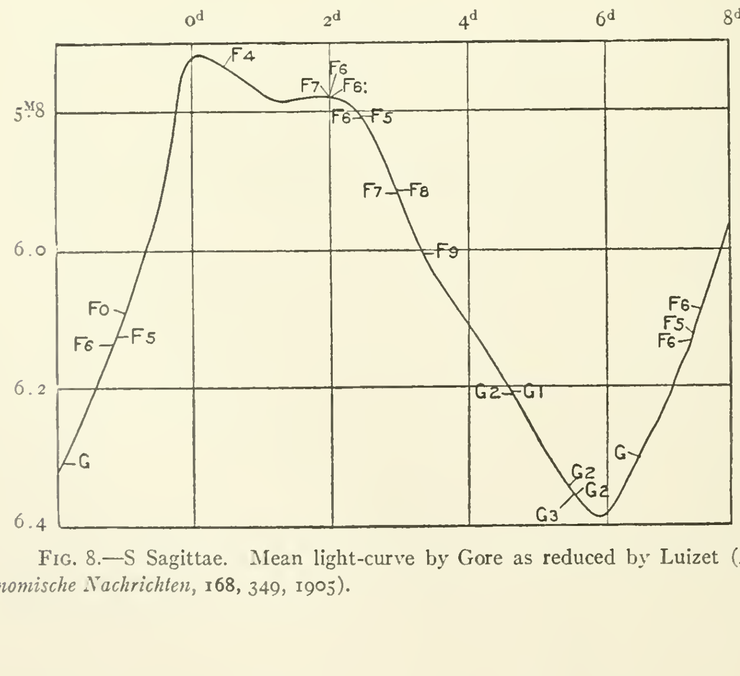

FIG. 8.—S Sagittae. Mean light-curve by Gore as reduced by Luizet (*Astronomische Nachrichten*, 168, 349, 1905).

TABLE XVIII

T VULPECULAE

Max.=J.D. 2409849.04+4ᵈ435521·E*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
	1916					
154......	Apr. 9	14^h31^m	10^m	3^d918	F4	
		40	7	3.924	F5	
163......	25	13 54	7	2.150	G1	
		14 00	5	2.154	G0	Faint
171......	27	14 12	8	4.162	F0	
		18	3	4.167	A9	
180......	28	13 02	8	0.679	F4	
		10	9	0.685	F3	
186*a*.....	July 9	10 56	9	1.623	F6	
192......	10	10 26	3	2.602	F7	
		33	8	2.607	F8	
214......	29	15 56	12	4.089	F2	Hδ very faint
217......	30	8 52	3	0.359	F0	Hδ stronger
		9 00	11	0.365	F0	
233*a*.....	31	9 22	15	1.380	F5	
260......	Aug. 2	14 20	3:	3.587:	F6	Time uncertain
		40	9	3.601	F6	Thick

* Hellerich, *op. cit.*, p. 7.

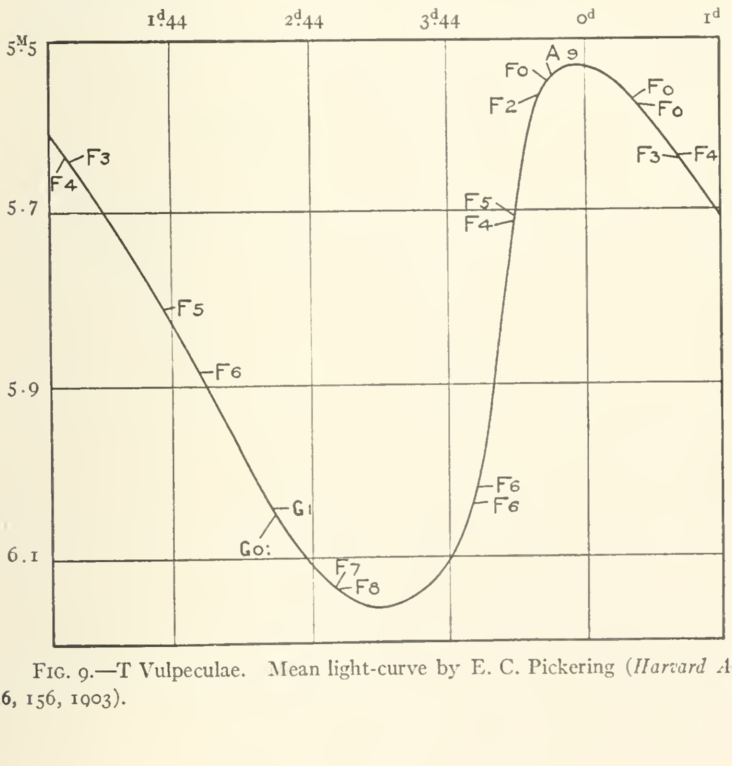

FIG. 9.—T Vulpeculae. Mean light-curve by E. C. Pickering (*Harvard Annals*, 46, 156, 1903).

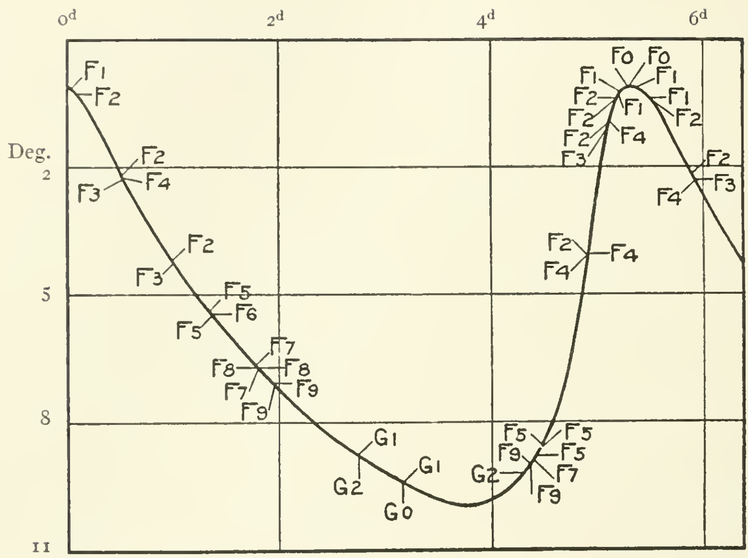

FIG. 10.—δ Cephei. Mean light-curve by Luizet (*op. cit.*). The curve printed in *Popular Astronomy*, 25, 355, 1916, has been revised and supplemented by later data.

TABLE XIX

δ CEPHEI

Max. = J.D. 2393659.856 + $5^{d}.366386 \cdot E$*

Plate	Date	P.S.T.	Duration	Phase	Spectrum	Remarks
	1915					
γ 4571...	Dec. 23	7^h04^m	225^m	$4^d.294$	G2	60-inch reflector
γ 4578...	24	14 07	120	5.254	F4	60-inch reflector
	1916					
60......	Jan. 31	7 28	5	0.379	F	Bad focus
66......	Feb. 1	6 59	4	1.359	F5	Haze
		7 02	1	1.361	F5	
		04	3	1.362	F6:	
		14	4	1.369	F6	
80......	5	6 28	3	5.338	F :	Clouds
		32	3	5.341	F1	
		34	3	5.342	F0	
83......	5	8 18	2	0.048	F1	
		20	2	0.049	F2	
98......	6	7 10	10	1.001	F5:	Seed "23" plate
		18	5	1.006	F5:	Faint
107......	7	7 06	12	1.998	F9	Seed "23" plate
		16	5	2.005	F9	
143......	Apr. 8	16 02	8	4.340	F9	
		07	2	4.343	F9	
157......	9	16 02	5	5.340	F2	Wratten "M" plate
		10	10	5.345	F1	
165......	25	16 02	5	5.241	F2	Seed "23" plate
		06	3	5.244	F2	
		11	6	5.247	F3	
173......	27	14 48	1.5	1.823	F8	Bad seeing
		50	1	1.824	F7	
		51	1	1.825	F8	
		52	2	1.826	F7	
182......	28	14 04	5	2.792	G1	Seed "23" plate
		07	2	2.794	G2	
186*b*.....	July 9	11 10	1	4.908	F4	
		14	2	4.911	F4	
		18	5	4.914	F2	
193......	10	10 45	6	0.525	F2	
		48	1	0.527	F3	
		52	4	0.530	F4	
218......	30	9 26	8	4.371	F7	45°
		36	12	4.378	F5	
		44	3	4.383	F5:	
219*b*.....	30	11 12	5	4.444	F5	45°
		19	8	4.449	F5	
232......	31	9 02	5	5.354	F0	Seed "23" plate
		06	3	5.357	F1	
245......	Aug. 1	9 28	9	1.006	F3	45°
		34	4	1.010	F2	
271......	3	14 14	11	3.204	G0	45°
		23	8	3.210	G1	

* Luizet, *Annales de l'Université de Lyon*, Nouvelle Série, Fascicule 33, 1912.

MOUNT WILSON SOLAR OBSERVATORY
September 1916

THE DENSITIES OF VISUAL BINARY STARS[1]

By E. ÖPIK

1. The density of a visual binary can be determined if the surface brightnesses of the components are known. Let R be the absolute radius; A, the apparent brightness; I, the surface brightness of the principal component (the corresponding values for the sun $=1$); π, the parallax in seconds of arc; then

$$R=\frac{206265}{\pi}\sqrt{\frac{A}{I}}\,. \tag{1}$$

If M and M_1 are the masses ($\odot=1$) of the components; a, the major axis; t, the period of revolution, we obtain the relation

$$M+M_1=\frac{a^3}{t^2\pi^3}\,. \tag{2}$$

Let δ be the density of the first component; then

$$M=\delta R^3,\ M+M_1=\delta R^3\left(1+\frac{M_1}{M}\right),$$

and from (2)

$$\delta=\frac{a^3\sqrt{\left(\frac{I}{A}\right)^3}}{t^2\left(1+\frac{M_1}{M}\right)\cdot 206265^3}\,. \tag{3}$$

Let m be the stellar (apparent) magnitude of the component; i, the surface brightness, expressed in stellar magnitudes ($i_\odot=0.0$); then, assuming the stellar magnitude of the sun to be -26.60, we obtain:

$$A=10^{-0.4(m+26.6)} \tag{4}$$

$$I=10^{-0.4i}. \tag{5}$$

After substituting these values in (3), it is easy to obtain the following formula:

$$\log\delta=\log\frac{a^3}{t^2}+0.6(m-i)+0.02-\log\left(1+\frac{M_1}{M}\right). \tag{6}$$

[1] *Publications of the Russian Astronomical Society*, No. 3, p. 49, 1915.

The density δ_1 of the other component can be computed if we in (6) instead of m, i, and $\frac{M_1}{M}$ substitute m_1, i_1 and $\frac{M}{M_1}$.

2. The surface brightness depends upon the absolute temperature, and, consequently, upon the spectral type; such a dependence can be derived from the color-index, as has been done by H. N. Russell and Harlow Shapley.[1] I have preferred to use for this purpose the effective temperatures of 109 stars, as determined by Wilsing and Scheiner;[2] the mean temperatures for various spectral classes are given in Table I (the Harvard classification is introduced

TABLE I

Spectral Class	No. of Stars	T Effective Temperature	Spectral Class	No. of Stars	T Effective Temperature
B0–B5......	6	9030°	K.........	35	3970°
B8–A4......	29	8880	M.........	8	2960
A5–F8......	20	5780	Sun.......	(1)	5130
G..........	11	4450			

instead of Vogel's, used by the authors). If L and L_1 denote the wave-lengths of maximum spectral energy of two luminous sources, we obtain from Planck's formula for the relation of their surface brightnesses in the spectral region λ

$$\frac{I}{I_1}=\frac{e^{\frac{4.965L_1}{\lambda}}-1}{e^{\frac{4.965L}{\lambda}}-1}. \tag{7}$$

For "visual" light $\lambda_{\text{eff}}=0.56\mu$, and assuming $\frac{4.965}{\lambda}=8.87=c$, it is easy to obtain for the difference (in stellar magnitude) of visual surface brightnesses of star and sun the equation

$$i=2.5 \log \frac{e^{cL}-1}{e^{cL_\odot}-1}. \tag{8}$$

From Wien's formula, $LT=2940$, L can be computed if T is known.

[1] *Astrophysical Journal*, 40, 417, 1914.

[2] *Astronomische Nachrichten*, 183, 97, 1909.

The equation (8) can be judged as true only when the *emissive powers* for star and sun are equal; that condition will be assumed in the following computations. It is probable that the order of magnitude of the radiating power for such dense and compact cosmical bodies does not differ much from unity in the majority of cases, but for stars of very low density and for nebulae exception must be made. In the case of the sun it is not difficult to confirm our assumption: it is sufficient to compare the solar constant of about 2 calories with the theoretical value corresponding to a temperature of about 6000° (with aid of the Stefan law).

The effective temperatures given by Wilsing and Scheiner are probably systematically too low, e.g., the temperature 5130°, found for the sun, corresponds to a value of the solar constant = 1.16 cal., which is smaller than the amount of radiation directly observable (at Mount Whitney, 1.6–1.7 calories). The much more numerous and direct measures of the sun's spectral energy-distribution, made by Abbot and Fowle, led to a value of $L_\odot = 0.47\mu$ or $T_\odot = 6250°$. We will assume this value; the observations of Wilsing and Scheiner we will consider as differential, and correct them so that the reduced temperature of the sun will equal 6250°.

In the Potsdam observations the intensity of radiation in a given spectral region of the star was compared with a terrestrial source of known energy-distribution or temperature; the star was directly compared with an electric lamp, the lamp with a furnace. In this manner, from the observations, the first part of equation (7) was obtained (I_1 and L_1 denote in this case the intensity of radiation and the spectral maximum of the furnace); that equation gives L if L_1 is known. Let us suppose that the assumed value of L_1 is in error; from (7) it is easy to obtain the differential formula

$$\Delta L = \frac{I_1}{I}\,\frac{e^{cL_1}}{e^{cL}}\Delta L_1,\ c = \frac{4.965}{\lambda}.$$

If cL and cL_1 are not too small (e.g. >3), one can write

$$\frac{e^{cL_1}}{e^{cL}} = \frac{e^{cL_1} - 1}{e^{cL} - 1} = \frac{I}{I_1}\ \text{and}\ \Delta L = \Delta L_1.$$

All values of L are modified by a nearly constant amount; that is approximately true for L not less than 0.3–0.4μ.

According to Wilsing and Scheiner $L_{\odot}=\frac{2940}{5130}\mu=0.57\mu$; according to Abbot, 0.47μ; thus we have $\Delta L=-0.10\mu$=approximately ΔL_1, which corresponds to an error of the absolute temperature of the electric furnace equal to 90° (at a temperature about 1600°). There can doubtless be other sources of error (or, if preferable, systematic difference), but we will for convenience assume the foregoing hypothesis; our only intention is to find a plausible formula for reducing the observations.

Let L' and $L'_{\odot}$ be the maximum-energy wave-lengths corresponding to Wilsing and Scheiner's effective temperatures for star and sun, L and $L_{\odot}$, the reduced values; then, assuming I/I_1 to be correct, we obtain the equation

$$\frac{e^{cL'}-1}{e^{cL'_{\odot}}-1}=\frac{e^{cL}-1}{e^{cL_{\odot}}-1}, \left(c=\frac{4.965}{\lambda}\right) \tag{9}$$

from which L can be found if L, $L_{\odot}$, and $L'_{\odot}$ are given. Theoretically for various λ the result must be the same if Planck's formula can be applied to stars and the observations are correct. The computation was made for $\lambda=0.56\mu$, $c=8.87$, $L_{\odot}=0.47\mu$, $L'_{\odot}=0.57\mu$, $L'=\frac{2940}{T'}$, where T' is the effective temperature; the result is given in Table II.

TABLE II

T' Effective Temperature According to Wilsing and Scheiner	Reduced Values		T' Effective Temperature According to Wilsing and Scheiner	Reduced Values	
	L	T		L	T
2500°	1.090μ	2700°	6000°	0.387μ	7590°
3000	0.886	3320	7000	0.314	9350
3500	0.746	3940	8000	0.258	11,400
4000	0.638	4600	9000	0.213	13,800
4500	0.554	5300	10,000	0.173	16,900
5000	0.488	6030	12,000	0.115	25,500
5500	0.434	6780	13,000	0.089	33,000

With aid of that table the reduced value of L for each star of Wilsing and Scheiner's list was found and the means for various spectral subdivisions were formed. The results are given in

Table III; in the third column L is the mean reduced wave-length of the spectral energy-maximum, the second and fourth columns give the number of stars n and the mean deviation Δ of L for one star; in the last column i denotes the surface brightness, computed with aid of equation (8) and the value of L from the third column. An inspection of the table leads to an interesting conclusion. The variation of temperature with spectral type goes on very irregularly; but it is not permissible to smooth out these

TABLE III

Spectral Class	n	L	Δ	$T=\left(\frac{2940}{L}\right)^\circ$	i Surface Brightness
Bo, B2, B5	5	0.196μ	$\pm$0.07μ	15,000°	−3.0
B8	5	0.194	0.03	15,100	−3.0
Ao	18	0.218	0.04	13,500	−2.8
A2	6	0.260	0.07	11,300	−2.2
A5	2	0.310	....	9480	−1.8
[A8p	1	0.410	....	7160]	(−0.7)
Fo	7	0.350	0.05	8390	−1.2
F5	5	0.422	0.04	6970	−0.6
F8	5	0.422	0.04	6970	−0.6
Go	6	0.548	0.07	5360	+0.7
G5	5	0.586	0.08	5010	+1.1
Ko	28	0.614	0.08	4790	+1.3
K2	4	0.714	0.05	3930	+2.5
K5	3	0.877	0.07	3350	+3.9
Ma	8	0.895	0.03	3280	+4.1
Sun (Go)	....	0.470	....	6250	0.0

numbers, because the spectral classification is not quantitative, but merely qualitative, and it seems not to be correct to assume, for instance, that the spectral classes correspond to equal differences of temperature. The abrupt changes between some stellar classes seem to be real, e.g., between F8 and Go or Ko and K2. If we assume that various temperatures indicate different stages of stellar evolution in a cooling process, and that the fall of temperature goes on more or less uniformly with time, we are led to the conclusion that at some stages the variation of spectral qualities determining the type is very slow (as between F8 and Go, Ko and K2); at others, more rapid.

The sun can be placed between F8 and Go; its temperature is higher than the mean for the Go stars investigated.

3. To compute the density of a binary system the mass-ratio of the components must be known, which in but few cases is known with much precision; for our purposes, however, an approximate mass-ratio is sufficient. In cases when no reliable determinations of M_1/M were available, the masses were assumed to be equal if the difference of magnitudes of the components was $m_1 - m \leqq 0^{M}.8$. For the other cases the following process was used: for 11 stars[1] with known mass-ratio (determined generally by Struve, Auwers, Boss, and in one case each by Lewis and by Seeliger), the values of M_1/M were plotted as ordinates, while the abscissae were $m_1 - m$ (difference of visual magnitude of components); a smoothed curve was drawn, the points of which are represented in Table IV.

TABLE IV

Difference of Magnitude $m_1 - m$	Mass Ratio $M_1 : M$
$0^{M}.0$	1.00
0.8	1.00
1.0	0.95
2.0	0.88
3.0	0.76
5.0	0.60
10.0	0.30

4. The entire data for computing the density are divided into two groups. In the first group are the binaries with $m_1 - m \leqq 0^{M}.8$; assuming the ratio $M_1/M = 1$, we can compute the mean logarithm of density for both components with aid of the modified equation (6):

$$\log \delta = \log \frac{a^3}{t^2} + 0.6\left(\frac{m + m_1}{2} - i\right) - 0.28, \qquad (10)$$

where m and m_1 are the visual magnitudes; i the surface brightness, from Table III, corresponding to the mean spectral type of the system.

For $m_1 - m > 0^{M}.8$ in the majority of cases no data concerning the spectral type of the fainter component were available, and only the density of the brighter component was computed with aid of equation (6).

[1] The stars were: ε Hydrae, ζ Herculis, α Can. Maj., η Cassiopeiae, α Can. Min., ζ Cancri, ξ Ursae Maj., α Centauri, 70 Ophiuchi, ξ Boötis, γ Virginis.

It is evident that the stellar magnitude of star and sun must be referred to the same photometric system; the magnitude of the sun $= -26.60$ corresponds to the Potsdam system, and for this reason the magnitudes of the components were so chosen that the equations

$$10^{-0.4m_0} = 10^{-0.4m} + 10^{-0.4m_1} \tag{11}$$

and

$$m_1 - m = \Delta \tag{12}$$

were fulfilled, where m_0 is the stellar magnitude of the whole binary system according to the Potsdam scale; Δ, the difference of magnitude of the components.

In Tables V and VI are given the results of the computation; in the sixth column the parentheses indicate that the corresponding mass-ratio is found with aid of the empirical relation given in Table IV.

TABLE V

Star	m_0 P. D.	Spectral Type	Stellar Magnitude of Component		Mass-ratio $M_1:M$	$\log \frac{a^3}{t^2}$	Density	
			m	m_1			$\log \delta$	δ ($\odot=1$)
α_1 Centauri	......	G	0.3	......	0.96	9.924–10	−0.60	0.25
α_2 Centauri	......	K5	......	2.1			−1.44	0.036
Stars with $m_1 - m \leqq 0^{\text{M}}8$; mean density of the system								
γ Virginis	2.7	F	3.5	3.5	1.0	7.227–10	−0.23	0.59
δ Equulei	4.7	F5	5.2	5.7	(1.0)	6.682	0.03	1.07
13 Ceti	5.8	F	6.2	6.9	(1.0)	6.226	0.60	4.00
κ Pegasi	4.3	F5	4.7	5.5	(1.0)	6.274	−0.59	0.26
ζ Sagittarii	2.6	A2	3.2	3.4	(1.0)	6.592	−0.39	0.41
β 612	5.8	A	6.5	6.6	(1.0)	5.424	0.75	5.6
9 Argus	5.3	F8	5.8	6.4	(1.0)	6.621	0.36	2.3
82 Ceti	5.6	K	6.3	6.4	(1.0)	6.700	−0.55	0.28
42 Comae	4.6	F5	5.4	5.4	(1.0)	6.604	−0.08	0.83
β Delphini	4.0	F5	4.5	4.9	(1.0)	6.312	−0.79	0.16
20 Persei	5.7	F	6.1	6.9	(1.0)	4.500	−1.16	0.069
η Coronae Bor	5.2	G	5.7	6.2	(1.0)	6.611	−0.52	0.30
ξ Scorpii	4.2	F8	4.8	5.0	(1.0)	6.239	−0.74	0.18
Σ 2173	5.5	G	6.0	6.4	(1.0)	6.845	−0.14	0.72
8 Sextantis	5.3	A2	5.8	6.1	(1.0)	4.956	−0.43	0.37
γ Centauri	......	A	3.2	3.2	(1.0)	6.139	−0.54	0.29
ϕ Ursae Maj	4.7	A2	5.2	5.8	(1.0)	4.517	−1.14	0.072
ω Leonis	5.6	G	6.0	6.8	(1.0)	5.704	−1.16	0.069
γ Coronae Austr	......	F8	5.0	5.0	(1.0)	6.799	−0.12	0.76
Σ 2	6.3	A	6.9	7.2	(1.0)	4.780	0.41	2.6
ζ Cancri	4.8	F	5.2	5.9	1.0	6.258	0.03	1.07

Only in one case, for α Centauri, was it possible to obtain separate densities for both components, because in this case their individual spectral types are known.

In forming the means it is preferable to use the logarithms of δ instead of the densities themselves, because the data on which our computations are based are logarithms (m, m_1, and i), and the

TABLE VI

STARS WITH $m_1 - m > 0^M.8$

Star	m_0	Spectral Type	m	m_1	$M_1 : M$	$\log \frac{a^3}{P^2}$	Density of the Brighter Component: log δ	δ (⊙=1)
ε Hydrae	3.6	F8	3.7	5.7	0.90	5.748−10	−1.93	0.012
ζ Herculis	3.2	G	3.2	6.6	0.43	7.326	−1.31	0.049
Sirius	−1.4	A	−1.4	10.0	0.42	9.252	−0.04	0.91
η Cassiopeiae	3.7	F	3.7	8.0	0.76	8.053	0.77	5.9
Procyon	0.8	F5	0.8	9.6	0.26	8.915	−0.32	0.48
ξ Ursae Maj	3.9	G	4.2	5.1	1.0	7.640	−0.54	0.29
70 Ophiuchi	4.2	K	4.3	6.3	0.82	8.082	−0.36	0.44
ξ Boötis	4.8	K5p	4.9	6.8	0.87	7.750	−1.90	0.013
85 Pegasi	6.0	G	6.0	11.0	0.60	6.856	−0.14	0.72
β 416	5.9	K5	6.0	8.0	(0.89)	7.534	−1.47	0.034
A Cassiopeiae	4.7	A2	4.8	7.3	(0.81)	5.907	−0.13	0.74
τ Cygni	4.0	F	4.0	10.0	(0.53)	6.744	−0.30	0.50
99 Herculis	5.3	F8	5.3	11.0	(0.55)	6.701	0.07	1.17
OΣ235	5.7	F	6.0	7.3	(1.0)	6.117	0.16	1.45
γ Coronae Bor.	4.0	A	4.1	6.9	(0.9)	5.881	−0.24	0.58
Σ 3062	6.1	F	6.4	7.5	(1.0)	6.369	0.65	4.5
25 Can. Venat.	5.0	F	5.1	8.6	(0.72)	5.629	−0.81	0.16

deviations caused by errors of observation, are logarithmic deviations; if converted into numbers, the positive deviations would be greater than the negative for equal errors of observation, and the means would be greater than the true one.

The principal source of error is probably the uncertainty in spectral classification; an error amounting to one spectral class will alter the result some 4 times. In the foregoing tables are four exceptionally high values of δ: 4.0, 5.6, 5.9 (η Cassiopeiae), 4.5. Such densities are very improbable; they correspond to 5.6–8.2 times the density of water, and are probably due to errors in the assumed surface brightness (spectral type); in the case of η Cassiopeiae the elements of the orbit are too uncertain.

As appears from the tables, the densities cover a wide range, but there are a great number of binaries approaching the solar density. From this point of view the visual binaries differ substantially from the Algol variables, which give generally very low densities; it is probable that the visual systems approach much more the mean conditions of the stellar universe, and the study of their densities, masses, etc., can give us a fairly approximate idea of the constitution of the great number of stars with low or moderate luminosity, which form the prevailing mass in the galactic system. The eclipsing variables, however, are known to be exceptions to the general rule; their luminosity is much higher than the average, the stars of early spectral type prevail among them, and numerically they represent but a small portion of all the stars.

In Table VII the number of binaries is arranged according to the density; 10 binaries, or 25 per cent of the total number, have densities greater than the sun. The numbers of Table VII are plotted on Fig. 1 as rectangles; the line represents the hypothetical "density-curve," or the number of stars with a given log δ; the maximum of the curve is at log $\delta = -0.23$ or $\delta = 0.59$.

TABLE VII

Limits of log δ	No. of Binaries	Percentage	Limits of log δ	No. of Binaries	Percentage
−2.0 to −1.5...	2	5	−0.5 to 0.0....	13	32
−1.5 −1.0...	6	15	0.0 0.5....	6	15
−1.0 −0.5...	9	23	0.5 1.0....	4	10

In Table VIII are given the mean values of log δ and the corresponding δ for different spectral classes. There is a marked decrease of density in passing from the early to the later types.

TABLE VIII

Spectral Type	No. of Stars	Mean log δ	Corresponding δ
A0–A5........	9	−0.19	0.65
F0–F8........	19	−0.23	0.59
G............	7	−0.63	0.23
K, K5........	5	−1.14	0.072
Total.....	40	−0.407	0.39

The foregoing results are only rough, because the data are strongly affected by observational errors. Assuming a probable error $=0^{M}.1$ for the visual magnitude and one-fourth spectral class for the spectral data, which corresponds to an error in i equal to $0^{M}.25$, we obtain the probable error of a density-logarithm $= \pm 0.6\sqrt{0.1^2+0.25^2} = \pm 0.16$; that corresponds to 45 per cent of the value to be determined.

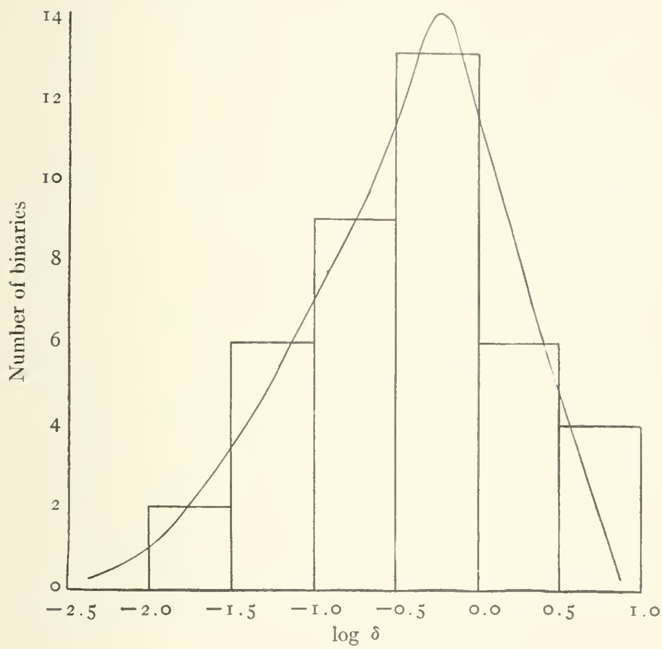

FIG. 1

It is desirable that more accurate determinations of magnitude and spectral type of visual binaries with known orbital elements should be made; instead of spectral type it is sufficient to determine the color-index. When such data for each component of the binary become available, it will be possible to obtain separately the densities of the components, and a very important and interesting region of stellar statistics will be opened.

Among the binaries with known orbital elements there is one which is not included in the preceding discussion, o_2 Eridani; the

components are $9^{M}.2$ and $10^{M}.9$ respectively, and, according to Adams and Pease,[1] of the spectral type A0; the parallax is $0''.17$, and the absolute luminosity of the components must be $\frac{1}{100}$ and $\frac{1}{800}$, respectively, of the sun's—values exceptionally small for A-type stars; the density according to equation (6) ($M_1/M=1$) would be 25,000. This impossible result indicates that in this case our assumptions are wrong; the only possible explanation is that, however high the temperature, the surface brightness or the radiating power is very low; probably o_2 Eridani is a pair of very rarified nebulae.

Moscow, Russia
May 1916

[1] *Publications of the Astronomical Society of the Pacific*, **26**, 258, 1914.

PRELIMINARY OBSERVATIONS OF THE SPECTRA OF CALCIUM AND IRON WHEN PRODUCED BY CATHODO-LUMINESCENCE[1]

By ARTHUR S. KING and EDNA CARTER

The excitation of a vapor to luminescence by means of a stream of cathode rays directed into it furnishes a source different from those in which the radiation is produced by high temperature or by the conduction of a current through the vapor. The purpose of the present investigation was to study the peculiarities of the spectrum given by the cathode-ray excitation for certain elements having groups of lines known to show different behavior in the ordinary sources.

The possibility of producing spectra in this way was shown by Hertz,[2] who obtained the stronger mercury lines, and by Lewis,[3] who examined the spectra of several of the more easily vaporized metals, the vapor being produced by heating the substance in a hard glass tube by means of a flame and directing a stream of cathode rays into the tube. In each case a spectrum of what seemed to be the more fundamental lines of the element was observed. The present writers have been aided by valuable suggestions from Professor Lewis, obtained during a discussion of the problem.

The heating effects regularly observed in X-ray tubes since the introduction of tungsten targets made it appear feasible to work with some of the more refractory metals by heating them with a concentrated stream of cathode particles. When brought to luminescence the vapor in the cathode stream could then be observed at a sufficient distance from the heated surface to avoid the direct effects of high temperature.

The first apparatus constructed was a modified X-ray bulb, with concave cathode and disk anode in the usual positions. The

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 125.

[2] *Wiedemanns Annalen*, **19**, 809, 1883.

[3] *Astrophysical Journal*, **16**, 31, 1902.

cathode stream was directed vertically downward upon an anti-cathode consisting of a tungsten dish containing metallic calcium, supported on the end of a silica tube passing into the bulb through the end opposite the cathode. When the bulb was pumped out to a vacuum sufficient to give in air a parallel spark 10–15 cm long, the calcium was readily vaporized and the pencil of cathode rays above the dish showed, when observed visually with a small spectroscope, a spectrum containing a considerable number of lines. The lines H, K, and λ 4227 were photographed with a 1-meter concave grating.

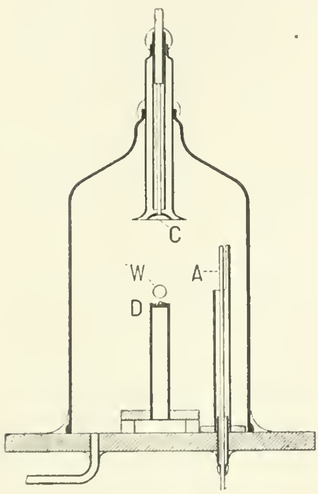

Fig. 1

A more substantial apparatus was then made from a bell-jar supported on a thick glass plate, as shown in Fig. 1. The cathode C is 25 mm in diameter, concave to such a degree as to focus the rays at 9.5 cm when the spark crosses a parallel air-gap of 15 cm. It is screwed to an aluminum rod passing through the top of the bell-jar, while a flaring glass tube around the electrode protects the wall of the jar from the cathode discharge. The anode A is a vertical aluminum rod 5 mm in diameter, passing through the glass base and inclosed by a glass tube, the inner side of which is cut away for a length of 4 cm below the end of the rod in order to expose the anode and at the same time protect the wall of the jar. The anti-cathode D is supported by a silica tube of 15 mm bore, held vertically by being set in a glass plate resting on the base plate. This tube holds at its top either a tungsten dish containing calcium or a shallow iron dish with iron filings. The dish in each case is at the focus of the cathode and is several centimeters away from the direct discharge between anode and cathode. As the experiments of Lewis with metals and with nitrogen[1] showed clearly the possibility of bringing gases outside the path of the conduction

[1] *Astrophysical Journal*, 17, 258, 1903.

current to luminescence by cathode rays, no special examination of this point was made during the present investigation. A tube through the base-plate for the pump connection and a side tube inserted at the opening *W* in the bell-jar and closed by a quartz window completed the fittings. The joints in the apparatus were cemented either with sealing-wax or with an odorless black wax of nearly the same consistency. The discharge was obtained from a large induction coil driven by an electrolytic interrupter of the Simon type, which, when the vacuum was sufficiently high, gave little inverse current.

When calcium was vaporized by the cathode stream, the vapor immediately absorbed the residue of air—an effect noted by Saunders[1]—which made the vacuum so high that the discharge would not pass until a little air was admitted. The lumps of metallic calcium were found to be almost consumed by the discharge, which could not be closely watched on account of an opaque deposit that blackened all parts of the chamber except the window toward the spectrograph. The iron was raised only to a white heat, the filings not being fully fused. However, sufficient iron vapor was given off in the high vacuum to enable the cathode rays to yield a spectrum. In focusing the light on the spectrograph, the image of the metal anti-cathode was kept well off the slit, the light photographed coming from the cathode stream about a centimeter above the heated metal.

As the experiments will be discontinued for a short time, pending the installation of apparatus which may be expected to give much stronger effects, the following results are presented in the belief that they are typical of what such a discharge will yield. The spectra were photographed in the first order of a 1-meter concave grating, the scale being 1 mm = 17 A. The metallic lines were very sharp and the measurements for identification usually agreed with the published values within 0.02 A. Two reasonably strong spectrograms were obtained for calcium, in which the 26 lines listed in Table I were identified. These were supported by three weaker photographs giving the same effects for such lines as appeared. Three photographs of the iron spectrum were made, which were

[1] *Ibid.*, 40, 377, 1914.

also concordant in the effects shown. While the 42 iron lines obtained are a small fraction from the rich region between λ 3400 and λ 4400, they show the class of lines which appear under these conditions.

RESULTS

Calcium.—Table I gives a comparison of the relative intensities of the lines in the luminescence spectrum and in that of the arc in air. An arc spectrum of very short exposure on the same kind of film was used in order to obtain a general intensity comparable with that of the spectrum excited by the cathode discharge. In the fourth column is entered the symbol used by Saunders[1] to denote the series to which the line belongs.

TABLE I

LUMINESCENCE SPECTRUM OF CALCIUM

λ (Exner and Haschek)	Arc in Air	Luminescence Spectrum	Series	λ (Exner and Haschek)	Arc in Air	Luminescence Spectrum	Series
3179.50....	1	1	P_1	4283.20...	15	....	p
3181.43....	trace	trace	P_1	4289.50...	15	....	T
3350.25....	2	....	T_1	4299.18...	10	....	T
3361.95....	4	....	T_1	4302.70...	20	1	p
3624.19....	4	....	T_1	4307.90...	15	....	......
3630.87....	8	2	T_1	4318.80...	15	....	T
3644.53....	15	4	T_1	4355.50...	2	7	SL_3
3706.18....	1	4	P_2	4425.60...	20	3	T_1
3737.06....	2	8	P_2	4435.17...	30	8	T_1
3933.81....	80	40	PH	4435.88...	8	1	T_1
3949.10....	1	....	T_2	4455.00...	40	15	T_1
3957.22....	4	trace	T_2	4456.10...	10	1	T_1
3968.63....	60	30	PH	4527.35...	3	8	SL_2
3973.91....	6	2	T_2	4578.88...	3	....	t
4093.00....	1	....	t	4581.77...	6	1	t
4095.30....	2	....	t	4586.22...	8	2	t
4098.9.....	4	....	t	4685.35...	trace	5	......
4108.60....	trace	4	SL_3	4878.38...	3	4	SL_3
4226.90....	100R	400	SL_1	5041.83...	1	1	SL_2
4240.61....	1	4	SL_2				

In the luminescence spectrum, the lines H, K, and λ 4227 are much the strongest, and perhaps the most striking feature is the phenomenal intensity, combined with sharpness, of λ 4227. In the arc and furnace, even in vacuum, this line widens and reverses

[1] *Astrophysical Journal*, 32, 154, 1910.

more strongly than any other line in the calcium spectrum. Its intensity, so far as this can be based on widening, has seemed a reliable indication of the vapor-density of the source. When excited by cathode rays in the present experiments, a strong central maximum is present with apparently no tendency toward reversal. This is illustrated by the fact that while the grating used does not usually give perceptible ghosts, even of very strong lines, the first-order ghosts of λ 4227 are now of about the same intensity as the H line. Apparently we have here a low vapor-density, resulting in an extremely narrow line, combined with other conditions highly favorable for the production of λ 4227. The strength of λ 4227 is the only resemblance to the furnace spectrum. The luminescence spectrum brings out strongly H and K and the enhanced pair λ 3706 and λ 3737. The latter have not been obtained in the furnace, and H and K are weak in the furnace as compared with the arc. λ 3159.01, belonging to the same series as the first two lines in Table I, is probably present, but blends with the diffuse head of a nitrogen band.

Another noteworthy feature of the luminescence spectrum is the weakness of the group of six lines near λ 4300, of which only the strongest, λ 4302.70, was photographed. This group is one of the least variable in the usual sources, including the furnace at different temperatures.

Table I shows further that the series triplets are uniformly weak in the cathode discharge as compared with the arc, while the members of the "single-line" (SL) series of Saunders are much stronger than in the arc. Some of these, such as λ 4108.60, are diffuse in the arc in air, and are rendered more distinct by any of the vacuum sources, but this does not account for the high relative intensity obtained here. Saunders[1] has recently placed λ 4227 also in a single-line series, of which the other members are in the ultraviolet.

If a general feature of this discharge is an excitation of the more fundamental vibrations, as the experiments of Lewis seemed to indicate, the several series, consisting of single lines, of pairs, and of triplets, respectively, are of increasing complexity in their

[1] *Ibid.*, 43, 234, 1916.

vibrations, since they are arranged in this order in the facility with which they are given by the cathode luminescence.

There is no evidence that new lines are produced by the cathode discharge and, with the exception of certain nitrogen bands referred to later on, all lines in the luminescence spectrum have been identified with known lines of calcium.

TABLE II

LUMINESCENCE SPECTRUM OF IRON

λ (Rowland)	Int.	λ (Rowland)	Int.
3440.762 } 3441.155 }	4	3767.341	trace
		3813.100	trace
3570.273	3	3815.987	1
3581.349	10	3820.586	4
3609.008	1	3824.591	2
3618.919	1	3826.027	2
3631.605	1	3834.364	1
3647.988	1	3840.580	trace
3680.069	trace	3856.524	3
3687.610	trace	3860.055	15
3705.708	1	3878.720	2
3720.084	20	3886.434	5
3722.729	1	3920.410	1
3727.778	trace	3923.054	1
3735.014	7	3928.075	1
3737.281	12	3930.450	1
3745.717	8	4045.975	1
3748.408	4	4308.081	trace
3749.631	4	4325.939	trace
3758.375	2	4383.720	2
3763.945	1	4404.927	trace

Iron.—The list of iron lines identified, with their estimated intensities, is given in Table II. Their general characteristic is that they are the strongest lines in the flame, furnace, arc, and spark spectra. Hemsalech and De Watteville[1] observed all of the lines listed in Table II, and a few others in the oxyhydrogen flame. Although considerable differences in relative intensity appear between the flame and the cathode excitation, the latter seeming stronger in the ultra-violet, we cannot tell how far differences in contrast and in the treatment of plates may account for the variations observed. The strongest lines of the luminescence spectrum

[1] *Comptes Rendus*, 146, 962, 1908.

are also the strongest in the flame, but many very faint luminescence lines are given as of considerable strength in the flame.

The furnace material available, part of which has not been published, shows that the lines of Table II appear at low temperatures, below 2000° C., and reverse readily at temperatures above 2300°. They would be placed uniformly in Class II of the furnace classification. In the arc and the condensed spark these are among the strongest lines, reversing readily with high vapor-density. The strongest lines of the luminescence spectrum, λλ 3581, 3720, 3737, 3860, are of great strength in all of the usual sources. The lines most characteristic of the furnace (Class I) and the enhanced lines of the spark are, however, absent from the luminescence spectrum. While this may be due in some degree to the general weakness of the latter in the photographs obtained, it can at least be said that such lines do not dominate the spectrum, as they do in the sources favorable to them. A fair statement would be that the lines thus far obtained are those of an under-exposed arc spectrum of iron. When more lines are photographed we may expect, among various groups, differences similar to those occurring in the calcium spectrum.

Band spectrum of nitrogen.—The following nitrogen bands appeared on the photographs of the luminescence spectrum:

Positive Pole Bands	Negative Pole Bands
3158.9	3581.5
3371.2	3914.4
3536.5	4236.3
3576.9	4278.0
3755.2	4708.6
3804.9	
4059.0	

While no attempt was made to locate closely the region of production, the appearance of the bands under these conditions is in harmony with the conclusions of Lewis,[1] that the negative pole bands are produced by the impact of cathode rays, while the positive pole bands are given in all parts of the chamber. The concentration of the cathode discharge should favor the negative bands, and

[1] *Astrophysical Journal*, 17, 258, 1903.

this is borne out by the fact that they are much the stronger. The heads of the positive pole bands appear as diffuse lines, while the negative bands λ 3914 and λ 4278 are strong enough to show considerable structure. These bands present the curious appearance of fading out a short distance from the head and then continuing again. This was observed by Deslandres[1] to be peculiar to low pressures, and to result from the suppression under these conditions of one of the series of lines composing the band, only a few of the first members of this series appearing, while at atmospheric pressure the full structure is present.

SUMMARY

1. A method, applicable to a wide variety of elements, has been developed for the production of metallic spectra through the luminescence excited by cathode rays.

2. The spectrum of calcium in this discharge consists of lines present in the arc, but differing in relative intensities from the arc and other sources. The lines of the several single-line series are strongest, followed by the lines occurring in pairs, while the triplet series are relatively faint.

3. The iron lines thus far obtained belong to what appears to be the fundamental group in the iron spectrum, strong in all of the usual sources. Lines characteristic of the low-temperature sources and of the spark are absent.

4. Bands of the positive and negative pole spectra of nitrogen have been photographed, the negative bands having high intensity.

Mount Wilson Solar Observatory
October 9, 1916

[1] *Comptes Rendus*, **139**, 1174, 1904.

OBSERVATIONAL EVIDENCE THAT THE RELATIVE POSITIONS OF FRAUNHOFER LINES ARE NOT SYSTEMATICALLY AFFECTED BY ANOMALOUS DISPERSION[1]

By CHARLES E. ST. JOHN

I. INTRODUCTION

The suggested mutual influence of Fraunhofer lines, a quasi-repulsion between closely adjacent lines, offers a ready means of testing the theory of anomalous dispersion in the solar atmosphere. Professor Julius[2] developed the deduction and applied it to the displacements between the center and limb edges of the penumbrae of sun-spots given in my paper on "Radial Motion in Sun-Spots."[3] He found what he considered positive evidence of the effect. I rediscussed[4] the data from this point of view, including a large number of omitted lines fulfilling the conditions of selection, and calling attention to errors involved in the method used by Julius which introduced systematic differences of the sign required by the hypothesis. Of this discussion Julius says:

> A few months later St. John published the elaborate article in which he criticized my treatment of his observations very severely, in several respects justly. I had not overcome the difficulties of handling rightly the Mount Wilson data, nor had I entirely avoided bias. St. John made certain alterations in the method of grouping and comparing the measured displacements, added a number of omitted and of new cases, and thus reached the conclusion that there was no indication at all of a mutual influence.
>
> Although I am satisfied by St. John's improved discussion of the data that, in the Evershed effect, mutual influence is not so conspicuous as my defective treatment of those measurements had made it appear, I still believe that future research will bring it to light.[5]

The magnitude of the effect in this case Julius now says can scarcely be expected, under the most favorable conditions, to exceed

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 123.

[2] *Astrophysical Journal*, 40, 1, 1914.

[3] *Mt. Wilson Contr.*, No. 69; *Astrophysical Journal*, 37, 327, 1913.

[4] *Mt. Wilson Contr.*, No. 93; *Astrophysical Journal*, 41, 28, 1915.

[5] *Astrophysical Journal*, 43, 49–50, 1916.

0.002 A, a quantity so near the limit of precision for such observations that its definitive establishment must prove a matter of extreme difficulty, a quantity so small that the question becomes one of theoretical rather than practical interest to the astrophysicist.

Albrecht[1] found by comparing the Rowland and International wave-lengths of iron lines that the violet and red components of close solar pairs showed, relative to the general displacement, specific displacements to the violet and red of 0.007 and 0.005 A, respectively, for a mean separation of 0.22 A from the companion line. These results he interpreted as effects of anomalous refraction, an interpretation approved by Julius, who accepts the data as definitely establishing the existence of mutual influence between the lines in the general solar spectrum.[2]

II. SUN-ARC DISPLACEMENTS

Quantities of the order given by Albrecht's investigations are well within the range of present instruments and methods. The displacements of the solar lines in sun-arc comparisons furnish a direct and suitable means of investigating the question, as a difference of 0.012 A between the displacements of lines with violet and red companions is greater than the average value of the sun-arc displacements, quantities determinable with high accuracy. Such data, center of sun *minus* center of arc, have been accumulating at this Observatory for three or four years. The early observations were made with the 75-foot spectrograph. The length of the plates (90 cm) and the scale (1 mm = 0.7 A) are such that a range of spectrum of 600 A is covered by a single exposure. This has insured such interlocking that the results may be considered free from serious relative errors. Recently a series of plates was begun under still more refined conditions with the remodeled 30-foot spectrograph of the 60-foot tower telescope. Measurements now in progress upon these plates are confirming the absolute values previously obtained for the groups of stable arc lines.

[1] *Astrophysical Journal*, 41, 333, 1915.

[2] *Ibid.*, 43, 53, 1916.

In the Mount Wilson classification of iron lines,[1] those whose wave-lengths are independent of arc conditions at constant pressure are distributed among groups *a*, *b*, and *c*4; to groups *c*5, *d*, and *e* are assigned the lines subject to pole-effect,[2] whose wave-lengths therefore vary with the current-strength and the part of the arc used.

A. *Groups a, b, and c*4.—In the present situation as to solar observations, conclusions based upon the relative behavior, in the sun and arc, of lines belonging to groups *a*, *b*, and *c*4 carry the maximum weight. Of these groups 211 lines, sufficiently separated from neighboring lines for measurement of precision and free from known blends, have been examined for sun-arc displacements.

An investigation entitled "The Accuracy Obtainable in the Measured Separation of Close Solar Lines; Systematic Errors in the Rowland Table for Such Lines,"[3] has recently been concluded. This has furnished the means for determining the minimum separation between lines in the solar spectrum, consistent with their accurate measurement upon spectrograms taken with the instrument employed in obtaining these sun-arc displacements.

The data relative to all lines of these groups having close companions are given in Tables I and II. To identify the lines common to Albrecht's list and mine, his values are shown in the last column. The first section of each table contains measurements free from known sources of error. The measurements in the second section are of little or no weight, the lines themselves being blends or the adjacent lines too near.

The 56 lines having companions to the violet, mean separation 0.275 A, and the 29 having companions to the red, mean separation 0.320 A, show displacements of +0.0036 and +0.0038 A, respectively, in agreement with the mean of 0.0038 A from the 211 lines, instead of approximately +0.009 A and −0.003 A, values required to harmonize with the observations of Albrecht,

[1] *Transactions of the International Union for Co-operation in Solar Research*, 4, 74, 1913.

[2] St. John and Babcock, "A Study of the Pole-Effect in the Iron Arc," *Mt. Wilson Contr.*, No. 106; *Astrophysical Journal*, 42, 231, 1915.

[3] St. John and Ware, *Mt. Wilson Contr.*, No. 120; *Astrophysical Journal*, 44, 15, 1916.

if the deviations found by him represent actual shifts of 0.005 and 0.007 to the red and violet relative to the general displacement of +0.004 A. The behavior of these 85 lines with closely adjacent companions, lines of high quality in both solar and arc spectra,

TABLE I

RELATIVE DISPLACEMENTS OF SOLAR AND ARC LINES OF IRON, GROUPS *a*, *b*, AND *c*4 COMPANIONS TOWARD THE VIOLET IN SOLAR SPECTRUM

SECTION A. MEASUREMENTS OF WEIGHT

λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun *minus* Arc	Albrecht	λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun *minus* Arc	Albrecht
3964.663...	2: 3	.247	+0.001	....	4443.365..	1:3	.369	+0.003	....
3969.413...	6:10	.527	+ .013	−0.004	4531.327..	2:5	.204	+ .004	−0.002
3977.009...	3: 2	.170	+ .003	....	4556.306..	3:4	.243	+ .004	+ .002
4000.611...	2: 2	.208	+ .002	....	4592.840..	2:4	.133	+ .004	− .010
4001.814...	1: 3	.219	+ .007	....	4741.718..	1:3	.456	+ .001	....
4067.429...	5: 3	.290	+ .007	....	4787.003..	3:2	.276	+ .003	− .012
4087.252...	1: 3	.391	+ .004	....	4789.849..	2:3	.321	+ .002	− .012
4123.907...	1: 5	.243	+ .006	....	4939.868..	2:4	.452	+ .004	− .002
4133.062...	1: 4	.199	+ .005	....	5041.255..	3:4	.186	.000	− .002
4134.840...	3: 5	.251	+ .004	+ .006	5041.936..	2:4	.141	+ .012	+ .001
4137.156...	4: 6	.478	+ .002	....	5051.825..	1:4	.142	+ .006	....
4146.225...	1: 3	.311	.000	....	5079.409..	3:4	.251	+ .003	− .006
4147.836...	2: 4	.334	.000	+ .002	5098.885..	1:3	.134	+ .002	− .005
4149.533...	2: 4	.173	− .001	....	5107.823..	4:4	.204	+ .006	− .004
4152.343...	2: 3	.235	− .006	....	5143.111..	2:3	.153	+ .004	....
4156.970...	1: 3	.139	+ .001	....	5227.362..	3:5	.319	+ .002	− .003
4172.923...	2: 4	.120	− .003	....	5250.817..	2:3	.432	+ .003	....
4174.095...	3: 3	.385	+ .008	....	5328.696..	2:2	.181	+ .006	....
4191.843...	6: 3	.248	+ .006	+ .003	5333.089..	1:4	.247	+ .006	+ .005
4202.198...	1: 8	.329	+ .006	....	5365.596..	5:3	.527	+ .002	+ .003
4220.509...	1: 3	.297	− .002	....	5405.989..	1:6	.435	+ .006	....
4242.897...	2: 2	.131	− .004	....	5429.911..	1:6	.194	+ .007	....
4282.565...	2: 5	.438	+ .004	....	5447.130..	2:6	.333	+ .004	+ .003
4290.542...	2: 1	.165	+ .002	....	6137.210..	8:3	.381	+ .006	....
4291.630...	1: 2	.255	+ .002	+ .007	6191.779..	6:9	.386	+ .005	−0.004
4315.262...	3: 4	.124	+ .006	− .007	6462.965..	5:3	.181	+0.008	....
4369.941...	1: 4	.373	+ .003	....					
4427.482...	2: 5	.216	+ .004	.000	Mean, 56 lines.......		.275	+0.0036	....
4430.785...	1: 3	.429	+ .001	....	Mean, 23 lines, Albrecht............		.283	−0.0018	....
4435.321...	5: 2	.192	+0.005	....					

SECTION B. MEASUREMENTS WITHOUT WEIGHT

4294.301...	2: 5	.097	−0.013	−0.005	5167.678..	15:5	.181	+0.019	−0.016
4407.871...	2: 4	.061	− .027	+0.011	6254.456..	1:5	.074	−0.014	+0.017
4547.192...	1: 2	.091	+ .003	....					
4552.725...	2: 1	.093	+ .006	....					
4633.100...	1: 4	.109	+0.001	....	Mean, 7 lines........		.101	−0.0036	....

fails to show the large differences found by Albrecht, but, on the other hand, furnishes strong observational evidence that the sun-arc shifts for such lines are the same as for isolated lines and are therefore not measurably affected by mutual influence.

B. *Groups c*5, *d*, *and e.*—Measurements upon iron lines of groups *c*5, *d*, and *e* are so easily affected by conditions in the arc, that con-

sistent results can be expected only when these conditions are constant. In obtaining the sun-arc displacements at Mount Wilson, the Pfund arc used had a length of 6 mm and was fed by a current of 6 amperes from a 110-volt storage battery. The slit was normal to the axis and in the equatorial zone of a two-and-one-half-fold enlarged image produced by an achromatic lens. Under

TABLE II

Relative Displacements of Solar and Arc Lines of Iron, Groups a, b, and $c4$ Companions toward the Red in the Solar Spectrum

SECTION A. MEASUREMENTS OF WEIGHT

λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun *minus* Arc	Albrecht	λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun *minus* Arc	Albrecht
4000.403...	2:2	.218	0.000	....	4454.552..	5:3	.401	+0.002	+0.003
4030.339...	5:2	.307	− .001	....	4461.818..	3:4	.347	+ .004	+ .008
4067.139...	3:5	.290	+ .011	....	4466.727..	1:5	.375	+ .008	....
4082.264...	3:2	.325	− .003	....	4489.911..	3:4	.342	+ .006	+ .004
4096.129...	2:3	.133	+ .003	....	4619.468..	1:3	.243	+ .004	....
4098.335...	4:5	.354	.000	....	5079.921..	1:4	.223	+ .003	....
4136.678...	6:4	.478	+ .005	....	5107.619..	4:4	.204	+ .004	+ .008
4140.089...	3:6	.469	+ .006	....	5131.642..	1:2	.300	+ .003	....
4142.025...	2:4	.305	+ .001	....	5195.113..	2:4	.534	+ .004	+ .006
4171.068...	4:4	.145	+ .006	....	5328.236..	2:8	.279	+ .011	− .007
4175.082...	1:4	.210	− .001	....	5341.213..	1:7	.124	+ .010	....
4224.337...	3:4	.336	+ .005	....	5397.344..	1:7	.478	+ .008	....
4226.584...	20:2	.320	+ .006	....	6136.829..	3:8	.381	+0 002	+0.004
4267.122...	2:3	.421	− .006	....					
4337.216...	3:5	.509	+ .004	+0.002	Mean, 29 lines.......		.320	+0.0038	....
4351.711...	5:2	.219	+0.004	....	Mean, 8 lines, Albrecht.............		.375	....	+0.0035

SECTION B. MEASUREMENTS WITHOUT WEIGHT

λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun *minus* Arc	Albrecht	λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun *minus* Arc	Albrecht
3997.547...	2:4	.091	+0.022	+0.001	4476.185	3:4	.068	+0.025	+0.008
4204.101...	4:3	.062	− .011	+ .048	5012.252..	1:4	.083	+ .017	− .002
4210.494...	3:4	.067	+ .002	+0.027	5169.069..	4:3	.151	+ .015	....
4245.422...	2:4	.098	+ .006	....	5476.500..	3:1	.278	0.000	+0.011
4367.749...	2:5	.090	+ .005	....					
4391.123...	1:2	.069	+0.003	....	Mean, 10 lines.......		.105	+0.0084	....

the constant conditions, the displacements for lines of groups $c5$, d, and e, relative to each other, are probably free from serious error, and comparison between the behavior of lines with companions, and that of isolated lines, can yield results of weight.

The Mount Wilson data include 125 lines of the related groups $c5$ and d, and 34 lines of group e. The mean sun-arc displacement for the 125 lines of groups $c5$ and d is -0.0063 Å. As the displacements for these lines show an increase for lines in the red, the mean based upon the lines in the region covered by the lines to be

tested is taken as the standard or normal displacement. The mean displacement for the 34 lines of group *e* is +0.0142 A. Lines of these groups were included by Albrecht in reaching the conclusion that mutual influence displaces the violet components 0.007 A less and the red components 0.005 A more than the average. It is evident that a comparison, to have definite meaning, must be between lines of the same or similar groups. For lines of groups *c*5 and *d* the displacements of the violet and red components of close pairs should then be of the order of −0.012 A and 0, respectively; for lines of group *e* +0.007 and +0.019 A. An inspection of Tables III, IV, and V shows no indication of such systematic differences, but rather a remarkable agreement between lines in the open and lines with close companions, an agreement, however, in large measure fortuitous for arc lines of this character.

TABLE III

RELATIVE DISPLACEMENTS OF THE SOLAR AND ARC LINES OF IRON, GROUPS *c*5 AND *d* COMPANIONS TOWARD THE VIOLET IN SOLAR SPECTRUM

SECTION A. MEASUREMENTS OF WEIGHT

λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun *minus* Arc	Albrecht	λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun *minus* Arc	Albrecht
4024.881...	3:4	.155	−0.007	....	4983.433..	2:3	.439	−0.021	....
4083.917...	4:1	.134	− .003	....	4985.730..	3:3	.298	− .006	....
4154.976...	4:4	.309	− .002	....	5006.306..	4:5	.410	− .005	....
4187.204...	2:6	.426	− .004	....	5137.558..	3:3	.308	+ .002	....
4227.606...	20:4	.702	− .004	−0.004	5208.776..	5:2	.180	− .005	−0.006
4233.772...	4:6	.444	− .004	+ .002	5615.877..	2:6	.357	.000	− .004
4247.591...	1:4	.127	− .007	....	5641.667..	1:2	.461	− .016	....
4250.287...	2:8	.490	− .003	....	5655.715..	1:2	.320	− .008	....
4469.545...	1:4	.229	.000	....	5659.052..	1:4	.299	−0.008	−0.003
4607.831...	1:4	.321	− .003	....					
4872.332...	1:4	.220	− .003	− .006	Mean, 21 lines.......		.333	−0.0052	....
4885.620...	2:3	.356	−0.003	+0.003	Standard displacement.		......	−0.0050	....

SECTION B. MEASUREMENTS WITHOUT WEIGHT

4668.331...	2:4	.088	−0.011	−0.010	5139.644*.	4:4	.217	+0.003	+0.002
4957.785*..	4:4	.305	+0.018	−0.004	Mean, 3 lines........		......	+0.003	....

* Blend.

A comprehensive view of the results of the discussion based upon the behavior of 372 iron lines for which the measurements of the relative sun-arc displacements are of sufficient weight to furnish conclusions of value, and of which 142 have close companions, may

be had from Table VI, where the deviations from the mean are accounted favorable to the anomalous-dispersion hypothesis when

TABLE IV

Relative Displacements of the Solar and Arc Lines of Iron, Groups *c*5 and *d*
Companions toward the Red in the Solar Spectrum

SECTION A. MEASUREMENTS OF WEIGHT

λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun *minus* Arc	Albrecht	λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun *minus* Arc	Albrecht
4101.421...	3:2	.419	+0.002	....	5125.300..	1:3	.123	−0.011	−0.002
4109.215...	1:3	.394	− .003	....	5139.427..	4:4	.217	− .002	+ .006
4158.959...	5:5	.394	− .007	....	5273.339..	2:3	.219	− .004	+ .004
4191.595...	3:6	.248	− .003	....	5283.802..	1:6	.479	− .002	....
4196.372...	2:4	.327	− .006	....	5476.778..	5:3	.345	− .004	+ .015
4225.619...	1:3	.255	− .005	....	5573.075..	1:6	.253	− .003	....
4236.112...	1:8	.242	− .003	....	5662.744..	1:4	.411	− .009	....
4859.928...	30:4	.599	− .004	....	5709.601..	5:5	.174	− .007	....
4910.198...	2:3	.307	− .005	....	5952.943..	1:4	.443	− .016	....
4938.997...	2:4	.419	− .005	+0.005	6246.535..	2:8	.239	− .008	....
4957.480...	8:5	.305	− .008	+ .003	6400.217..	2:8	.321	−0.007	+0.001
4982.682...	2:4	.312	− .023	+ .013					
4985.432...	3:3	.298	− .007	+ .009	Mean, 25 lines.......		.326	−0.0062	....
5005.896...	5:4	.410	−0.005	+0.002	Standard displacement.		......	−0.0063	....

SECTION B. MEASUREMENTS WITHOUT WEIGHT

λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun *minus* Arc	Albrecht	λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun *minus* Arc	Albrecht
4125.776...	1:3	.074	0.000	....	4707.457..	2:5	.215	−0.009	+0.006*
4210.494...	3:4	.067	+0.002	+0.027	4727.582..	2:3	.094	+0.014	+0.003
					Mean, 4 lines........		......	+0.002	....

* Blend.

TABLE V

Relative Displacements of Solar and Arc Lines of Iron, Group *e*

Companions toward Violet					Companions toward Red				
λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun *minus* Arc	Albrecht	λ Rowland	Int. Ratio Comp. to Line	Sep.	Sun *minus* Arc	Albrecht
5195.647...	4:2	.534	+0.007	−0.001	5365.069..	3:5	.527	+0.018	0.000
5370.166...	1:6	.384	+ .013	....	5411.124..	1:4	.304	+ .016	....
5404.357...	2:5	.329	+ .015	− .012	5463.174..	3:3	.320	+ .014	+ .009
5463.494...	3:3	.320	+ .018	− .002	5598.524..	4:4	.187	+ .016	+ .008
5930.406...	2:6	.508	+ .009	....	6078.710..	2:5	.517	+0.014	−0.002
6020.401...	2:4	.169	+0.004	−0.008					
Mean..............		.374	+0.0110	....	Mean................		.371	+0.0156	....

Standard displacement.............................. +0.0142 A

they show that the displacements of the red and violet components of close solar pairs are respectively greater and less than the mean for all lines of the group, as should be the case if mutual influence

were superposing upon the general displacement a measurable shift of the red and violet components of close pairs to the red and violet, respectively. In sign the mean result is unfavorable to the anomalous-dispersion hypothesis, but in magnitude it is practically zero, being 0.0003 A. On the assumption that the Albrecht deviations are real shifts, the displacements of lines with violet companions should exceed those of lines with red companions by 0.010–0.012 A. They are, however, within the limits of error, identical.

TABLE VI

RÉSUMÉ OF BEARING OF SUN-ARC DISPLACEMENTS ON MUTUAL INFLUENCE

No. of Lines	Group	Companion	Sun *minus* Arc	Normal	Summation of Deviations	
					Favorable	Unfavorable
56.	*a, b, c*4	To Violet	+0.0036	+0.0038	. . . .	−0.0112
29.	*a, b, c*4	Red	+ .0038	+ .0038	0.0000	. . . .
21.	*c*5, *d*	Violet	− .0052	− .0050	. . . .	− .0042
25	*c*5, *d*	Red	− .0062	− .0063	. . . .	− .0025
6.	*e*	Violet	+ .0110	+ .0142	. . . .	− .0192
5.	*e*	Red	+0.0156	+0.0142	. . . .	−0.0070
142						

Mean deviation unfavorable to mutual influence. . . . . −0.0003A

The data for the lines in the second sections of Tables I, II, III, and IV are given in order to make the presentation of the Mount Wilson observations complete. Because of the recognized sources of possible error, they have not been considered in the discussion; their inclusion, however, would tend to increase the foregoing nfavorable deviation from the mean. Of these rejected lines the 17 used by Albrecht are more fully considered in the fourth division of this contribution.

The evidence of equality between the displacements of lines with companions to the violet and red, respectively, given by these 142 lines for which the conditions are favorable to the evoking of mutual influence, is clear and positive. It seems, therefore, a justifiable conclusion from this equality that mutual repulsion does not occur to a measurable amount, and that some other explanation applies to the Albrecht deviations from the mean.

III. RELATIVE SEPARATION IN SOLAR AND ARC SPECTRA

Mutual influence appearing as a quasi-repulsion requires that the difference in wave-length between a line and a closely adjacent companion be greater in the solar spectrum than when determined in terrestrial sources. If the interpretation adopted by Albrecht for his observations be the correct one, the separation in the solar spectrum, when the mean distance between the components is 0.22 A, should exceed that in terrestrial sources by 0.012 A, a quantity easily within the range of measurement. The measurements considered in this section were made upon separate plates for the sun and arc. The solar measurements, being more difficult than the corresponding ones for the arc, have been made upon plates of higher dispersion, with a scale sufficient to yield reliable results for the particular combination of intensity and spacing.[1] The conditions for the sun-arc measurements and for the determinations of separation considered in this section are so different, that it seems hardly probable that personal equation enters in the same manner or to the same degree in both, or that different observers would obtain concordant results, as they do, if personal equation introduced appreciable systematic errors. Four measurers have worked upon the sun-arc plates, and two of these have made the major part of the determinations recorded in Table VII, which form the subject-matter of this section.

For the 45 pairs the mean difference between the separations in the solar and arc spectra is zero within the limit of precision. According to the theory of mutual influence, the repulsion should be the more marked, the closer the adjacent lines. Of the pairs in Table VII, 13 have separations under 0.2 A, the mean being 0.148 A. For these pairs the mean separation in the solar spectrum is less by 0.0015 A than in terrestrial sources; for the remaining 32 pairs with a mean separation of 0.349 A, it is greater by 0.0003 A. In both cases the mean separations in sun and arc are practically identical.

The mean difference in separation without regard to sign, 0.003 A, and individual differences are larger, however, than one

[1] St. John and Ware, *Mt. Wilson Contr.*, No. 120; *Astrophysical Journal*, **44**, 15, 1916.

would expect from accidental errors of measurement. The following considerations show that these large differences are systematic in sign and probably real. Three things appear distinctly in the Mount Wilson sun-arc observations:[1] (1) in groups *a* and *b* strong lines are displaced more than weak; (2) lines of groups

TABLE VII

COMPARATIVE SEPARATION IN SOLAR AND ARC SPECTRA

λ Rowland	Intensity, Element, or Group V	R	Separation Sun	Arc	Sun *minus* Arc	λ Rowland	Intensity, Element, or Group V	R	Separation Sun	Arc	Sun *minus* Arc
3647.561...	4	12	.422	.415	+0.007	4489.911..	Fe	Mn	.340	.340	0.000
3707.959...	5	5	.105	.102	+ .003	4654.672..	*a*	*d*	.120	.132	− .012
3711.364...	4	3	.184	.184	.000	4938.997..	4	2	.423	.426	− .003
3722.071...	3	2	.102	.100	+ .002	4939.416..	*d*	*a*	.449	.442	+ .007
3735.014...	40	4	.459	.460	− .001	4985.432..	3	3	.296	.294	+ .002
3743.508...	Fe	Ti	.108	.106	+ .002	5005.896	4	5	.406	.405	+ .001
3745.717...	8	6	.337	.340	− .003	5079.158..	*e*	*b*	.248	.245	+ .003
3748.408...	10	1	.227	.228	− .001	5107.619..	4	4	.191	.193	− .002
4000.403...	2	2	.204	.200	+ .004	5139.427..	4	4	.211	.212	− .001
4058.915...	Fe	Mn	.170	.173	− .003	5148.222..	*e*	*d*	.185	.190	− .005
4063.436...	4	20	.314	.310	+ .004	5169.069..	3	4	.129	.129	.000
4067.139...	5	3	.291	.296	− .005	5208.596..	Cr	Fe	.169	.170	− .001
4079.996...	3	3	.370	.370	.000	5227.043..	3	5	.322	.323	− .001
4106.420...	2	2	.171	.168	+ .003	5273.339..	3	2	.215	.214	+ .001
4136.678...	4	6	.475	.474	+ .001	5365.069..	*e*	*a*	.527	.533	− .006
4143.572...	4	15	.452	.450	+ .002	5404.028..	*a*	*e*	.319	.303	+ .016
4154.667...	4	4	.308	.310	− .002	5446.797..	Ti	Fe	.332	.334	− .002
4191.595...	6	3	.240	.240	.000	5463.174..	3	3	.319	.316	+ .003
4226.904...	Ca	4	.701	.710	− .009	5455.671..	*e*	*a*	.156	.162	− .006
4315.138...	Ti	Fe	.115	.115	.000	5476.500..	*a*	*d*	.280	.284	− .004
4427.266...	Ti	Fe	.213	.215	− .002	6136.829..	8	3	.373	.373	.000
4430.356...	1	3	.425	.422	+ .003	6400.217	8	2	.310	.311	−0.001
4454.552...	Fe	Ca	.399	.399	0.000						

45 lines, mean separation	0.291 A
17 lines, sum of positive differences	+ .064
20 lines, sum of negative differences	− .070
8 lines, zero difference	.000
Mean Δλ Sun − Δλ Arc	−0.0002 A

*c*5 and *d* are displaced to the violet; (3) lines of group *e* show the maximum displacement to the red. If, therefore, the components of a pair differ greatly in intensity or belong to different groups, whether the separation in the solar spectrum is greater or less than in the arc depends upon the configuration of the pair, provided the behavior of lines with closely adjacent companions is similar to that of free-standing lines. For example, the pair at λ 4939, with components *d* and *a*, should be wider

[1] *Mt. Wilson Contr.*, No. 93, p. 35; *Astrophysical Journal*, 41, 63, 1915.

apart in the sun, while the pair at λ 4654, with components *a* and *d*, should be closer in the sun than in the arc, as the observations show. Taking as a lower limit a 50 per cent difference in intensity and considering the characteristic behavior of the groups, one can predict the relative separation in solar and arc spectra for 23 pairs as shown in Table VIII. The homogeneity of the two classes seems conclusive evidence that lines with close companions conform to the behavior of isolated lines of the same group and that their relative position in the solar spectrum is, therefore, not determined by mutual influence.

TABLE VIII

Classification of Solar Separations Based upon the Normal Behavior of Isolated Lines

Predicted Separation, Solar > Arc

λ Rowland	Sep.	Intensity or Group V	Intensity or Group R	Sep. Sun *minus* Sep. Arc
3647.561......	.422	4	12	+0.007
4063.436......	.314	4	20	+ .004
4136.678......	.475	4	6	+ .001
4143.572......	.452	4	15	+ .002
4430.356......	.425	1	3	+ .003
4939.416......	.449	*d*	*a*	+ .007
5227.043......	.322	3	5	− .001
5404.028......	.319	*a*	*e*	+0.016
Mean.........	.397	....		+0.005 A

Predicted Separation, Solar < Arc

λ Rowland	Sep.	Intensity or Group V	Intensity or Group R	Sep. Sun *minus* Sep. Arc
3735.014......	.459	40	4	−0.001
3745.717......	.337	8	6	− .003
3748.408......	.227	10	1	− .001
4067.139......	.291	5	3	− .005
4191.595......	.240	6	3	.000
4226.904......	.701	20	4	− .009
4654.672......	.120	*a*	*d*	− .012
4938.997......	.423	4	2	− .003
5079.158......	.248	*e*	*b*	+ .003
5148.222......	.185	*e*	*d*	− .005
5365.069......	.527	*e*	*a*	− .006
5455.671......	.156	*e*	*a*	− .006
5476.500......	.280	*a*	*d*	− .004
6136.829......	.373	8	3	.000
6400.217......	.310	8	2	−0.001
Mean.........	.325	....		−0.0035A

IV. THE SYSTEMATIC DEVIATIONS OF ALBRECHT

Dr. Albrecht plotted the differences between the Rowland and the "corrected" International wave-lengths of the iron lines against wave-lengths as abscissae,[1] and found that the values for lines with violet and red companions were, respectively, above and below the curve, requiring, when the average distance between the lines and their companions was 0.22 A, corrections of −0.005 and +0.007 A to reduce them to the mean. He saw no reason

[1] *Astrophysical Journal*, 44, 14, 1916.

for considering these systematic deviations to be due to the rôle played by personal equation in the measurement of close pairs. He regarded them, therefore, as evidence of anomalous dispersion in the sun.

As the extensive Mount Wilson data for sun-arc displacements and for the comparative separations of the components of close pairs in solar and terrestrial spectra indicate, within the limits of precision, a total absence of mutual influence, it seems necessary to conclude either that the Mount Wilson data, though depending upon concordant results from several observers and methods, are affected by a personal equation introducing systematic errors just balancing the effects of mutual influence, or that the Rowland wave-lengths for lines in close pairs, depending upon a single observer, are systematically in error. A slight over-spacing of such pairs, displacing the components in opposite directions, would introduce an effect of the sign required by the theory. An investigation of the accuracy obtainable in the measurement of close solar pairs and of possible errors in the Rowland table has lately been carried out at this Observatory.[1] For the 30 pairs composed of lines of intensities 3 and 4 which formed the basis of the investigation, the Rowland separations are larger than those found at Mount Wilson; the sign of the Rowland errors is such as would explain the Albrecht deviations as errors in the Rowland tables. For separations of 0.274, 0.145, and 0.075 A the errors are +0.003, +0.005, and +0.013 A, respectively, a march of magnitude with proximity of components similar to that observed by Albrecht.

The investigation showed that spectrograms of the finest definition yield the lowest values for the separation of the components of doublets near the limit of resolution, that whatever decreases the intensity of the common region relatively to that of the continuous spectrum produces a tendency on the part of the measurer toward increased separation.

Professor Aitken finds in measures upon close double stars with 12- and 36-inch telescopes the same tendency toward over-

[1] St. John and Ware, *Mt. Wilson Contr.*, No. 120; *Astrophysical Journal*, 44, 15, 1916.

separation with increased overlap of the stellar images, as appears in measures of incompletely resolved solar pairs. I am under obligation to him for the following data:

SEPARATIONS OF DOUBLE STARS

Under 1."0 12-Inch *minus* 36-Inch			Between 1."0 and 2."0 12-Inch *minus* 36-Inch		
33 plus	residuals	+2."47	10 plus	residuals	+0."85
7 minus	"	−0.45	18 minus	"	−1.46
2 zero	"	0.00	4 zero	"	0.00
42	"	+2.02	32	"	−0.61
Mean	"	+0.05	Mean	"	−0.02

Concerning these data, Professor Aitken says:

The theoretical resolving power of the 12-inch is 0."39, hence, taking distorted images into account, due to bad seeing, pairs with angular separations less than 1."0 would have little space between the two images and the closer pairs would usually be in contact, or would overlap. For pairs over 1."0 and certainly for pairs over 1."25, there would always be clear space between the images on any night I would use for work. I think the tabulation shows that there is a systematic tendency to overmeasure small distances with the 12-inch telescope—assuming that the 36-inch measures are exact—a tendency which disappears with distances of 1″ or more. The small negative residual (12-inch−36-inch) for the larger distances is obviously of a more accidental character than the positive residual for the smaller distances, since we have 18 minus and 10 plus signs in the one case against 33 plus and 7 minus signs in the other. This result is in harmony with many other similar comparisons I have made both of my own measures and of measures by different observers at about the same epoch with telescopes of different apertures.

These results point to an explanation of the systematic deviations observed by Albrecht, based upon errors in the Rowland wave-lengths for lines in close pairs. An extensive investigation of all the cases occurring in Albrecht's tables was undertaken. From experience gained in obtaining the data in *Contribution* No. 120, it has been possible to take advantage of the best available conditions for observations upon such difficult objects as close solar pairs. The wave-lengths of the solar lines used by Albrecht have been referred to those of the neighboring free-standing lines. All measurements have been made by two observers upon spectrograms of a scale and dispersion that experience has shown to be the most

reliable in each particular case. In every instance the data depend upon three or more concordant plates.

From the characteristic behavior of the Fe lines in the arc, it is evident that conclusions drawn from the differences between the Rowland and International wave-lengths must carry the maximum weight when based upon the stable lines of groups *a*, *b*, and *c*4. In the discussion of the Albrecht deviations, these lines, therefore, are considered separately from the unstable lines of groups *c*5, *d*, and *e*, for which the errors in the International wave-lengths are known to be systematic in sign and relatively large. According to the theory the effect of mutual influence increases with decreasing separation of the reacting solar lines; at the same time, however, the difficulties of measurement become increasingly great. As the evidence in such cases is relatively of great importance, pairs for which the separation from the companion is less than 0.1 A are given separate consideration. The discussion of the two series of data follows under heads, A, B, C, and D.

A. *Groups a, b, and c*4.—The general result from Table IX is that for 88 lines the sum of the Albrecht deviations is 0.377 A, and that of the counterbalancing Rowland errors is 0.351 A, a mean unbalanced deviation of 0.0003 A. The data, however, vary in reliability. For the 54 lines of greater weight, groups *a*, *b*, and *c*4 in Part I, the sum of Albrecht deviations is 0.226 A, that of the Rowland errors, 0.255 A. That the Albrecht deviations and the Rowland errors are mutually explanatory appears clearly from the parallelism between them. In Table X the largest Albrecht deviations are in the first column and the smallest in the third, the Rowland errors for the same lines being in the second and fourth columns of the two parts of the table. For the 27 lines showing the smallest Albrecht deviations the mean mutual influence is zero, and the Rowland error correspondingly small. If the larger deviations are due to mutual influence, a lack of parallelism should be conspicuous for them, but here the correspondence is particularly close. Among the 27 lines with the largest deviations, discrepancies are pronounced for λ 5195 and λ 5476 indicated by "?" in the table. These lines have companions to the red for which, on the hypothesis of mutual influence, large displacements are characteristic. Their

TABLE IX

THE ALBRECHT SYSTEMATIC DEVIATIONS AND ERRORS IN THE ROWLAND TABLE

PART I. LINES OF GROUPS *a*, *b*, AND *c*4. SECTION A. COMPANION TO VIOLET

λ Rowland	Int. Ratio Comp. to Line	ΔλR.	λ Mt.W.	Mt.W.−R.	Albrecht	Remarks
3647.988.....	4:12	0.427	.986	−0.002	+0.002	
3680.069.....	2: 9	.248	.062	− .007	− .007	
3737.281.....	5:30	.222	.280	− .001	+ .003	
3746.058.....	8: 6	.341	.054	− .004	− .008	
3748.650.....	10: 1	.242	.646	− .004	− .008	
3887.196.....	3: 7	.254	.198	+ .002	.000	
3888.671.....	2: 5	.111	.663	− .008	− .008	
3895.803.....	3: 7	.220	.805	+ .002	.000	
3969.413.....	6:10	.527	.410	− .003	− .004	Comp. in shade of λ 3968
4132.235.....	2:10	.135	.222	− .013	− .012	
4134.840.....	3: 5	.251	.836	− .004	+ .006	
4144.038.....	4:15	.466	.029	− .009	− .005	
4147.836.....	2: 4	.334	.831	− .005	+ .002	
4191.843.....	6: 3	.248	.839	− .004	+ .003	
4291.630.....	2: 2	.354	.625	− .005	+ .007	
4308.081.....	3: 6	.174	.065	− .016	− .006	
4315.262.....	3: 4	.124	.252	− .010	− .007	
4427.482.....	2: 5	.216	.476	− .006	.000	
4531.327.....	2: 5	.204	.326	− .001	− .002	
4556.306.....	3: 4	.243	.309	+ .003	+ .002	
4592.840.....	2: 4	.133	.831	− .009	− .010	
4787.003.....	3: 2	.276	.994	− .009	− .012	
4789.849.....	2: 3	.321	.840	− .009	− .012	
4939.868.....	2: 3	.452	.867	− .001	− .002	
5028.308.....	1: 2	.369	.310	+ .002	.000	
5041.255.....	3: 4	.186	.250	− .005	− .002	
5041.936.....	2: 4	.141	.943	+ .007	+ .001	
5079.409.....	3: 4	.251	.402	− .007	− .006	
5098.885.....	1: 3	.134	.876	− .009	− .005	
5107.823.....	4: 4	.204	.820	− .003	− .004	Quality excellent in sun and arc
5167.678.....	15: 5	.181	.669	− .009	− .016	Very difficult*
5227.362.....	3: 5	.319	.355	− .007	− .003	
5270.558.....	3: 4	.120	.541	− .017	− .026	Very difficult
5273.558.....	3: 2	.219	.555	− .003	− .014	
5333.089.....	1: 4	.240	.088	− .001	+ .005	
5365.596.....	5: 3	.527	.593	− .003	+ .003	
5447.130.....	2: 6	.333	.125	− .005	+ .003	
5455.834.....	2: 4	.163	.810	− .024	− .007	
6191.779.....	6: 9	0.386	.775	−0.004	−0.004	
Mean, 39 lines..........................				−0.0054	−0.0039	

SECTION B. COMPANION TO RED

λ Rowland	Int. Ratio Comp. to Line	ΔλR.	λ Mt.W.	Mt.W.−R.	Albrecht	Remarks
3705.708.....	2: 9	0.141	.714	+0.006	+0.007	
3735.014.....	4:40	.471	.013	− .001	+ .004	
3743.508.....	2: 6	.118	.511	+ .007	+ .006	
3834.364.....	4:10	.142	.377	+ .013	+ .009	
4337.216.....	3: 5	.509	.216	.000	+ .002	
4454.552.....	5: 3	.401	.553	+ .001	+ .003	
4461.818.....	3: 4	.347	.824	+ .006	+ .008	
4489.911.....	3: 4	.342	.910	− .001	+ .004	
4679.027.....	2: 6	.382	.029	+ .002	+ .005	
5107.619.....	4: 4	.204	.627	+ .008	+ .008	Quality excellent in sun and arc*
5195.113.....	2: 4	.534	.113	.000	+ .006	Arc λ depends on unstable standards
5208.596.....	2: 5	.180	.598	+ .002	+ .003	
5328.236.....	2: 8	.279	.235	− .001	− .007	
5476.500.....	3: 1	.278	.496	− .004	+ .011	Weak line 0.11 to violet vitiates measures
6136.829	3: 8	0.381	.835	+0.006	+0.004	
Mean, 15 lines..........................				+0.0029	+0.0049	

* *Astrophysical Journal*, 44, 21 and 30, 1916.

TABLE IX—*Continued*

PART II. LINES OF GROUPS $c5$ AND d. SECTION A. COMPANION TO VIOLET

λ Rowland	Int. Ratio Comp. to Line	ΔλR.	λ Mt.W.	Mt.W.−R.	Albrecht	Remarks
4227.606.....	20:4	0.702	.596	−0.010	−0.004	
4233.772.....	4:6	.444	.769	− .003	+ .002	
4872.332.....	1:4	.220	.329	− .003	− .006	
4957.785.....	5:8	.305	.785	.000	− .004	
4985.730.....	3:3	.298	.729	− .001	+ .003	
5006.306.....	4:5	.410	.303	− .003	− .002	
5139.644.....	4:4	.217	.642	− .002	+ .002	
5208.770.....	5:2	.180	.766	− .010	− .006	
5603.186.....	3:4	.103	.178	− .008	− .011	
5615.877.....	2:6	.357	.870	− .007	− .004	
5659.052.....	1:4	0.299	.042	−0.010	−0.003	
Mean, 11 lines..........				−0.0052	−0.0030	

SECTION B. COMPANION TO RED

λ Rowland	Int. Ratio Comp. to Line	ΔλR.	λ Mt.W.	Mt.W.−R.	Albrecht	Remarks
4191.595.....	3:6	0.248	.598	+0.003	+0.011	
4637.685.....	4:5	.508	.685	.000	+ .004	
4707.457.....	2:5	.215	.459	+ .002	+ .006	
4938.997.....	2:4	.419	.996	− .001	+ .005	
4957.480.....	8:5	.305	.481	+ .001	+ .003	
4982.682.....	2:4	.312	.682	.000	+ .013	
4985.432.....	3:3	.298	.433	+ .001	+ .009	
5005.896.....	5:4	.410	.898	+ .002	+ .002	
5125.300.....	1:3	.123	.296	− .004	− .002	
5139.427.....	4:4	.217	.431	+ .004	+ .006	
5273.339.....	2:3	.219	.343	+ .004	+ .004	
5476.778.....	5:3	.345	.775	− .003	+ .015	
6400.217.....	2:8	0.321	.221	+0.004	+0.001	
Mean, 13 lines..........				+0.0010	+0.0059	

PART III. LINES OF GROUP e. SECTION A. COMPANION TO VIOLET

λ Rowland	Int. Ratio Comp. to Line	ΔλR.	λ Mt.W.	Mt.W.−R.	Albrecht	Remarks
5195.647.....	4:2	0.534	.644	−0.003	−0.001	
5404.357.....	2:5	.329	.344	− .013	− .012	
5463.494.....	3:3	.320	.488	− .006	− .002	
5594.884.....	4:1	.193	.882	− .002	+ .009	
6020.401.....	2:4	0.169	.391	−0.010	−0.008	
Mean, 5 lines..........				−0.0068	−0.0028	

SECTION B. COMPANION TO RED

λ Rowland	Int. Ratio Comp. to Line	ΔλR.	λ Mt.W.	Mt.W.−R.	Albrecht	Remarks
5365.069.....	3:5	0.527	.068	−0.001	0.000	
5463.174.....	3:3	.320	.169	− .005	+ .009	
5598.524.....	4:1	.187	.525	+ .001	+ .008	
6008.186.....	6:4	.599	.186	.000	+ .012	
6078.710.....	2:5	0.517	.707	−0.003	−0.002	
Mean, 5 lines..........				−0.0016	+0.0054	

significance in this case is, however, open to question, for λ 5195, intensity 4, is 0.534 A from its companion of intensity 2, and λ 5476, intensity 1, stands between two lines, a weaker one, 0.113 A to the violet, a stronger one, 0.278 A to the red. The arc wavelength of λ 5195 is derived from unstable standards of group d,

and is subject to the errors attaching to them. Of the arc wave-length of λ 5476 St. John and Ware remark, "Near diffuse line λ 5476.587. Separation difficult."[1] Notes upon the observing records for its solar wave-length indicate that the measurements are influenced by the weak violet companion which, even on plates of high dispersion, is contiguous to the violet edge and tends to produce a fictitious shift to the violet.

TABLE X

CORRESPONDENCE BETWEEN ALBRECHT DEVIATIONS AND ROWLAND ERRORS BASED UPON LINES OF GROUPS *a*, *b*, AND *c*4 IN TABLE IX

COMPANION TO VIOLET				COMPANION TO RED			
20 Largest Deviations		19 Smallest Deviations		7 Largest Deviations		8 Smallest Deviations	
Albrecht	λMt.W. *minus* λR.	Albrecht	λMt.W. *minus* λR.	Albrecht	λMt.W. *minus* λR.	Albrecht	λMt.W. *minus* λR.
−0.007	−0.007	+0.002	−0.002	+0.007	+0.006	+0.004	−0.001
− .008	− .004	+ .003	− .001	+ .006	+ .007	+ .002	.000
− .008	− .004	.000	+ .002	+ .009	+ .013	+ .003	− .001
− .008	− .008	.000	+ .002	+ .008	+ .006	+ .004	− .001
− .004	− .003	+ .006	− .004	+ .008	+ .008	+ .005	− .002
− .012	− .013	+ .002	− .005	+ .006?	.000?	+ .003	+ .002
− .005	− .009	+ .003	− .004	+0.011?	−0.004?	− .007	− .001
− .006	− .016	+ .007	− .005	....	....	+0.004	−0.006
− .007	− .010	.000	− .006				
− .010	− .009	− .002	− .001				
− .012	− .009	+ .002	+ .003				
− .012	− .009	− .002	− .001				
− .006	− .007	.000	+ .002				
− .005	− .009	− .002	− .005				
− .004	− .003	+ .001	+ .007				
− .016	− .009	+ .003	− .007				
− .026	− .017	+ .005	− .001				
− .014	− .003	− .003	− .003				
− .007	− .024	+0.003	−0.005				
−0.004	−0.004	....	....				
SUMS AND MEANS							
−0.181 −0.0090	−0.177 −0.0088	+0.028 +0.0015	−0.034 −0.0018	+0.055 +0.0079 (7) +0.0076 (5)	+0.036 +0.0051 (7) +0.0080 (5)	+0.018 +0.0022	+0.008 +0.0010

B. *Groups c*5, *d*, *and e*.—As to the 34 lines of the unstable groups *c*5, *d*, and *e*, the situation is so complicated that an attempt to follow its intricacies would be profitless. Four considerations are involved: (1) The systematic errors in the Rowland values tending to account for the deviations found by Albrecht. The average error is 0.003 A, somewhat smaller than for lines of groups

[1] *Mt. Wilson Contr.*, No. 61, p. 28; *Astrophysical Journal*, **36**, 41, 1912.

a, *b*, and *c*4. Such a difference is consistent with the relative appearance of the lines in arc and solar spectra, for the lines of groups *c*5, *d*, and *e*, diffuse, unsymmetrical, and difficult to measure in the arc, are, in general, the sharpest and most accurately measurable in the solar spectrum. (2) The undetermined systematic errors in the International wave-lengths of these lines, positive for groups *c*5, and *d*, negative for group *e*. In recognition of these errors standard conditions for the arc were adopted at the Bonn meeting of the International Solar Union and redeterminations of the iron standards have been undertaken in various laboratories. Preliminary results at this Observatory indicate that the errors are systematic and of the sign expected. (3) The systematic errors in the data for pressure-shift. At the time of publication of the pressure data, the sensitiveness of these lines to varying arc conditions had not been recognized and the needed precautions against pole-effect were not taken. Recent observations under standard arc conditions give 0.008 A per atmosphere for groups *c*5 and *d* instead of the 0.022 A previously published. For the lines of group *e* the later result is +0.002 A per atmosphere instead of the negative value −0.016 A.[1] (4) Albrecht's effort to make the International and Rowland wave-lengths homogeneous by "correcting" the International values to a pressure of half an atmosphere. That this operation still left systematic differences appears in Table XI based upon data from Albrecht's Table I. For reasons given later and because the points fall so far from the curve that they have had little if any influence on its course, λ 4204, λ 4210, and λ 5371 have not been taken into account.

A reference-curve based upon $\Delta\lambda'$ for all lines is on the average too high for the lines of groups *c*5 and *d*, whose proportional representation is small. This tends to increase their deviations to the violet and to decrease those to the red. Such a tendency shows in Part II of Table IX, where the mean deviation to the violet is 0.006 A, that to the red 0.003 A, while the Rowland errors are 0.001 and 0.005 A, respectively. A lowering of the curve by 0.004 A would bring the deviations and errors into practical agreement. As a systematic effect of this order is indicated by the

[1] *Mt. Wilson Contr.*, No. 106, p. 16; *Astrophysical Journal*, 42, 231, 1915.

negative residuals in Table XI, curves have been drawn for the lines of groups *c*5 and *d* alone, using Δλ and Δλ′ from Albrecht's Table I as ordinates. The mean displacement for the same lines of these groups with companions to the red, deduced from either curve, is 0.002 A instead of 0.006 A, a result in close agreement with the corresponding Rowland error of 0.001, while the mean for the lines with companions to the violet remains practically unchanged. This systematic effect, introduced by referring the lines of groups *c*5 and *d* to a curve based largely upon lines of other

TABLE XI

Systematic Variation from Homogeneity in Δλ′

Region	Group	Δλ	Δλ′	Δλ′ Group *c*5, *d* *minus* Δλ′ Group *a*, *b*
4200–4300	*c*5, *d*......	.159	.165	+0.001
	a, *b*.......	.163	.164	
5000–5100	*c*5, *d*......	.165	.173	−0.007
	a.........	.178	.180	
5100–5200	*d*.........	.155	.168	−0.004
	a.........	.170	.172	
5200–5300	*d*.........	.164	.175	−0.002
	a.........	.175	.177	
5300–5400	*d*.........	.177	.187	−0.009
	a.........	.194	.196	
6300–6500	*d*.........	.195	.212	−0.004
	b.........	.212	.216	

groups, is apparently of a sign and magnitude to account for the inequality of the displacements for lines with red and violet companions, which Albrecht considered the principal objection to an explanation based upon personality in the Rowland measures.[1]

C. *Lines less than 0.1 A from companion.*—Of the 104 lines considered by Albrecht, the remaining 16 are separated from the companion line by less than 0.1 A. The data relative to these very close pairs are given in Table XII. The wide range in the deviations, +0.048 to −0.002 A for the 6 lines with companions

[1] *Astrophysical Journal*, 41, 355, 1915.

TABLE XII

Lines Less than 0.1 A from Companion

PART I. COMPANION TO VIOLET

λ Rowland	Int.	Element and Class	ΔλR	λ Mt.W.	Mt.W. *minus* Rowland	Albrecht	Remarks
3722.639	3...	Ni					
(.692)	10..		0.090				
.729	6...	Ti–Fe, *a*		.714	−0.015	−0.014	Resolved only with best definition. Fe line a blend with titanium
4294.204	2...	Ti					
....			0.097				
.301	5...	Fe, *b*		.287	−0.014	−0.005	
4407.810	2...	V					
....			0.061				
.871	4...	Fe, *c4*		.878	+0.007	+0.011	
4668.243	2...						
....			0.088				
.331	4...	Fe, *d*		.322	−0.009	−0.010	
4727.582	3...	Fe					
....			0.094				
.676	2...	Mn		.662	−0.014	−0.029	In arc, Mn λ 4727.476 Kilby, .478 Janicki, .464 Burns; Fe .408 Janicki, .410 Burns; blend .438 Kayser, .434 Goos, .435 "unreliable" St. John and Ware; weight 0, Albrecht
4878.313	3...	Ca					
....			0.094				
.407	4...	Fe, *c5*		.406	−0.001	−0.008	
5202.439	2...	Fe?					
....			0.077				
.516	4...	Fe, *b*		.510	−0.006	−0.006	
5371.656	4...	Cr?					
....			0.078	.692			
.734	3...	Fe, *a*			−0.042	−0.042	Misidentification by Rowland. Not a pair in solar spectrum but a single line due to Fe
6147.950	2...						
....			0.090				
8.040	3...	Fe, *d*		.038	−0.002	−0.005	
6254.382	1...						
....			0.074				
.456	5...	Fe, *b*		.479	+0.023	+0.017	Doubtful, Albrecht

PART II. COMPANION TO RED

λ Rowland	Int.	Element and Class	ΔλR	λ Mt.W.	Mt.W. *minus* Rowland	Albrecht	Remarks
3997.547	4...	Fe, *b*		.549	+0.002	+0.001	
....			0.091				
.638	2...						
4204.101	3...	Fe, *b*		.106	+0.005	+0.048	Only incipiently separated from red companion by the highest resolving power used. Blend in solar spectrum. High dispersion shows weak line contiguous to violet edge. Burns gives two lines in arc 4203.953, intensity 1, and .985, intensity 3, of which the stronger alone was used by Albrecht. Both things contribute to the abnormal displacement to violet
....			0.062				
.163	4...	La					

TABLE XII—*Continued*

PART II (*Continued*). COMPANION TO RED

λ Rowland	Int.	Element and Class	$\Delta\lambda R$	λ Mt.W.	Mt.W. *minus* Rowland	Albrecht	Remarks
4210.494	4...	Fe, *c*5	....	.499	+0.005	+0.027	Resolution incipient. Blend in solar spectrum. With high dispersion a line contiguous to the violet edge is conspicuous. The measurements, being upon the blend of the two, give the apparent shift to the violet
....	......	....	0.067	....	....	....	
.561	3...	....	....	....	....	....	
4476.185	4...	Fe, *b*	....	.182	−0.003	+0.008	Resolution incipient. In the arc, "Unreliable. Hazy on the red edge," St. John and Ware; "Ur," Goos; "Not suitable for standard," Janicki; "Close double in vacuum," Babcock
....	......	....	0.008	....	....	....	
.253	3...	Ag	....	....	....	....	
4727.582	3...	Fe, *d*	....	.584	+0.002	+0.003	
....	......	....	0.094	....	....	....	
.676	2...	Mn	....	....	....	....	
5012.252	4...	Fe, *a*	....	.250	−0.002	−0.002	
....	......	....	0.083	....	....	....	
.335	1...	....	....	....	....	....	

to the red, and −0.041 to +0.017 A for the 10 lines with companions to the violet, indicates at once the low weight to be assigned to the data as evidence of mutual influence. Three pairs of lines, companions to the red, λ 4204, λ 4210, and λ 4476, are at the limit of spectrographic resolution for lines of these intensities. The probable errors are therefore large.[1] The record of observations in the last column furnishes sufficient grounds for omitting them from a definitive discussion. The three other pairs with companions to the red are measurable with high precision and the conditions are apparently favorable to the appearance of mutual influence; the mean displacement to the violet, however, is minute, 0.0007 A, and balanced by the Rowland errors.

Of the 10 lines with companions to the violet, 4 are of doubtful or no weight in a definitive discussion; λ 3722 itself is a blend and is in a complex very difficult of resolution; the arc wave-length of λ 4727 is uncertain; the indicated pair at λ 5371 is a single line; λ 6254 is not separable from its companion upon a fourth-order spectrogram, upon which the head of the oxygen band at λ 6276 is completely resolved, a resolution not reached by the Rowland

[1] St. John and Ware, *Mt. Wilson Contr.*, No. 120; *Astrophysical Journal*, 44, 15, 1916.

plates for that region. The deviations for the remaining 6 lines are balanced by Rowland errors.

The most favorable condition for the appearance of mutual influence is in the case of lines with companions at the closest distance consistent with precision of measurement, as the probability is then greatest that the affected lines are situated upon a steep portion of the "anomaly-curve" due to the adjacent line.[1] Observations upon lines that, aside from closeness to a companion line, are free from known sources of error are consequently of crucial importance in their bearing upon the hypothesis of anomalous

TABLE XIII

Correspondence between Albrecht Deviations and Rowland Errors for the Smallest Measurable Separations, Lines Free from Extraneous Sources of Error

Companion to Red				Companion to Violet			
λ Rowland	Δλ	λMt.W. *minus* λR.	Albrecht	λ Rowland	Δλ	λMt.W. *minus* λR.	Albrecht
3997.547...	0.085	+0.002	+0.001	4294.301..	0.078	−0.014	−0.005
4727.582...	0.084	+0.002	+0.003	4407.871..	.068	+ .007	+ .011
5012.252...	0.082	−0.002	−0.002	4668.331..	.077	− .009	− .010
				4878.407..	.092	− .001	− .008
Means.....	0.083	+0.0007	+0.0007 A	5202.516..	.077	− .006	− .006
				6148.040..	0.099	−0.002	−0.005
				Means....	0.082	−0.0042	−0.0038 A

dispersion in the solar atmosphere, and failure here would indicate that, within the present attainable precision in measurement, anomalous dispersion does not systematically alter the relative positions of the Fraunhofer lines. Data for the 9 lines fulfilling these conditions are given in Table XIII. Their average separation is 0.082 A; ratio of companion to line, 3.2:2.7; weight, according to Albrecht, 1.7. The displacement to the violet for the lines with companions to the red should be larger than the displacement to the red for lines with companions to the violet. The deviation to the violet is, however, vanishingly small, much less than that to the red, and both are balanced by the corresponding Rowland errors.

[1] *Astrophysical Journal*, 43, 53, note, 1916.

D. *Correlation.*—The correspondence between the Albrecht deviations and the Rowland errors appears distinctly in Tables X and XIII. A line-to-line comparison for these 63 lines, to which a maximum of weight may be ascribed, shows a striking parallelism. A direct correspondence between the 104 Albrecht deviations and the Rowland errors for the same lines is shown to be a practical certainty by the large correlation coefficient $+0.56$ and its small probable error ± 0.05.

The facts observed by Albrecht and the corresponding results for this investigation are strikingly complementary.

ALBRECHT	ST. JOHN
Fraunhofer lines as given in Rowland's table are displaced when they have close companions.	Rowland wave-lengths of lines with close companions are systematically in error.
a) The displacement is to the violet when the adjacent line is to the red.	*a′*) The sign of the error is negative when the adjacent line is to the red.
b) The displacement is to the red when the adjacent line is to the violet.	*b′*) The sign of the error is positive when the adjacent line is to the violet.
c) The displacement in (*a*) is greater than in (*b*).	*c′*) A systematic excess in the Albrecht displacement to the violet for lines of groups *c*5 and *d* tends to make (*a′*) greater than (*b′*).
d) The displacement increases as the separation between the lines diminishes.	*d′*) The Rowland error increases as the separation of the lines diminishes.
e) The displacement is inappreciable for separation somewhat under 0.7 A.	*e′*) The Rowland errors cease to be systematic for separation somewhat under 0.5 A.

There seems to be no explanation of the *pari passu* march of the Albrecht observations and those recorded in Section IV of this paper other than that they are two phases of the same phenomenon.

V. ADJACENT LINES DUE TO DIFFERENT ELEMENTS

Under the title, "Mutual Repulsion of Contiguous Spectrum Lines," Sir Joseph Larmor says:

> Thus *very close spectrum lines ought to repel each other*, to a degree that experience alone can reveal.

But this conclusion implies that the adjacent lines represent independent vibrations. If they are two components of a single compound vibration of the molecule, the argument is not applicable.

If an iron line in the solar spectrum has a very close adjacent line *due to another substance*, while in the arc spectrum that substance is not present, then a displacement of the solar line relative to the arc line may be looked for.[1]

If such a mutual repulsion is operative to a measurable degree in the solar atmosphere, the sun-arc displacements of the Fe lines should be greater for the red and less for the violet components of a pair when the adjacent line is due wholly or in part to another substance than when the influencing line is iron or weak and unknown.[2] The most dependable results should be given by lines of groups *a*, *b*, and *c*4, both because of their stability in the arc and because of their greater number. For these 85 lines a residual of 0.002 A would have considerable weight. For the lines of groups *c*5, *d*, and *e*, the errors due to their sensitiveness to arc conditions are relatively large, and definite indications of characteristic behavior require correspondingly larger residuals for a like number of lines. The comparison between the displacements of 72 Fe lines, Table XIV, under the influence of adjacent lines in which other substances are concerned with the 70 for which mutual influence is supposed to be smaller shows no differences greater than the limit of error.

Among the 45 pairs in Table VII for which the separations were measured in solar and arc spectra are 8 close pairs of lines that fulfil

[1] *Observatory*, 497, 103, 1916.

[2] Note.—According to Sir Joseph Larmor an increase of refractive power is accompanied by a lowering of the aethereal elasticity which results in an increase of the free period and consequently a real displacement of the red component of a close solar pair toward longer wave-length. Similar considerations indicate a displacement of the violet component toward shorter wave-length. Such changes in vibration-frequency occur only when the anomalous refractive power is due to an adjacent line originating in an independent vibrating system, that is, in general, to a line of another element. The view of Professor Julius is that, owing to a mutual influence of two close Fraunhofer lines, the refractive power of the medium for the spectral region between them is decreased, less R- and V-light is anomalously refracted or scattered away from the observer on their opposed edges, and the centers of gravity of the dispersion bands are consequently farther apart. As the mutual influence consists simply in the superposition of the anomalous refractive powers due to the two absorption lines, the displacements should be independent of the elements involved. Julius adds the proviso that the elements coexist in the mixture at the same levels.

the Larmor conditions, as the arc wave-lengths for the different elements were determined in separate arcs. For these 8 pairs, mean separation 0.23 A (Table XV), there is no systematic difference between the separations in the sun and in independent

TABLE XIV

SUN-ARC DISPLACEMENTS IN RELATION TO ORIGIN OF THE ADJACENT LINE

(1) Companion	(2) Group	(3) No. of Lines	(4) Sun-Arc	(5) No. of Lines	(6) Sun-Arc	(7) Remarks
To violet.....	*a, b, c4*	30	+0.0040	26	+0.0031	Favorable by 0.001 A
To red.......	*a, b, c4*	19	+ .0041	10	+ .0031	Unfavorable by 0.001 A
To violet.....	*c5, d*	10	− .0031	11	− .0072	Three abnormal values included
		9	− .0037	9	− .0047	Three abnormal values omitted
To red.......	*c5, d*	10	− .0066	15	− .0060	Equal within limit of error
To violet.....	*e*	1	+ .013	5	+ .011	Equal within limit of error
To red.......	*e*	2	+0.016	3	+0.015	Equal within limit of error

(4) Adjacent line wholly or in part not iron.
(6) Adjacent line iron or weak and unknown.

TABLE XV

SEPARATIONS IN THE SOLAR SPECTRUM COMPARED WITH SEPARATIONS DETERMINED FROM INDEPENDENT ARCS WHEN THE ADJACENT LINE IS DUE TO ANOTHER ELEMENT

λ Rowland	Elements	Separation		Sep. Sun *minus* Sep. Arc
		Sun	Arc	
3743.508........	Fe, Ti	.108	.106	+0.002
4058.915........	Fe, Mn	.170	.173	− .003
4315.138........	Ti, Fe	.115	.115	.000
4427.266........	Ti, Fe	.213	.215	− .002
4454.552........	Fe, Ca	.399	.399	.000
4489.911........	Fe, Mn	.340	.340	.000
5208.596........	Cr, Fe	.169	.170	− .001
5446.797........	Ti, Fe	.332	.334	−0.002
Means..........	....	.231	.232	−0.0008

terrestrial sources, and the absence of mutual repulsion is still shown by the 4 closer pairs, mean separation 0.14 A. It is evident that the effect is not within the present means of observation.

In a recent number of the *Astrophysical Journal*[1] Albrecht deduces the following unweighted results from the systematic

[1] *Astrophysical Journal*, 44, 1, 1916.

deviations between the Rowland and the "corrected" International wave-lengths for Fe lines with close companions:

Companion not Fe	Sum of Deviations	Companion Fe	Sum of Deviations
20 lines, comp. to R. 0.008.....	0.160	18 lines, comp. to R. 0.0055....	0.099
41 " " " V. 0.0047....	0.193	23 " " " V. 0.003.....	0.069
61 " total sum............	0.353	41 " total sum............	0.168
Mean....................	0.0055	Mean....................	0.0041

and concludes that "this material reduction of the displacement for pairs in which both components are due to iron is distinctly in line with Larmor's theory."

The data show an apparent Larmor effect of 0.0014 A. There are included, however, 3 lines whose evidence in a definitive discussion is open to question, namely, λ 4204, λ 4210, and λ 5371. The Albrecht deviations for them are 0.048, 0.027, and 0.042 A. These average 8 times the mean for the 102 lines. Aside from their extraordinary magnitudes, other grounds for assigning them a very low weight have been given in a previous section and are here briefly restated. λ 5371 is not in a solar pair with Cr but is a single line due to iron; λ 4204 and λ 4210 are only partially resolved from their red companions even on excellent fifth-order spectrograms; each has, moreover, a hitherto unnoted weak line contiguous to its violet edge; λ 4204 is double in the arc, the red component only was used in the comparison, and λ 4210 is subject to pole-effect. These conditions all combine to produce abnormal violet displacements. If these three lines are omitted, the indication of a Larmor effect disappears, as the following tabulation shows:

Companion not Fe	Sum of Deviations	Companion Fe	Sum of Deviations
20 − 2 = 18 lines comp. to R.....	0.085	18 lines comp. to R...........	0.099
41 − 1 = 40 " " " V.....	0.151	23 " " " V............	0.069
58 " total sum......	0.236	41 " total sum.............	0.168
Mean....................	0.0041	Mean....................	0.0041

Other lines giving extraordinary values, namely, λ 3722, λ 5167, λ 5270, and λ 6254, are practically in the same category as the three mentioned above. Their omission does not change the evidence against the Larmor effect. The inconsistency between

0.004 A given by the 99 lines and 0.039 A given by 3 lines is so pronounced that one seems compelled to choose between them. The straight means only are here considered, as the system of weighting used by Albrecht unfortunately assigns the greater weight to observations presenting the greater difficulties of execution. For example, a combined weight of 8 is assigned to these 3 questionable lines in obtaining his final result, the average weight of the 102 lines being 1.25, so that in the weighted mean their influence is more than doubled.

Albrecht remarks further:

> As the displacement has not entirely disappeared for pairs in which both lines are due to iron, we must conclude that the components of these pairs represent only in part actual physical connection in the molecule, and in part entirely independent vibrations.[1]

It is implied that if the components are due to one compound vibration, that is, to a single element, and originate at the same level, no displacement would occur. This seems to be a relinquishment of the Julius point of view upheld in the former paper and opposed to the assumption of Julius that mutual influence is operative only when the lines originate at the same level.[2]

VI. PRESSURE IN THE SOLAR ATMOSPHERE

The pressure of 0.5 atmosphere found by Albrecht for the solar atmosphere was based upon the pressure-shifts for the iron lines, published previously to the appearance of his paper. *Contribution* No. 106 from this Observatory,[3] in which attention was called to the errors in the published data for groups $c5$, d, and e, due to pole-effect, was issued later. Using the old pressure-shifts of $+0.022$ A for groups $c5$ and d, and -0.016 A for group e, Albrecht obtained results of which he says:

> . . . . The correction of the wave-lengths in the International system to a pressure of 0.5 atmosphere has brought in toward the curve the lines which are strongly affected by pressure—namely, those of groups c, d, and e. Without the application of these corrections, the lines of groups c and d are decidedly

[1] *Astrophysical Journal*, **44**, 8, 1916. [2] *Ibid.*, **43**, 49, 1916.

[3] St. John and Babcock, "A Study of the Pole-Effect in the Iron Arc," *Mt. Wilson Contr.*, No. 106; *Astrophysical Journal*, **42**, 231, 1915.

below the curve, and those of group e are above the curve. This "drawing in" of the points toward the curve is gratifying, as it shows that the reduction of the wave-lengths in the International system to a pressure of 0.5 atmosphere has made them practically homogeneous with Rowland's wave-lengths in the sun—except, of course, for the systematic differences between the two systems as represented by the curve.[1]

A homogeneity attained by applying corrections based upon pressure data in which the errors are comparable to the original lack of homogeneity can hardly be regarded, in the light of fuller knowledge, as a justification of the method. That the practical homogeneity is still affected by systematic deviations for the lines of groups $c5$ and d is apparent in Table XI, and the lack of homogeneity, when the later pressure-shifts are used, is shown in Table XVI. Even an assumed pressure of zero fails to make them homogeneous with the Rowland wave-lengths, but they become so at a

TABLE XVI

NON-HOMOGENEITY OF $\Delta\lambda'$ AT 0.5 ATMOSPHERE

Region	Group	$\Delta\lambda$	$\Delta\lambda'$ at 0.5 Atm.		$\Delta\lambda'$ at 5 Atm.
			Old Pressure Observations	Later Pressure Observations	
5202–5341	a	0.182	0.184	0.184	0.198
	d	0.166	0.177	0.170	0.198
5365–5476	a	0.208	0.210	0.210	0.224
	e	0.217	0.209	0.218	0.225

pressure of 5 atmospheres, a pressure in the solar atmosphere inconsistent, however, with the long series of sun-arc observations with which comparison is made in Table XVII. The discrepancies between the observed and calculated displacements are so large that it is evident that a homogeneity depending upon a correction for pressure only does not rest upon a substantial basis. In fact, so many elements enter into the problems raised by the differences between solar and arc wave-lengths that it seems advisable to obtain a much wider range of sun-arc and other data before entering upon a definitive discussion of the question of pressure in the solar

[1] *Astrophysical Journal*, 41, 351, 1915.

atmosphere. Of four suggested explanations of the differences between solar and arc wave-lengths, (1) pressure, (2) motion in line of sight, (3) difference in the gravitational fields of the sun and earth, (4) anomalous dispersion, Albrecht considers only the first. The omission of the fourth, a fundamental deduction from the hypothesis according to Julius, is remarkable in a paper purporting to establish anomalous dispersion in the sun.

TABLE XVII

SUN-ARC DISPLACEMENTS INCONSISTENT WITH A PRESSURE OF 5 ATMOSPHERES IN THE SUN

Group	No. of Lines	Calculated at 5 Atm.	Observed	O−C
a and *b*........	211	+0.016	+0.004	−0.012
c_5 and *d*........	125	+0.032	−0.006	−0.038
e..............	34	+0.007	+0.014	+0.007

The outstanding indications that appear to be established by solar observations are that no one cause furnishes a satisfactory explanation of the differences between the wave-lengths in solar and arc spectra, and that pressure, though intimately concerned, does not play the supremely predominant rôle formerly attributed to it. A conclusion that the pressure in the solar atmosphere where the Fe lines originate is 0.5 atmosphere appears questionable when no account has been taken of variation with the solar intensity of the lines and other suggested effects.

I wish to express my appreciation of the assistance in this long and exacting investigation rendered by Miss Ware, whose unflagging interest and conscientious work have made the investigation possible and greatly increased the weight of the observations.

SUMMARY AND CONCLUSIONS

1. The mean sun-arc displacements for 211 Fe lines of the stable groups *a*, *b*, and c_4 is +0.0038 A; that for 56 lines with companions to the violet, mean separation 0.275 A, is +0.0036 A; that for 29 lines with companions to the red, mean separation 0.320 A, is +0.0038 A.

2. The mean sun-arc displacement for 125 lines of groups $c5$ and d is -0.0063 A; that for 25 lines in the same region with companions to the red is -0.0062 A. The mean displacement for 21 lines with companions to the violet is -0.0052 A; that for all $c5$ and d lines in the same region is -0.0050 A.

3. The mean sun-arc displacement for 34 lines of group e is $+0.0142$ A; that for 5 lines with companions to the red and 6 with companions to the violet is $+0.0156$ and $+0.0110$ A, respectively.

4. The mean separation in the solar spectrum for 45 pairs is, within the limits of error, identical with that in arc spectra.

5. The behavior of lines with companions is like that of similar isolated lines. Whether the separation in the solar spectrum is greater or less than in arc spectra depends upon the configuration of the pair. For 8 pairs of the 45 it should be larger, for 15 smaller; it is respectively 0.005 A greater and 0.0035 A less than in arc spectra.

6. For 54 lines of groups a, b, and $c4$, separation >0.1 A, the sum of the systematic displacements found by Albrecht is 0.226 A, that of the counterbalancing errors in the Rowland wave-lengths is 0.255 A.

7. For 34 lines of groups $c5$, d, and e, separation >0.1 A, the sum of the systematic displacements is 0.151 A; corrected for systematic error it is 0.099 A; that of the balancing errors is 0.096 A.

8. Of the 16 remaining lines, separation <0.1 A, 7 are without weight in a definitive discussion. The sum of the systematic displacements for the remaining 9 lines, separation from companions 0.082 A, is 0.025 A; that of the balancing Rowland errors is 0.027 A.

9. The coefficient of correlation between the 104 displacements attributed by Albrecht to mutual influence and the 104 errors in Rowland wave-lengths found in this investigation is $+0.56\pm0.05$.

10. The sun-arc displacements for iron lines are independent of the origin of closely adjacent lines.

11. The separations of iron lines from those due to another substance are the same in solar and arc spectra.

12. From 102 lines Albrecht finds a small Larmor effect which disappears when 3 lines giving inconsistently large values are omitted.

13. The later recognized systematic errors in the published data on pressure-shift invalidate conclusions based upon them.

14. The correspondence between the errors in the Rowland wave-lengths of lines with close companions and the displacements noted by Albrecht is so complete that it appears a practical certainty that these displacements are another measurement of the errors.

15. They therefore furnish no valid evidence that the relative positions of the Fraunhofer lines are systematically displaced by mutual influence. On the other hand, the sun-arc displacements (370 lines) and the relative separations of the components of 45 close pairs in solar and arc spectra indicate that, within the limits of error, evidence of mutual influence is absent from the general solar spectrum, and in so far as mutual influence is a necessary corollary of anomalous dispersion evidence for the existence of the latter is also absent.

Mount Wilson Solar Observatory
August 1916

REVIEWS

A Meteorological Treatise on the Circulation and Radiation in the Atmospheres of the Earth and of the Sun. By F. H. BIGELOW. New York: John Wiley & Sons, Inc., 1915. Pp. xi+431. 78 figures. $5.00.

At present there is a lack of textbooks on meteorology in which an adequate account of the mathematical theory is given, and a student who is laboriously endeavoring to piece together the fragments of theory which are scattered in the numerous periodicals will warmly welcome a mathematical treatise written by an experienced hand. Professor Bigelow's book contains not only an account of work which has become classical; it is devoted chiefly to a presentation of the results of his own researches and deals with a large number of problems which are of present-day interest. It deserves, then, to be reviewed as an important contribution to the elucidation of these questions.

The author first points out that Boyle's law for a perfect gas in which the pressure P, the density ρ, and the absolute temperature T are connected by the relation $P=\rho RT$ does not agree with observations made at different heights unless R is supposed to vary with the height above sea-level. The physical reason for this variation is indicated on p. 80, where it is stated that air is not an ideal gas but rather a mixture of gases which are undergoing rapid changes of condition through variations in the heat contents by insolation and radiation. The processes taking place in the atmosphere are thus not generally adiabatic, and in the formula $dT=-adz$, for the variation of the temperature with the height, the non-adiabatic gradient a is generally less than the adiabatic gradient a_0.

The formulae of the kinetic theory of gases for a mixture of several gases are given in chap. i: they indicate that R varies with the composition of the air. This idea of the variation of the gas coefficient R is fundamental in Mr. Bigelow's thermodynamical theory. The relation $R=C_p-C_v$, between R and the specific heats at constant pressure and constant volume, is adopted as usual and the ratio k of the two specific heats is assumed to be constant, although strictly this also varies slightly

with the composition of the air, as is indicated by Capstick's formula (*Philosophical Transactions*, 1894), which has been applied to a mixture of air and water-vapor by G. Schweikert (*Annalen der Physik*, 48, 593, 1915) and by Leduc's modified formula (*Comptes Rendus*, 160, 316, 1915), which holds when the gases are not perfect.

The author proceeds on the foregoing assumptions to develop some useful formulae for the computation of P, ρ, and R from the observed values of T and gives tables to illustrate his methods of computation. In the derivation of the non-adiabatic formulae, in which $n=a_0/a$ occurs as a variable, there are some points which are a little obscure. The approximation

$$\int \frac{dP}{\rho}=\frac{P_1-P_0}{\rho_{10}}, \qquad \rho_{10}=\tfrac{1}{2}(\rho_1+\rho_0)$$

which is made on p. 63 requires justification, and on p. 58 the term involving $\log T_0$ seems to be dropped in the transition from equation (188) to (189).

By introducing a term representing the heat lost by radiation into the hydrodynamical equation for the kinetic energy, and combining this equation with his thermodynamical equations, Mr. Bigelow obtains a number of interesting relations which he applies to the study of radiation and circulation in the atmosphere; the radiation being expressed by a formula analogous to that occurring in Stefan's law, except that the index is not equal to four.

With regard to the isothermal layer Mr. Bigelow says: "The principal fact to be explained is the slow rate of loss of heat in the convectional region as compared with that in the isothermal region." To explain this the author makes use of his equation $Q_1'-Q_0'=C_{p10}\,(T_1-T_0)$ for the evolution of heat during the vertical convection of air from a place where the temperature is T_0 to a place where it is T_1; C_{p10} here denotes the mean value of the specific heat at constant pressure. Thus if Q_1-Q_0 is the natural loss of heat by radiation without convection, this quantity represents the loss of heat in the isothermal region, but in the convectional region the loss of heat is $(Q_1-Q_0)-(Q_1'-Q_0')$. The author illustrates his ideas by discussing the diurnal convection and the semi-diurnal waves in the lower strata and the thermodynamic structure of cyclones and anticyclones.

The hydrodynamical equations of motion are next discussed and the author finds that two interesting types of vortices, which he describes as funnel-shaped and dumb-bell-shaped, may be specified by means

of the current functions $\psi = A\tilde{\omega}^2 z$ and $\psi = A\tilde{\omega}^2 \sin az$, z and $\tilde{\omega}$ being cylindrical co-ordinates and A and a constants. The motions in certain waterspouts, tornadoes, and cyclones which are described in chap. iv appear to resemble closely these two types of vortex motion. In the next two chapters the author deals with a variety of topics, such as the measurement of the intensity of solar radiation by means of the pyrheliometer and bolometer, the ionization of the atmosphere and measurement of conductivity, the diurnal convection in the earth's atmosphere, the diurnal variations of the meteorological, electrical, and magnetic elements, the laws of evaporation, polarization of sunlight, and solar physics. The author describes and discusses the theory of some of the instruments used in these branches of physics. His remarks on Bouguer's formula of depletion are interesting, the author being inclined to the view that the true "solar constant" is more than four calories per square centimeter per minute instead of two, the number which is usually adopted. The treatment of atmospheric electricity is to some extent unorthodox.

It is impossible to do justice in a short review to a work which covers such a vast field of research. The numerous tables and instructions for making computations will be invaluable to the student in observational work. There are also many interesting diagrams, all of which are well drawn.

In his tables of constants the author is generally up to date. In Table 7, p. 31, the formula for the conduction coefficient, viz., $L = 1.667\ \eta C_v$, may perhaps need correction (cf. S. Chapman, *Philosophical Transactions* [A], **211**, 462, 1912).

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May 17, 1916

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